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PE | Chemical

Reference Handbook
Version 2.0

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PREFACE

About the Handbook

The Principles and Practice of Engineering (PE) Chemical exam is computer-based, and the *PE Chemical Reference Handbook* is the only resource material you can use during the exam. Reviewing it before exam day will help you become familiar with the charts, formulas, tables, and other reference information provided. You will not be allowed to bring your personal copy of the *PE Chemical Reference Handbook* into the exam room. Instead, the computer-based exam will include a PDF version of the handbook for your use. No printed copies of the handbook will be allowed in the exam room.

The PDF version of the *PE Chemical Reference Handbook* that you use on exam day will be very similar to this one. However, pages not needed to solve exam questions—such as the cover, introductory material, and exam specifications—will not be included in the exam version. In addition, NCEES will periodically revise and update the handbook, and each PE Chemical exam will be administered using the updated version.

The *PE Chemical Reference Handbook* does not contain all the information required to answer every question on the exam. Theories, conversions, formulas, and definitions that examinees are expected to know have not been included. The handbook is intended solely for use on the NCEES PE Chemical exam.

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Errata

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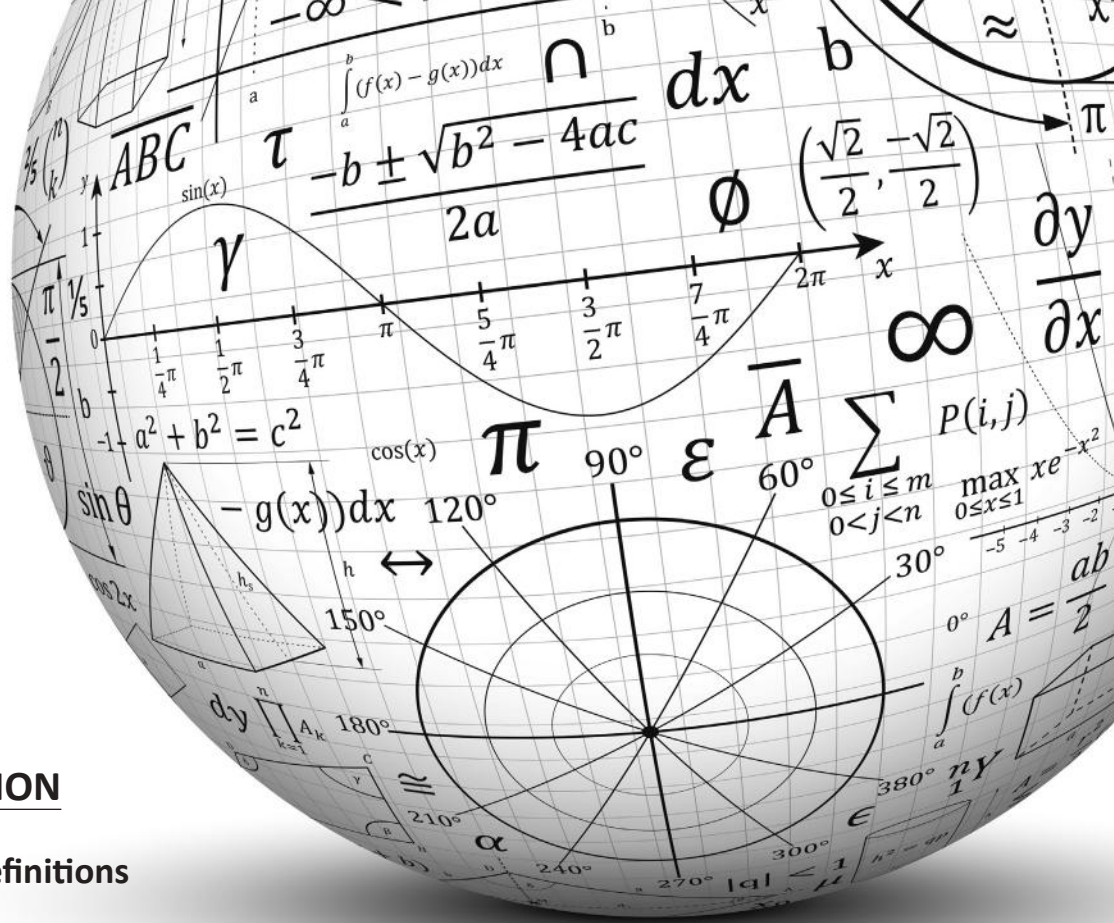
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1 GENERAL INFORMATION

1.1 Terms, Symbols, and Definitions

Symbols

Symbol	Description	Units (U.S.)	Units (SI)
A	Area <i>or</i> surface area	ft ²	m ²
a	Acceleration	$\frac{\text{ft}}{\text{sec}^2}$	$\frac{\text{m}}{\text{s}^2}$
c_i	Molar concentration	$\frac{\text{lb mole}}{\text{ft}^3}$	$\frac{\text{mol}}{\text{m}^3}$
c_p	Heat capacity	$\frac{\text{Btu}}{\text{lbm}^\circ\text{F}}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}} = \frac{\text{m}^2}{\text{s}^2 \cdot \text{K}}$
D	Diameter	ft <i>or</i> in.	m
D_{AB}	Mass diffusivity	$\frac{\text{ft}^2}{\text{hr}}$	$\frac{\text{m}^2}{\text{s}}$
d	Distance <i>or</i> diameter <i>or</i> diagonal	ft <i>or</i> in.	m
f	Friction factor (Darcy-Weisbach)	dimensionless	
f	Frequency	$\frac{1}{\text{sec}}$	$\frac{1}{\text{s}}$
g	Gravitational acceleration	$\frac{\text{ft}}{\text{sec}^2}$	$\frac{\text{m}}{\text{s}^2}$
h	Height	ft <i>or</i> in.	m
h	Convection heat-transfer coefficient	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{m}^2 \cdot \text{K}} = \frac{\text{kg}}{\text{s}^3 \cdot \text{K}}$
h_m	Mass-transfer coefficient	$\frac{\text{ft}}{\text{hr}}$	$\frac{\text{m}}{\text{s}}$
Δh_{fusion}	Latent heat of fusion	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}} = \frac{\text{m}^2}{\text{s}^2}$
Δh_{vap}	Latent heat of vaporization	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}} = \frac{\text{m}^2}{\text{s}^2}$

Symbols (cont'd)

Symbol	Description	Units (U.S.)	Units (SI)
k	Thermal conductivity	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{m} \cdot \text{K}} = \frac{\text{kg} \cdot \text{m}}{\text{s}^3 \cdot \text{K}}$
L	Length	ft <i>or</i> in.	m
MW	Molecular weight	$\frac{\text{lbm}}{\text{lb mole}}$	$\frac{\text{kg}}{\text{mol}}$
N	Number of moles	lb mole	mol
n	Impeller speed (revolutions per time)	$\frac{1}{\text{sec}}$	$\frac{1}{\text{s}}$
m	Mass	lbm	kg
P	Power	$\frac{\text{Btu}}{\text{hr}}$	$\text{W} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3}$
P	Pressure	$\frac{\text{lbf}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
P	Perimeter	ft <i>or</i> in.	m
P	Probability	dimensionless	
r, R	Radius	ft <i>or</i> in.	m
R	Universal gas constant	$\frac{\text{psi} \cdot \text{ft}^3}{\text{lb mole} \cdot ^\circ\text{R}}$ <i>or</i> $\frac{\text{Btu}}{\text{lb mole} \cdot ^\circ\text{R}}$	$\frac{\text{J}}{\text{mol} \cdot \text{K}}$
RD	Relative density	dimensionless	
SG	Specific gravity	dimensionless	
T	Temperature	$^\circ\text{F}$ <i>or</i> $^\circ\text{R}$	$^\circ\text{C}$ <i>or</i> K
t	Time	hr <i>or</i> sec	s
u	Velocity	$\frac{\text{ft}}{\text{sec}}$	$\frac{\text{m}}{\text{s}}$
u_{sound}	Local speed of sound	$\frac{\text{ft}}{\text{sec}}$	$\frac{\text{m}}{\text{s}}$
V	Volume	ft^3	m^3
W_i	Mass ratio	dimensionless	
w_i	Mass fraction <i>or</i> weight fraction	dimensionless	
X_i	Molar ratio	dimensionless	
x_i	Mole fraction	dimensionless	
x	Distance	ft <i>or</i> in.	m
$\alpha, \beta, \theta, \phi, \varphi$	Angle	degree <i>or</i> radians	
α	Thermal diffusivity	$\frac{\text{ft}^2}{\text{hr}}$	$\frac{\text{m}^2}{\text{s}}$
β	Coefficient of thermal expansion	$\frac{1}{^\circ\text{R}}$	$\frac{1}{\text{K}}$
γ_i	Mass concentration	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{kg}}{\text{m}^3}$

Symbols (cont'd)

Symbol	Description	Units (U.S.)	Units (SI)
γ	Surface tension	$\frac{\text{lbf}}{\text{in.}}$	$\frac{\text{N}}{\text{m}} = \frac{\text{kg}}{\text{s}^2}$
λ	Molecular mean free path	ft or in.	m
μ	Dynamic viscosity	cP or $\frac{\text{lbf-sec}}{\text{ft}^2}$	$\text{Pa} \cdot \text{s} = \frac{\text{kg}}{\text{m} \cdot \text{s}}$
ν	Kinematic viscosity	$\frac{\text{ft}^2}{\text{hr}}$	$\frac{\text{m}^2}{\text{s}}$
ρ	Density	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{kg}}{\text{m}^3}$
τ	Shear stress	$\frac{\text{lbf}}{\text{in}^2}$	$\frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
ϕ_i	Volume fraction	dimensionless	
ϕ_i	Volume concentration	$\frac{\text{ft}^3}{\text{ft}^3}$	$\frac{\text{m}^3}{\text{m}^3}$

1.1.1 Constants

Physical Constants

Symbol	Value	Units	Description
c_o	$6.706 \cdot 10^8$	$\frac{\text{miles}}{\text{hr}}$	Speed of light
	$2.998 \cdot 10^8$	$\frac{\text{m}}{\text{s}}$	
G	$3.44 \cdot 10^{-8}$	$\frac{\text{ft}^4}{\text{lbf-sec}^4}$	Gravitational constant
	$6.674 \cdot 10^{-11}$	$\frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$	
g	32.174	$\frac{\text{ft}}{\text{sec}^2}$	Gravitational acceleration (Earth)
	9.8067	$\frac{\text{m}}{\text{s}^2}$	
g_c	32.174	$\frac{\text{lbm-ft}}{\text{lbf-sec}^2}$	Gravitational conversion factor
	1	$\frac{\text{kg} \cdot \text{m}}{\text{N} \cdot \text{s}^2} = 1$	
k	$5.66 \cdot 10^{-24}$	$\frac{\text{ft-lbf}}{^\circ\text{R}}$	Boltzmann constant
	$1.3806 \cdot 10^{-23}$	$\frac{\text{J}}{\text{K}} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2 \cdot \text{K}}$	
	$1.3806 \cdot 10^{-16}$	$\frac{\text{erg}}{\text{K}}$	

Chapter 1: General Information

Physical Constants (cont'd)

Symbol	Value	Units	Description
N_A	$2.731 \cdot 10^{26}$	$\frac{1}{\text{lb mole}}$	Avogadro's Number
	$6.022 \cdot 10^{23}$	$\frac{1}{\text{mol}}$	
R	8.314	$\frac{\text{J}}{\text{mol} \cdot \text{K}} = \frac{\text{m}^3 \cdot \text{Pa}}{\text{mol} \cdot \text{K}}$	Universal gas constant
	83.14	$\frac{\text{cm}^3 \cdot \text{bar}}{\text{mol} \cdot \text{K}}$	
	8314	$\frac{\text{m}^3 \cdot \text{Pa}}{\text{kmol} \cdot \text{K}}$	
	82.06	$\frac{\text{cm}^3 \cdot \text{atm}}{\text{mol} \cdot \text{K}}$	
	0.0821	$\frac{\text{liter} \cdot \text{atm}}{\text{mol} \cdot \text{K}}$	
	62.36	$\frac{\text{liter} \cdot \text{Torr}}{\text{mol} \cdot \text{K}}$	
	62,360	$\frac{\text{cm}^3 \cdot \text{Torr}}{\text{mol} \cdot \text{K}}$	
	10.73	$\frac{\text{psi} \cdot \text{ft}^3}{\text{lb mole} \cdot ^\circ\text{R}}$	
	1.987	$\frac{\text{Btu}}{\text{lb mole} \cdot ^\circ\text{R}} = \frac{\text{cal}}{\text{mol} \cdot \text{K}}$	
	1545	$\frac{\text{ft} \cdot \text{lbf}}{\text{lb mole} \cdot ^\circ\text{R}}$	
	0.7302	$\frac{\text{atm} \cdot \text{ft}^3}{\text{lb mole} \cdot ^\circ\text{R}}$	
σ	$1.71 \cdot 10^{-9}$	$\frac{\text{Btu}}{\text{ft}^2 \cdot \text{hr} \cdot ^\circ\text{R}^4}$	Stefan-Boltzmann constant (radiation)
	$5.67 \cdot 10^{-8}$	$\frac{\text{W}}{\text{m}^2 \cdot \text{K}^4} = \frac{\text{kg}}{\text{s}^3 \cdot \text{K}^4}$	

Mathematical Constants

Symbol	Value	Description
π	3.14159	Archimedes constant (Pi)
e	2.71828	Base of the natural log
γ	0.57722	Euler's constant

Chapter 1: General Information

Standard Values

*Note: The definitions for STP (standard temperature and pressure) vary between industries.
The table below contains several conditions as specified.*

Property	Conditions	U.S. Units	SI Units
Molar standard volume, ideal gas (STP)	$P = 1 \text{ atm} = 14.696 \text{ psia}$ $T = 0^\circ\text{C} = 32^\circ\text{F}$	$359 \frac{\text{ft}^3}{\text{lb mole}}$	$0.0224 \frac{\text{m}^3}{\text{mol}}$ $22.41 \frac{\text{liter}}{\text{mol}}$
Molar standard volume, ideal gas (ambient)	$P = 1 \text{ atm} = 14.696 \text{ psia}$ $T = 15^\circ\text{C} = 59^\circ\text{F}$		$0.02365 \frac{\text{m}^3}{\text{mol}}$ $23.645 \frac{\text{liter}}{\text{mol}}$
Standard cubic foot (scf)	$P = 1 \text{ atm} = 14.696 \text{ psia}$ $T = 15.6^\circ\text{C} = 60^\circ\text{F}$	$379.49 \frac{\text{ft}^3}{\text{lb mole}}$	
Density of air (STP)	$P = 1 \text{ atm} = 14.696 \text{ psia}$ $T = 0^\circ\text{C} = 32^\circ\text{F}$	$0.0805 \frac{\text{lbm}}{\text{ft}^3}$	$1.29 \frac{\text{kg}}{\text{m}^3}$
Density of air (ambient)	$P = 1 \text{ atm} = 14.696 \text{ psia}$ $T = 15.6^\circ\text{C} = 60^\circ\text{F}$	$0.0764 \frac{\text{lbm}}{\text{ft}^3}$	$1.22 \frac{\text{kg}}{\text{m}^3}$
Density of air (ambient)	$P = 1 \text{ atm} = 14.696 \text{ psia}$ $T = 20^\circ\text{C} = 68^\circ\text{F}$	$0.0749 \frac{\text{lbm}}{\text{ft}^3}$	$1.20 \frac{\text{kg}}{\text{m}^3}$
Density of mercury	$P = 1 \text{ atm} = 14.696 \text{ psia}$ $T = 20^\circ\text{C} = 68^\circ\text{F}$	$848 \frac{\text{lbm}}{\text{ft}^3}$	$13,579 \frac{\text{kg}}{\text{m}^3}$
Density of water	$P = 1 \text{ atm} = 14.696 \text{ psia}$ $T = 4^\circ\text{C} = 39.2^\circ\text{F}$	$62.4 \frac{\text{lbm}}{\text{ft}^3}$	$1000 \frac{\text{kg}}{\text{m}^3}$
Density of water	$P = 1 \text{ atm} = 14.696 \text{ psia}$ $T = 15.6^\circ\text{C} = 60^\circ\text{F}$	$62.37 \frac{\text{lbm}}{\text{ft}^3}$	$999.0 \frac{\text{kg}}{\text{m}^3}$
Atmospheric pressure	Sea level	$14.696 \frac{\text{lbm}}{\text{in}^2}$	$1.013 \cdot 10^5 \text{ Pa}$
Triple point of water		32.02°F 0.0887 psia	0.01109°C 0.6123 kPa
Speed of sound in air (STP)	$P = 1 \text{ atm} = 14.696 \text{ psia}$ $T = 0^\circ\text{C} = 32^\circ\text{F}$	$1090 \frac{\text{ft}}{\text{sec}}$	$330 \frac{\text{m}}{\text{s}}$
Speed of sound in air (ambient)	$P = 1 \text{ atm} = 14.696 \text{ psia}$ $T = 20^\circ\text{C} = 68^\circ\text{F}$	$1130 \frac{\text{ft}}{\text{sec}}$	$343 \frac{\text{m}}{\text{s}}$
Energy of visible light	Wavelength: 555 nm	$1 \text{ cd} \cdot \text{sr} = 4.98 \cdot 10^{-3} \frac{\text{Btu}}{\text{hr}} *$	$1 \text{ cd} \cdot \text{sr} = 1.46 \cdot 10^{-3} \text{ W}$

* $\text{cd} \cdot \text{sr}$ = candela steradian; see derived SI units for definition

1.1.2 Dimensional Analysis

A dimensionally homogeneous equation has the same dimensions on the left and the right sides of the equation. Dimensional analysis involves the development of equations that relate dimensionless groups of variables to describe physical phenomena.

1.1.2.1 Buckingham Pi Theorem

The number of independent dimensionless groups that may be used to describe a phenomenon known to involve n variables is equal to the number $(n-\bar{r})$, where $\bar{r}$ is the number of basic dimensions (e.g., mass, length, time) needed to express the variables dimensionally.

1.1.2.2 Similitude

To use a model to simulate the conditions of the prototype, the model must be geometrically, kinematically, and dynamically similar to the system that is modeled. Systems that have the same dimensionless numbers are similar.

Dimensionless Numbers¹

Symbol	Definition	Name	Description
Ar	$\frac{gD_p^3\rho_f(\rho_p - \rho_f)}{\mu^2}$	Archimedes	Ratio of buoyancy forces to viscous forces for a particle (p) in a fluid (f)
Bi	$\frac{hL}{k} \text{ or } \frac{hV}{kA}$	Biot	Ratio of internal thermal resistance of a solid body to its surface thermal resistance (used for heat transfer)
Bi_m	$\frac{h_m L}{D_{AB}}$	Biot (mass transfer)	Ratio of the internal species transfer resistance to the boundary layer species transfer resistance (used for mass transfer)
Bm	$\frac{\tau_y g_c L}{\mu u}$	Bingham	Ratio of yield stress (τ_y) to viscous stress for Bingham fluids in laminar flow
Bo	$\frac{(\rho_l - \rho_v) g L^2}{\gamma}$	Bond	Ratio of buoyancy force to surface tension (used for boiling and condensation)
Br	$\frac{\mu u^2}{k \Delta T}$	Brinkman	Ratio of viscous dissipation to enthalpy change (for use in high-speed flow)
C_f	$\frac{\tau}{\frac{1}{2} \rho u^2}$	Drag or friction coefficient	Ratio of surface shear stress to free-stream kinetic energy; dimensionless surface shear stress
Ca	$\frac{\mu u}{\gamma} = \frac{We}{Re}$	Capillary	Ratio of viscous forces to surface tension (for use in two-phase flow)
Ca	$\frac{u^2}{u_{\text{sound}}^2} = Ma^2$	Cauchy	Ratio of inertia forces to compression forces (for use in compressible flow)
Ca	$\frac{P_{\text{ref}} - P_{\text{vap}}}{\frac{1}{2} \rho u^2}$	Cavitation	Ratio of pressure forces to inertia forces for pumps (special case of Euler number) with P_{ref} local absolute reference pressure
Ec	$\frac{u^2}{c_p \Delta T}$	Eckert	Kinetic energy of flow relative to boundary-layer enthalpy difference (for use in high-speed flow)
Eu	$\frac{\Delta P}{\rho u^2}$	Euler	Ratio of upstream and downstream pressure difference to inertia force
Fo	$\frac{\alpha t}{L^2}$	Fourier	Dimensionless time; ratio of rate of heat conduction to rate of internal energy storage in a solid (for use in transient heat-transfer problems)
Fo_m	$\frac{D_{AB} t}{L^2}$	Fourier (mass transfer)	Dimensionless time; ratio of the rate of species diffusion to the rate of species storage (for use in transient mass-transfer problems)
Fr	$\frac{u^2}{gL}$	Froude	Ratio of flow inertia to gravitational forces (for flow over a free surface)
f	$\frac{\Delta P}{\frac{L}{D} \rho \frac{1}{2} u^2}$	Friction factor	Ratio of shear force to inertia force; dimensionless pressure drop for internal flow

Dimensionless Numbers (cont'd)

Symbol	Definition	Name	Description
Ga	$\frac{g L^3}{\nu^2}$	Galilei	Ratio of gravitational forces to viscous forces
Gr	$\frac{g \beta \Delta T L^3}{\nu^2}$	Grashoff	Ratio of buoyancy to viscous forces (for use in natural convection)
Gz	$\frac{Re Pr}{\frac{x}{D}} = \frac{u L}{\alpha \frac{x}{D}}$	Graetz	Ratio of enthalpy flow rate to axial heat conduction
Ja_l	$\frac{c_{p,l} \Delta T}{\Delta h_{vap}}$	Jakob	Ratio of sensible heat to latent heat (for use in film condensation and boiling)
Ja_v	$\frac{c_{p,v} \Delta T}{\Delta h_{vap}}$		
j_H	$St Pr^{\frac{2}{3}}$	Colburn factor (heat)	Dimensionless heat-transfer coefficient
j_m	$St_m Sc^{\frac{2}{3}}$	Colburn factor (mass)	Dimensionless mass-transfer coefficient
Ka	$\frac{g \mu^4}{\rho \gamma^3}$	Kapitza	Ratio of surface tension forces to viscous forces (used for waves on a liquid film)
Kn	$\frac{\lambda}{L}$	Knudsen	Ratio of mean free path to a characteristic length (for use in non-continuum flow)
Le	$\frac{\alpha}{D_{AB}}$	Lewis	Ratio of molecular thermal diffusivity to mass diffusivity
Ma	$\frac{u}{u_{sound}}$	Mach	Dimensionless velocity; ratio of velocity to speed of sound (for use in compressible flow)
Nu	$\frac{h L}{k}$	Nusselt	Dimensionless heat-transfer coefficient; ratio of convection heat transfer to conduction in a fluid layer of thickness L (for use in convective heat transfer)
N_p	$\frac{P}{\rho n^3 D^5}$	Power number	Ratio of drag force to inertial force for power consumption calculation of a mixing impeller (where P is the impeller power)
Pe	$\frac{u_{\infty} L}{\alpha} = Re Pr$	Peclet	Ratio of enthalpy flow rate to heat conduction rate (for use in forced convection heat transfer)
Pe_m	$\frac{u_{\infty} L}{D_{AB}} = Re Sc$	Peclet (mass transfer)	Ratio of mass convection rate to mass diffusion rate (for use in forced convection mass transfer)
Pr	$\frac{c_p \mu}{k}$	Prandtl	Relative effectiveness of molecular transport of momentum and energy within the boundary layer; ratio of molecular momentum diffusivity to thermal conductivity (for use in convective heat transfer)
Ra	$\frac{g \beta \Delta T L^3}{\nu^2} Pr$	Rayleigh	Product of Grashoff and Prandtl numbers (for use in natural convection)
Re	$\frac{\rho u D}{\mu}$	Reynolds	Ratio of inertia and viscous forces (for use in forced convection and fluid flow)
Sc	$\frac{\nu}{D_{AB}}$	Schmidt	Ratio of molecular momentum diffusivity to mass diffusivity (for use in convective mass transfer)
Sh	$\frac{h_m L}{D_{AB}}$	Sherwood	Ratio of convection mass transfer to diffusion in a slab of thickness L (for use in convective mass transfer)

Dimensionless Numbers (cont'd)

Symbol	Definition	Name	Description
Sk	$\frac{\Delta P L}{\mu u}$	Stokes	Ratio of pressure force to viscous force
St	$\frac{Nu}{Re Pr} = \frac{h}{\rho u c_p}$	Stanton	Dimensionless heat-transfer coefficient, ratio of actual convection heat flux to enthalpy energy heat flux (for use in forced convection heat transfer)
St_m	$\frac{Sh}{Re Sc} = \frac{h_m}{u}$	Stanton (mass)	Dimensionless mass-transfer coefficient (for use in forced convection mass transfer)
Ste	$\frac{c_p \Delta T}{\Delta h_{\text{fusion}}}$	Stefan	Ratio of sensible heat to latent heat for the solid/liquid transition (for use in melting and solidification)
Sr	$\frac{L f}{u}$	Strouhal	Time characteristics of fluid flow (for use in oscillating flow)
We	$\frac{\rho u^2 L}{\gamma}$	Weber	Ratio of inertial to surface tension forces (for use in liquid/vapor phase change)

¹Verify whether gravitational conversion factor (g_c) is required before using.

1.2 Units of Measurement

1.2.1 Metric Prefixes

Metric Prefixes and Their Symbols

Multiple	Prefix	Symbol
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^1	deka	da
10^2	hecto	h
10^3	kilo	k
10^6	mega	M
10^9	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E

1.2.2 Base and Derived SI Units

Base SI Units

Quantity	Name	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	Kelvin	K
Amount of a substance	mol	mol
Luminous intensity	candela	cd

Derived SI Units With Special Names

Quantity		Unit		
Name	Symbol	Name	Symbol	Definition
Electric capacitance	C	farad	F	$F = \frac{C}{V} = \frac{A^2 \cdot s^2}{J} = \frac{A^2 \cdot s^4}{kg \cdot m^2}$
Electric charge	Q	coulomb	C	$C = A \cdot s$
Electric conductance	G	siemens	S	$S = \frac{1}{\Omega} = \frac{A}{V} = \frac{A^2 \cdot s}{J} = \frac{A^2 \cdot s^3}{kg \cdot m^2}$
Energy <i>or</i> work <i>or</i> heat	H	joule	J	$J = N \cdot m = \frac{kg \cdot m^2}{s^2}$
Force	F	newton	N	$N = \frac{kg \cdot m}{s^2}$
Frequency	f	hertz	Hz	$Hz = \frac{1}{s}$
Inductance	L	henry	H	$H = \Omega \cdot s = \frac{V \cdot s}{A} = \frac{kg \cdot m^2}{A^2 \cdot s^2}$
Electric potential	E	volt	V	$V = \frac{J}{A \cdot s} = \frac{kg \cdot m^2}{A^2 \cdot s^3}$
Power <i>or</i> energy flux	P	watt	W	$W = \frac{J}{s} = \frac{N \cdot m}{s} = \frac{kg \cdot m^2}{s^3}$
Pressure <i>or</i> stress	P	pascal	Pa	$Pa = \frac{N}{m^2} = \frac{kg}{m \cdot s^2}$
Electric resistance	R	ohm	Ω	$\Omega = \frac{V}{A} = \frac{kg \cdot m^2}{A^2 \cdot s^3}$
Illuminance		lux	lx	$lx = \frac{lm}{m^2} = \frac{cd \cdot sr}{m^2}$
Luminous flux	Φ_V	lumen	lm	$lm = cd \cdot sr$
Magnetic flux	Φ_E	weber	Wb	$Wb = V \cdot s = \frac{kg \cdot m^2}{s^2 \cdot A}$
Magnetic flux density		tesla	T	$T = \frac{Wb}{m^2} = \frac{V \cdot s}{m^2} = \frac{kg}{s^2 \cdot A}$

Note: Steradian or square radian (sr) is dimensionless and represents a solid angle in three-dimensional space (angle at the tip of a cone).

1.2.3 Unit Conversion Tables (U.S. and Metric)

1.2.3.1 Time

Time						
Time	Second (sec)	Minute (min)	Hour (hr)	Day	Week	Year
1 sec =	1	0.01667	2.7778E-04	1.1574E-05	1.6534E-06	3.1710E-08
1 min =	60	1	0.01667	6.9444E-04	9.9206E-05	1.9026E-06
1 hr =	3600	60	1	0.04167	5.9524E-03	1.1416E-04
1 day =	8.6400E+04	1440	24	1	0.14286	2.7397E-03
1 week =	6.0480E+05	1.0080E+04	168	7	1	0.01918
1 year =	3.1536E+07	5.2560E+05	8760	365	52.143	1

Additional Unit Conversions for Time

1 fortnight = $3.4560 \cdot 10^5$ sec = 14 days

1 astronomical year = 365.2422 days

1.2.3.2 Angle

Conversion Table for Common Units of an Angle

Angle	Degree (°)	rad	Minute (')	Second (")	Revolution
1° =	1	0.01745	60	3600	2.7778E-03
1 rad =	57.296	1	3437.7	2.0626E+05	0.15915
1' =	0.01667	2.9089E-04	1	60	4.6296E-05
1" =	2.7778E-04	4.8481E-06	0.01667	1	7.7161E-07
1 rev =	360	6.2832	2.1600E+04	1.2960E+06	1

1.2.3.3 Length

Conversion Table for Common Units of Length

Length	m	in.	ft	yd	mile	mil
1 m =	1	39.370	3.2808	1.0936	6.2135E-04	3.9370E+04
1 in =	0.0254	1	0.0833	0.02778	1.5782E-05	1000
1 ft =	0.3048	12	1	0.3333	1.8939E-04	1.2000E+04
1 yd =	0.9144	36	3	1	5.6816E-04	3.6000E+04
1 mile =	1609.4	6.3362E+04	5280.2	1760.1	1	6.3362E+07
1 mil =	2.5400E-05	0.001	8.3333E-05	2.7778E-05	1.5782E-08	1

Additional Unit Conversions for Length

1 league	=	4828.2 m	=	3 miles
1 μ (micron)	=	$1 \cdot 10^{-6}$ m	=	$3.937 \cdot 10^{-5}$ in.
1 mile (nautical)	=	1853.3 m	=	1.1515 miles
1 nautical league	=	5559.9 m	=	3 nautical miles
1 furlong	=	201.17 m	=	$\frac{1}{8}$ mile
1 perch = 1 rod = 1 pole	=	5.292 m	=	5.5 yds = $\frac{1}{4}$ chain
1 fathom	=	1.8288 m	=	6 ft = 2 yds
1 cable length (U.S. Survey)	=	219.456 m	=	120 fathoms = 240 yd
1 chain (U.S. Survey)	=	20.117 m	=	0.1 furlong
1 link	=	0.2012 m	=	0.001 furlong
1 cubit	=	0.4572 m	=	$\frac{1}{2}$ yard = 18 in.
1 bolt	=	36.576 m	=	40 yd
1 skein	=	109.728 m	=	120 yd
1 span	=	0.2286 m	=	9 in.
1 hand (horses)	=	0.1016 m	=	4 in.
1 caliber	=	$2.54 \cdot 10^{-4}$ m	=	$\frac{1}{100}$ in.
1 Å (Angström)	=	$1 \cdot 10^{-10}$ m	=	$3.937 \cdot 10^{-9}$ in.
1 fermi	=	$1 \cdot 10^{-15}$ m	=	$3.937 \cdot 10^{-14}$ in.
1 astronomical unit	=	$1.496 \cdot 10^{11}$ m	=	$9.2954 \cdot 10^7$ miles
1 light year	=	$9.4605 \cdot 10^{15}$ m	=	$5.8783 \cdot 10^{12}$ miles
1 mm	=	0.001 m	=	0.03937 in.
1 cm	=	0.01 m	=	0.3937 in.
1 km	=	1000 m	=	0.62135 mile

1.2.3.4 Area

Conversion Table for Common Units of Area

Area	m ²	in ²	ft ²	yd ²	acre	sq mile
1 m ² =	1	1550	10.764	1.196	2.4710E-04	3.8610E-09
1 in ² =	6.4516E-04	1	6.9444E-03	7.7160E-04	1.5942E-07	2.4910E-12
1 ft ² =	0.09290	144	1	0.1111	2.2957E-05	3.5870E-10
1 yd ² =	0.83613	1296	9	1	2.0661E-04	3.2283E-09
1 acre =	4046.9	6.2727E+06	4.3560E+04	4840	1	1.5625E-05
1 sq mile =	2.5900E+06	4.0145E+09	2.7879E+07	3.0976E+06	640	1

Additional Unit Conversions for Area

1 circ mil	=	5.067 • 10 ⁻¹⁰ m ²	=	$\frac{\pi}{4}$ sq.mil = 7.8539 • 10 ⁻⁷ in ²
1 circ inch	=	5.0671 • 10 ⁻⁴ m ²	=	$\frac{\pi}{4}$ in ² = 0.78539 in ²
1 ha (hectare)	=	1 • 10 ⁴ m ²	=	2.471 acres
1 township	=	9.324 • 10 ⁷ m ²	=	144 homesteads
1 homestead	=	6.475 • 10 ⁵ m ²	=	160 acres
1 rood	=	1011.725 m ²	=	0.25 acre
1 sq rod	=	25.2926 m ²	=	30.25 sq. yd
1 section	=	2.59 • 10 ⁸ m ²	=	1 sq. mile
1 barn (bn)	=	1 • 10 ⁻²⁸ m ²	=	100 fm ² (femtometer)
1 are	=	100 m ²	=	119.6 sq. yd
1 centiare	=	1 m ²	=	10.764 ft ²
1 mm ²	=	1 • 10 ⁻⁶ m ²	=	1.55 • 10 ⁻³ in ²
1 cm ²	=	1 • 10 ⁻⁴ m ²	=	0.155 in ²
1 km ²	=	1 • 10 ⁶ m ²	=	0.3861 sq. mile

1.2.3.5 Volume

Conversion Table for Common Units of Volume

Volume	m ³	in ³	ft ³	gal	barrel (oil)	liter
1 m ³ =	1	6.1024E+04	35.314	264.20	6.2901	1000
1 in ³ =	1.6387E-05	1	5.7870E-04	4.3295E-03	1.0308E-04	0.01639
1 ft ³ =	0.02832	1728	1	7.4814	0.17812	28.317
1 gal =	3.7850E-03	231	0.13367	1	0.02381	3.7850
1 barrel =	0.15898	9701.6	5.6143	42.0	1	158.98
1 liter =	0.001	61.024	0.03531	0.2642	6.2901E-03	1

Additional Unit Conversions for Volume

1 yd ³	=	0.7646 m ³	=	27 ft ³
1 register ton	=	2.8317 m ³	=	100 ft ³
1 dry gal (U.S.)	=	4.405 • 10 ⁻³ m ³	=	1.164 gal (U.S.)
1 U.S. bushel	=	0.0353 m ³	=	8 dry gal (U.S.) = 9.31 gal (U.S.)
1 quart (U.S.)	=	9.4625 • 10 ⁻⁴ m ³	=	$\frac{1}{4}$ gal (U.S.)
1 pint (U.S.)	=	4.7313 • 10 ⁻⁴ m ³	=	$\frac{1}{8}$ gal (U.S.) = $\frac{1}{2}$ quart (U.S.)
1 cup (U.S.)	=	2.3656 • 10 ⁻⁴ m ³	=	$\frac{1}{2}$ pint = $\frac{1}{16}$ gal (U.S.)
1 gill (U.S.)	=	1.1828 • 10 ⁻⁴ m ³	=	$\frac{1}{4}$ pint = $\frac{1}{32}$ gal (U.S.)
1 fl oz (U.S.)	=	2.9570 • 10 ⁻⁵ m ³	=	$\frac{1}{8}$ cup = $\frac{1}{128}$ gal (U.S.)
1 fl dram (U.S.)	=	3.6963 • 10 ⁻⁶ m ³	=	$\frac{1}{8}$ fl oz = $\frac{1}{1024}$ gal (U.S.)
1 minim (U.S.)	=	6.1605 • 10 ⁻⁸ m ³	=	$\frac{1}{60}$ dram = $\frac{1}{480}$ fl oz
1 cm ³ = 1 mL	=	1 • 10 ⁻⁶ m ³	=	0.06102 in ³
1 mm ³	=	1 • 10 ⁻⁹ m ³	=	6.1024 • 10 ⁻⁵ in ³
1 hectoliter	=	0.1 m ³	=	26.42 gal (U.S.)
1 hogshead	=	0.2385 m ³	=	63 gal (U.S.)
1 UK bushel	=	0.0364 m ³	=	8 dry gal (UK)
1 Imperial gal (UK)	=	0.0045 m ³	=	1.201 gal (U.S.)
1 quarter (UK)	=	0.291 m ³	=	64 gal (UK)
1 peck (UK)	=	0.0091 m ³	=	2 gal (UK)
1 quart (UK)	=	0.0011 m ³	=	$\frac{1}{4}$ gal (UK)
1 pint (UK)	=	5.6826 • 10 ⁻⁴ m ³	=	$\frac{1}{8}$ gal (UK)
1 barrel (UK)	=	0.1637 m ³	=	36 gal (UK) = 43 gal (U.S.)
1 barrel (U.S. liq)	=	0.1192 m ³	=	31.503 gal (U.S.) = 26 gal (UK)
1 barrel (U.S. dry)	=	0.1156 m ³	=	30.55 gal (U.S.)
1 cord (lumber)	=	3.625 m ³	=	128 ft ³
1 stere (lumber)	=	1 m ³	=	1.308 yd ³
1 boardfoot (lumber)	=	2.3597 • 10 ⁻⁶ m ³	=	$\frac{1}{12}$ ft ³

U.S. Conversion for Liquid Volume

1 gal (U.S.) = 4 quarts
 1 quart = 2 pints
 1 pint = 2 cups
 1 cup = 8 fl oz

U.S. Conversion for Dry Volume

1 cup = 16 tablespoons (Tbsp)
 1 Tbsp = 3 teaspoons (tsp)
 1 tsp = 8 pinches
 1 pinch = 2 dashes

1.2.3.6 Mass

Conversion Table for Common Units of Mass

Mass	kg	lbm	oz	ton (short)	ton (long)	slug
1 kg =	1	2.2046	35.273	1.1023E-03	9.8420E-04	0.06852
1 lbm =	0.45359	1	16	5.0000E-04	4.4642E-04	0.03108
1 oz =	0.02835	0.0625	1	3.1251E-05	2.7902E-05	1.9426E-03
1 ton (short) =	907.18	2000	3.1999E+04	1	0.89285	0.62162
1 ton (long) =	1016.1	2240	3.5840E+04	1.12	1	69.622
1 slug =	14.594	32.174	514.78	0.01608	0.014363	1

Additional Unit Conversions for Mass

1 hundredweight (short) = 45.3592 kg = 100 lbm
 1 hundredweight (long) = 50.8023 kg = 112 lbm
 1 tonne (metric) = 1000 kg = 2204.6 lbm
 1 centner = 100 kg = 220.5 lbm
 1 dram = $1.7719 \cdot 10^{-3}$ kg = 0.0625 oz
 1 grain = $6.4799 \cdot 10^{-5}$ kg = $2.2857 \cdot 10^{-3}$ oz
 1 carat = $2.0000 \cdot 10^{-4}$ kg = $7.0547 \cdot 10^{-3}$ oz
 1 atomic mass unit = $1.6605 \cdot 10^{-27}$ kg = $3.6608 \cdot 10^{-27}$ lbm
 $1 \frac{\text{kgf} \cdot \text{s}^2}{\text{m}}$ = 9.8067 kg = 21.62 lbm
 1 stone = 6.3503 kg = 14 lbm
 1 firkin = 40.8231 kg = 90 lbm
 1 lb (apothecary/troy) = 0.3732 kg = 13.166 oz = 12 oz (ap/troy) = 0.8229 lbm
 1 oz (apothecary/troy) = $3.1103 \cdot 10^{-2}$ kg = 1.0971 oz
 1 dram (apothecary) = $3.8879 \cdot 10^{-3}$ kg = 0.13714 oz
 1 scruple (apothecary) = $1.2960 \cdot 10^{-3}$ kg = 0.04571 oz
 1 grain (apothecary/troy) = $6.4799 \cdot 10^{-5}$ kg = $2.2857 \cdot 10^{-3}$ oz = $1.4286 \cdot 10^{-4}$ lbm
 1 carat (troy) = $2.0500 \cdot 10^{-4}$ kg = $7.231 \cdot 10^{-3}$ oz
 1 pennyweight (troy) = $1.5552 \cdot 10^{-3}$ kg = 0.05486 oz
 1 mite (troy) = $3.2400 \cdot 10^{-6}$ kg = $1.1428 \cdot 10^{-4}$ oz
 1 doite (troy) = $1.3500 \cdot 10^{-7}$ kg = $4.7618 \cdot 10^{-6}$ oz

Apothecary Measures

1 lb = 373.242 grain
 1 lb = 12 oz
 1 oz = 8 drams
 1 dram = 3 scruples
 1 scruple = 20 grains

Troy Measures

1 lb = 373.242 grain
 1 lb = 12 oz (ozt)
 1 ozt = 20 pennyweight (dwt)
 1 dwt = 24 grains
 1 grain = 20 mites
 1 mite = 24 doites

1.2.3.7 Density

Conversion Table for Common Units of Density

Density	$\frac{\text{kg}}{\text{m}^3}$	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{lbm}}{\text{gal}}$	$\frac{\text{kg}}{\text{liter}}$	$\frac{\text{lbm}}{\text{in}^3}$	$\frac{\text{ton}}{\text{yd}^3}$
$1 \frac{\text{kg}}{\text{m}^3} =$	1	0.06243	8.3452E-03	0.001	3.6128E-05	7.5250E-04
$1 \frac{\text{lbm}}{\text{ft}^3} =$	16.018	1	0.13367	0.01618	5.7870E-04	0.01205
$1 \frac{\text{lbm}}{\text{gal}} =$	119.83	7.481	1	0.11983	4.3292E-03	0.09017
$1 \frac{\text{kg}}{\text{liter}} =$	1000	62.43	8.3452	1	0.036128	0.7525
$1 \frac{\text{lbm}}{\text{in}^3} =$	2.7679E+04	1728	231	27.679	1	20.829
$1 \frac{\text{ton}}{\text{yd}^3} =$	1328.9	82.963	11.09	1.3289	0.048011	1

Additional Unit Conversions for Density

$1 \frac{\text{slug}}{\text{ft}^3} = 515.379 \frac{\text{kg}}{\text{m}^3} = 32.175 \frac{\text{lbm}}{\text{ft}^3}$
 $1 \frac{\text{g}}{\text{liter}} = 1 \frac{\text{kg}}{\text{m}^3} = 0.06243 \frac{\text{lbm}}{\text{ft}^3}$
 $1 \frac{\text{oz}}{\text{gal}} = 7.4906 \frac{\text{kg}}{\text{m}^3} = 0.46764 \frac{\text{lbm}}{\text{ft}^3}$
 $1 \frac{\text{grain}}{\text{ft}^3} = 0.0023 \frac{\text{kg}}{\text{m}^3} = 1.4286 \cdot 10^{-4} \frac{\text{lbm}}{\text{ft}^3}$
 $1 \frac{\text{lbm}}{\text{UK gal}} = 99.978 \frac{\text{kg}}{\text{m}^3} = 6.2416 \frac{\text{lbm}}{\text{ft}^3}$

Chapter 1: General Information

Specific gravity (also called relative density): The ratio of the density of a substance to the density of water at 4°C (39°F):

$$SG = \frac{\rho}{1000 \frac{\text{kg}}{\text{m}^3}} = \frac{\rho}{62.4 \frac{\text{lbm}}{\text{ft}^3}}$$

API gravity:

$$API = \frac{141.5}{SG_{60^\circ\text{F}}} - 131.5 \quad ; \quad SG_{60^\circ\text{F}} = \frac{141.5}{API + 131.5}$$

1.2.3.8 Specific Volume

Conversion Table for Common Units of Specific Volume

Specific Volume	$\frac{\text{m}^3}{\text{kg}}$	$\frac{\text{liter}}{\text{kg}}$	$\frac{\text{ft}^3}{\text{lbm}}$	$\frac{\text{gal}}{\text{lbm}}$	$\frac{\text{in}^3}{\text{lbm}}$	$\frac{\text{ft}^3}{\text{kg}}$
$1 \frac{\text{m}^3}{\text{kg}} =$	1	1000	16.018	119.76	2.7680E+04	35.314
$1 \frac{\text{liter}}{\text{kg}} =$	0.001	1	0.01602	0.11976	27.68	0.03531
$1 \frac{\text{ft}^3}{\text{lbm}} =$	0.06243	62.428	1	7.4764	1728	2.2046
$1 \frac{\text{gal}}{\text{lbm}} =$	8.3500E-03	8.35	0.13375	1	231	0.29488
$1 \frac{\text{in}^3}{\text{lbm}} =$	3.6127E-05	0.03613	5.7870E-04	4.3266E-03	1	1.2758E-03
$1 \frac{\text{ft}^3}{\text{kg}} =$	0.02832	28.317	0.45359	3.3913	783.81	1

1.2.3.9 Velocity

Conversion Table for Common Units of Velocity

Velocity	$\frac{\text{m}}{\text{s}}$	$\frac{\text{ft}}{\text{sec}}$	$\frac{\text{ft}}{\text{min}}$	$\frac{\text{miles}}{\text{hr}}$	$\frac{\text{km}}{\text{hr}}$	knots
$1 \frac{\text{m}}{\text{s}} =$	1	3.2808	196.85	2.2369	3.6	1.9423
$1 \frac{\text{ft}}{\text{sec}} =$	0.3048	1	60	0.68182	1.0973	0.592
$1 \frac{\text{ft}}{\text{min}} =$	5.0800E-03	0.01667	1	0.01136	0.018288	9.8667E-03
$1 \frac{\text{mile}}{\text{hr}} =$	0.44704	1.4667	88	1	1.6093	0.86827
$1 \frac{\text{km}}{\text{hr}} =$	0.27778	0.91134	54.681	0.62137	1	0.53952
$1 \text{ knot} =$	0.51486	1.6892	101.35	1.1517	1.8535	1

1.2.3.10 Acceleration

Conversion Table for Common Units of Acceleration

Acceleration	$\frac{\text{m}}{\text{s}^2}$	$\frac{\text{ft}}{\text{sec}^2}$	$\frac{\text{in.}}{\text{sec}^2}$	$\frac{\text{cm}}{\text{s}^2}$	g	$\frac{\text{km}}{\text{hr} \cdot \text{s}}$
$1 \frac{\text{m}}{\text{s}^2} =$	1	3.2808	39.37	0.01	0.10197	3.569
$1 \frac{\text{ft}}{\text{sec}^2} =$	0.3048	1	12	3.0480E-03	0.03108	1.0878
$1 \frac{\text{in.}}{\text{sec}^2} =$	0.0254	0.08333	1	2.5400E-04	2.5901E-03	0.09065
$1 \frac{\text{cm}}{\text{s}^2} =$	100	328.08	3937	1	10.197	356.9
$1 \text{ g} =$	9.8067	32.174	386.09	0.09807	1	35
$1 \frac{\text{km}}{\text{hr} \cdot \text{s}} =$	0.28019	0.91926	11.031	2.8019E-03	0.02857	1

1.2.3.11 Volumetric Flow

Conversion Table for Common Units of Volumetric Flow

Volumetric Flow	$\frac{\text{m}^3}{\text{s}}$	$\frac{\text{gal}}{\text{min}}$	$\frac{\text{ft}^3}{\text{hr}}$	$\frac{\text{barrel}^*}{\text{day}}$	$\frac{\text{MMgal}^{**}}{\text{day}}$	$\frac{\text{ft}^3}{\text{sec}}$
$1 \frac{\text{m}^3}{\text{s}} =$	1	1.5851E+04	1.2713E+05	5.4345E+05	22.8	35.314
$1 \frac{\text{gal}}{\text{min}} =$	6.3089E-05	1	8.0207	34.286	1.4384E-03	2.2280E-03
$1 \frac{\text{ft}^3}{\text{hr}} =$	7.8658E-06	0.12468	1	4.2747	1.7934E-04	2.7778E-04
$1 \frac{\text{barrel}}{\text{day}} =$	1.8401E-06	0.02917	0.23394	1	4.2000E-05	6.4982E-05
$1 \frac{\text{MMgal}}{\text{day}} =$	0.04386	695.2	5576	2.3835E+04	1	1.5489
$1 \frac{\text{ft}^3}{\text{sec}} =$	0.02832	448.84	3600	1.5389E+04	0.64563	1

* 1 barrel of oil = 42 gallons ** million gallons

Additional Unit Conversions for Volumetric Flow

$$\begin{aligned}
 1 \frac{\text{ft}^3}{\text{min}} &= 4.7195 \cdot 10^{-4} \frac{\text{m}^3}{\text{s}} = 7.4807 \frac{\text{U.S. gal}}{\text{min}} \\
 1 \frac{\text{gal}}{\text{hr}} &= 1.0515 \cdot 10^{-6} \frac{\text{m}^3}{\text{s}} = 0.01667 \frac{\text{U.S. gal}}{\text{min}} \\
 1 \frac{\text{UK gal}}{\text{min}} &= 7.5766 \cdot 10^{-5} \frac{\text{m}^3}{\text{s}} = 1.201 \frac{\text{U.S. gal}}{\text{min}} \\
 1 \frac{\text{UK gal}}{\text{hr}} &= 1.2628 \cdot 10^{-6} \frac{\text{m}^3}{\text{s}} = 0.02 \frac{\text{U.S. gal}}{\text{min}} \\
 1 \frac{\text{MM gal (UK)}}{\text{day}} &= 0.0526 \frac{\text{m}^3}{\text{s}} = 834.01 \frac{\text{U.S. gal}}{\text{min}} \\
 1 \frac{\text{m}^3}{\text{hr}} &= 2.778 \cdot 10^{-4} \frac{\text{m}^3}{\text{s}} = 4.403 \frac{\text{U.S. gal}}{\text{min}} \\
 1 \frac{\text{liter}}{\text{min}} &= 1.667 \cdot 10^{-5} \frac{\text{m}^3}{\text{s}} = 0.2642 \frac{\text{U.S. gal}}{\text{min}} \\
 1 \frac{\text{liter}}{\text{s}} &= 0.001 \frac{\text{m}^3}{\text{s}} = 15.852 \frac{\text{U.S. gal}}{\text{min}} \\
 1 \frac{\text{mliter}}{\text{s}} &= 10^{-6} \frac{\text{m}^3}{\text{s}} = 0.01585 \frac{\text{U.S. gal}}{\text{min}} \\
 1 \frac{\mu\text{liter}}{\text{s}} &= 10^{-9} \frac{\text{m}^3}{\text{s}} = 1.5851 \cdot 10^{-5} \frac{\text{U.S. gal}}{\text{min}}
 \end{aligned}$$

1.2.3.12 Mass Flow

Conversion Table for Common Units of Mass Flow

Mass Flow	$\frac{\text{kg}}{\text{s}}$	$\frac{\text{lbm}}{\text{hr}}$	$\frac{\text{lbm}}{\text{min}}$	$\frac{\text{kg}}{\text{hr}}$	$\frac{\text{MMlbm}^*}{\text{year}}$	$\frac{\text{ton (short)}}{\text{day}}$
$1 \frac{\text{kg}}{\text{s}} =$	1	7936.5	132.28	3600	69.524	95.238
$1 \frac{\text{lbm}}{\text{hr}} =$	1.2600E-04	1	0.01667	0.4536	8.7600E-03	0.012
$1 \frac{\text{lbm}}{\text{min}} =$	7.5600E-03	60	1	27.216	0.5256	0.72
$1 \frac{\text{kg}}{\text{hr}} =$	2.7778E-04	2.2046	0.03674	1	0.01931	0.02646
$1 \frac{\text{MMlbm}}{\text{year}} =$	0.01438	114.16	1.9026	51.781	1	1.3699
$1 \frac{\text{ton (short)}}{\text{day}} =$	0.0105	83.33	1.3889	37.8	0.73	1

* million pounds

Additional Unit Conversions for Mass Flow

$$\begin{aligned}
 1 \frac{\text{ton (long)}}{\text{day}} &= 0.0118 \frac{\text{kg}}{\text{s}} &= 930,333 \frac{\text{lbm}}{\text{hr}} \\
 1 \frac{\text{ton (short)}}{\text{hr}} &= 0.2520 \frac{\text{kg}}{\text{s}} &= 2000 \frac{\text{lbm}}{\text{hr}} \\
 1 \frac{\text{ton (long)}}{\text{hr}} &= 0.2822 \frac{\text{kg}}{\text{s}} &= 2240 \frac{\text{lbm}}{\text{hr}} \\
 1 \frac{\text{slug}}{\text{hr}} &= 4.0539 \cdot 10^{-3} \frac{\text{kg}}{\text{s}} &= 32.174 \frac{\text{lbm}}{\text{hr}} \\
 1 \frac{\text{lbm}}{\text{sec}} &= 0.4536 \frac{\text{kg}}{\text{s}} &= 3600 \frac{\text{lbm}}{\text{hr}}
 \end{aligned}$$

1.2.3.13 Mass Flux

Conversion Table for Common Units of Mass Flux

Mass Flux	$\frac{\text{kg}}{\text{s} \cdot \text{m}^2}$	$\frac{\text{kg}}{\text{hr} \cdot \text{m}^2}$	$\frac{\text{g}}{\text{s} \cdot \text{cm}^2}$	$\frac{\text{lbm}}{\text{hr} \cdot \text{ft}^2}$	$\frac{\text{lbm}}{\text{sec} \cdot \text{ft}^2}$	$\frac{\text{lbm}}{\text{sec} \cdot \text{in}^2}$
$1 \frac{\text{kg}}{\text{s} \cdot \text{m}^2} =$	1	3600	0.1	737.35	0.20482	1.4223E-03
$1 \frac{\text{kg}}{\text{hr} \cdot \text{m}^2} =$	2.7778E-04	1	2.7778E-05	0.20482	5.6894E-05	3.9510E-07
$1 \frac{\text{g}}{\text{s} \cdot \text{cm}^2} =$	10	3.6000E+04	1	7373.5	2.0482	0.01422
$1 \frac{\text{lbm}}{\text{hr} \cdot \text{ft}^2} =$	1.3562E-03	4.8823	1.3562E-04	1	2.7777E-04	1.9290E-06
$1 \frac{\text{lbm}}{\text{sec} \cdot \text{ft}^2} =$	4.8824	1.7577E+04	0.48824	3600	1	6.9444E-03
$1 \frac{\text{lbm}}{\text{sec} \cdot \text{in}^2} =$	703.07	2.5310E+06	70.307	5.1841E+05	144	1

1.2.3.14 Force

Conversion Table for Common Units of Force

Force	$\text{N} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$	lbf	$\text{pdl} = \frac{\text{lbm} \cdot \text{ft}}{\text{sec}^2}$	$\text{dyne} = \frac{\text{g} \cdot \text{cm}}{\text{s}^2}$	kgf = kilopond (kp)	ozf
1 N =	1	0.22481	7.233	1.0000E+05	0.10197	3.5969
1 lbf =	4.4482	1	32.174	4.4482E+05	0.45359	16
1 pdl =	0.13825	0.03108	1	1.3825E+04	0.01410	0.4973
1 dyne =	1.0000E-05	2.2481E-06	7.2330E-05	1	1.0197E-06	3.5969E-05
1 kgf =	9.8067	2.2046	70.932	9.8067E+05	1	35.274
1 ozf =	0.27801	0.0625	2.0109	2.7801E+04	0.02835	1

Additional Unit Conversions for Force

$$1 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = 1 \text{ dyne} = 1 \text{ N} = 0.22481 \text{ lbf}$$

$$1 \text{ dyne} = 1 \cdot 10^{-5} \text{ N} = 0.22481 \cdot 10^{-5} \text{ lbf}$$

$$1 \text{ tonf (long)} = 9964 \text{ N} = 2240 \text{ lbf}$$

$$1 \text{ tonf (short)} = 8896.44 \text{ N} = 2000 \text{ lbf}$$

$$1 \text{ kip} = 1 \text{ kilo lbf} = 4448.2 \text{ N} = 1000 \text{ lbf}$$

$$1 \text{ pond} = 1 \text{ p} = 0.0098 \text{ N} = 2.2046 \cdot 10^{-3} \text{ lbf}$$

1.2.3.15 Pressure/Stress

Conversion Table for Common Units of Pressure

Pressure	$\text{Pa} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$	$\text{psi} = \frac{\text{lbf}}{\text{in}^2}^*$	Torr = mmHg	in w. c.**	bar	atm
1 Pa =	1	1.4504E-04	7.5008E-03	4.0148E-03	1.0000E-05	9.8717E-06
1 psi*=	6894.8	1	51.716	27.681	0.06895	0.06806
1 Torr =	133.32	0.01934	1	0.53525	1.3332E-03	1.3161E-03
1 in w. c.** =	249.08	0.03613	1.8683	1	2.4908E-03	2.4588E-03
1 bar =	1.0000E+05	14.504	750.08	401.48	1	0.98717
1 atm =	1.0130E+05	14.696	759.83	406.7	1.013	1

*0 psig (gauge) = 14.696 psia (absolute) = 1 atm = $1.013 \cdot 10^5$ Pa

** inches water column

Additional Unit Conversions for Pressure and Stress

$$\begin{aligned}
 1 \frac{\text{N}}{\text{m}^2} &= 1 \frac{\text{kg}}{\text{m} \cdot \text{s}^2} = 1 \text{ Pa} &= 1.4504 \cdot 10^{-4} \text{ psi} \\
 1 \text{ in Hg} &= 3386.6 \text{ Pa} &= 0.49118 \text{ psi} \\
 1 \frac{\text{lbf}}{\text{ft}^2} &= 47.8803 \text{ Pa} &= 6.9444 \cdot 10^{-3} \text{ psi} \\
 1 \text{ at} &= 1 \frac{\text{kgf}}{\text{cm}^2} = 9.8067 \cdot 10^4 \text{ Pa} &= 14.223 \text{ psi} \\
 1 \text{ mm w. c.} &= 1 \frac{\text{kgf}}{\text{m}^2} = 9.8067 \text{ Pa} &= 1.4223 \cdot 10^{-3} \text{ psi} \\
 1 \text{ ft w. c.} &= 2988.98 \text{ Pa} &= 0.4335 \text{ psi} \\
 1 \frac{\text{dyne}}{\text{cm}^2} &= 0.1 \text{ Pa} &= 1.4504 \cdot 10^{-5} \text{ psi} \\
 1 \frac{\text{pdl}}{\text{ft}^2} &= 1.4882 \text{ Pa} &= 2.1584 \cdot 10^{-4} \text{ psi} \\
 1 \frac{\text{pdl}}{\text{m}^2} &= 0.1383 \text{ Pa} &= 2.0052 \cdot 10^{-5} \text{ psi} \\
 1 \frac{\text{tonf (long)}}{\text{in}^2} &= 1.5444 \cdot 10^7 \text{ Pa} &= 2240 \text{ psi} \\
 1 \frac{\text{tonf (short)}}{\text{in}^2} &= 1.3790 \cdot 10^7 \text{ Pa} &= 2000 \text{ psi} \\
 1 \frac{\text{N}}{\text{cm}^2} &= 1 \cdot 10^4 \text{ Pa} &= 1.4504 \text{ psi} \\
 1 \frac{\text{lbm-g}}{\text{ft}^2} &= 47.88 \text{ Pa} &= 6.9444 \cdot 10^{-3} \text{ psi} \\
 1 \text{ bar} &= 1 \cdot 10^6 \frac{\text{dyne}}{\text{cm}^2} &= 0.98692 \text{ atm}
 \end{aligned}$$

1.2.3.16 Energy and Torque

Conversion Table for Common Units of Energy (Work or Heat)

Energy	$J = \frac{kg \cdot m^2}{s^2}$	Btu	kcal	kWh	ft-lbf	hp-hr
1 J =	1	9.4778E-04	2.3885E-04	2.7778E-07	0.73757	3.7251E-07
1 Btu =	1055.1	1	0.25201	2.9308E-04	778.21	3.9303E-04
1 kcal =	4186.8	3.9682	1	1.1630E-03	3088.1	1.5596E-03
1 kWh =	3.6000E+06	3412	859.85	1	2.6553E+06	1.341
1 ft-lbf =	1.3558	1.2850E-03	3.2383E-04	3.7661E-07	1	5.0504E-07
1 hp-hr =	2.6845E+06	2544.3	641.19	0.7457	1.9800E+06	1

Additional Unit Conversions for Energy (Work or Heat) and Torque

$1 \frac{kg \cdot m^2}{s^2} = 1 N \cdot m = 1 W \cdot s$	= 1 J	= $9.4778 \cdot 10^{-4}$ Btu
1 therm	= $1.0551 \cdot 10^8$ J	= 10^5 Btu
1 cal = 0.001 kcal	= 4.1868 J	= $3.9682 \cdot 10^{-3}$ Btu
1 CHU	= 1899.1 J	= 1.8 Btu
1 ton-hr (refrigeration)	= $1.2661 \cdot 10^7$ J	= $1.2 \cdot 10^4$ Btu
1 PS • hr (metric)	= $2.6478 \cdot 10^6$ J	= 2509.5 Btu
1 kgf • m	= 9.8067 J	= $9.2946 \cdot 10^{-3}$ Btu
1 dyne • cm = 1 erg	= $1 \cdot 10^{-7}$ J	= $9.4778 \cdot 10^{-11}$ Btu
1 dyne • m	= 1 J	= $9.4778 \cdot 10^{-4}$ Btu
1 lbf-in	= 0.113 J	= $1.0708 \cdot 10^{-4}$ Btu
1 ft-pdl	= 0.0421 J	= $3.9938 \cdot 10^{-5}$ Btu
1 ton (explosives)	= $4.1840 \cdot 10^9$ J	= $3.9655 \cdot 10^6$ Btu
1 eV	= $1.6022 \cdot 10^{-19}$ J	= $1.5185 \cdot 10^{-22}$ Btu
1 hp-hr (UK)	= $2.5645 \cdot 10^6$ J	= 2430.6 Btu
1 psi-ft ³	= 195.2401 J	= 0.18504 Btu
1 atm • cm ³	= 0.1013 J	= $9.601 \cdot 10^{-5}$ Btu

1.2.3.17 Specific Enthalpy

Conversion Table for Common Units of Specific Enthalpy

Specific Enthalpy	$\frac{\text{J}}{\text{kg}}$	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{kcal}}{\text{kg}}$	$\frac{\text{hp-hr}}{\text{lbm}}$	$\frac{\text{kWh}}{\text{kg}}$	$\frac{\text{lbf-ft}}{\text{lbm}}$
$1 \frac{\text{J}}{\text{kg}} =$	1	4.2992E-04	2.3885E-04	1.6897E-07	2.7778E-07	0.33456
$1 \frac{\text{Btu}}{\text{lbm}} =$	2326	1	0.55556	3.9301E-04	6.4611E-04	778.18
$1 \frac{\text{kcal}}{\text{kg}} =$	4186.8	1.8	1	7.0743E-04	1.1630E-03	1400.7
$1 \frac{\text{hp-hr}}{\text{lbm}} =$	5.9184E+06	2544.4	1413.6	1	1.644	1.9800E+06
$1 \frac{\text{kWh}}{\text{kg}} =$	3.6000E+06	1547.7	859.85	0.60828	1	1.2044E+06
$1 \frac{\text{lbf-ft}}{\text{lbm}} =$	2.989	1.2851E-03	7.1392E-04	5.0505E-07	8.3029E-07	1

Additional Unit Conversions for Specific Enthalpy

$$\begin{aligned}
 1 \frac{\text{kcal}}{\text{kg}} &= 1 \frac{\text{CHU}}{\text{lbm}} = 1 \frac{\text{cal}}{\text{g}} = 4186.8 \frac{\text{J}}{\text{kg}} = 1.8 \frac{\text{Btu}}{\text{lbm}} \\
 1 \frac{\text{kgf} \cdot \text{m}}{\text{kg}} &= 9.8067 \frac{\text{J}}{\text{kg}} = 4.2161 \cdot 10^{-3} \frac{\text{Btu}}{\text{lbm}} \\
 1 \frac{\text{psi} \cdot \text{ft}^3}{\text{lbm}} &= 430.4329 \frac{\text{J}}{\text{kg}} = 0.18505 \frac{\text{Btu}}{\text{lbm}} \\
 1 \frac{\text{atm} \cdot \text{cm}^3}{\text{g}} &= 101.3 \frac{\text{J}}{\text{kg}} = 0.04355 \frac{\text{Btu}}{\text{lbm}}
 \end{aligned}$$

1.2.3.18 Calorific Value

Conversion Table for Common Units of Calorific Value

Calorific Value	$\frac{\text{J}}{\text{m}^3}$	$\frac{\text{Btu}}{\text{ft}^3}$	$\frac{\text{kcal}}{\text{m}^3}$	$\frac{\text{therm}}{\text{ft}^3}$	$\frac{\text{therm}}{\text{gal}}$	$\frac{\text{CHU}}{\text{ft}^3}$
$1 \frac{\text{J}}{\text{m}^3} =$	1	2.6838E-05	2.3885E-04	2.6838E-10	3.5971E-11	1.4910E-05
$1 \frac{\text{Btu}}{\text{ft}^3} =$	3.7260E+04	1	8.8994	1.0000E-05	1.3403E-06	0.55556
$1 \frac{\text{kcal}}{\text{m}^3} =$	4186.8	0.11237	1	1.1237E-06	1.5060E-07	0.06243
$1 \frac{\text{therm}}{\text{ft}^3} =$	3.7260E+09	1.0000E+05	8.8994E+05	1	0.13403	5.5556E+04
$1 \frac{\text{therm}}{\text{gal}} =$	2.7800E+10	7.4611E+05	6.6399E+06	7.4611	1	4.1451E+05
$1 \frac{\text{CHU}}{\text{ft}^3} =$	6.7067E+04	1.8	16.019	1.8000E-05	2.4125E-06	1

1.2.3.19 Entropy

Conversion Table for Common Units of Entropy

Entropy	$\frac{\text{J}}{\text{K}}$	$\frac{\text{Btu}}{^\circ\text{F}}$	$\frac{\text{kcal}}{^\circ\text{C}} = \text{Clausius}$	$\frac{\text{CHU}}{^\circ\text{C}}$	$\frac{\text{kcal}}{^\circ\text{F}}$
$1 \frac{\text{J}}{\text{K}} =$	1	5.2654E-04	2.3885E-04	5.2654E-04	4.2992E-04
$1 \frac{\text{Btu}}{^\circ\text{F}} =$	1899.2	1	0.45361	1	0.8165
$1 \frac{\text{kcal}}{^\circ\text{C}} =$	4186.8	2.2045	1	2.2045	1.8
$1 \frac{\text{CHU}}{^\circ\text{C}} =$	1899.2	1	0.45361	1	0.8165
$1 \frac{\text{kcal}}{^\circ\text{F}} =$	2326.0	1.2247	0.5556	1.2247	1

1.2.3.20 Power

Conversion Table for Common Units of Power

Power	W	$\frac{\text{Btu}}{\text{hr}}$	$\frac{\text{kcal}}{\text{hr}}$	hp	$\frac{\text{therm}}{\text{hr}}$	ton refrigeration
1 W =	1	3.4120	0.85985	1.3404E-03	3.4120E-05	2.8434E-04
$1 \frac{\text{Btu}}{\text{hr}} =$	0.29308	1	0.252	3.9285E-04	1.0000E-05	8.3335E-05
$1 \frac{\text{kcal}}{\text{hr}} =$	1.1630	3.9682	1	1.5589E-03	3.9682E-05	3.3069E-04
1 hp =	746.04	2545.5	641.48	1	0.02546	0.21213
$1 \frac{\text{therm}}{\text{hr}} =$	2.9308E+04	1.0000E+05	2.5200E+04	39.285	1	8.3335
1 ton refrigeration =	3516.9	1.2000E+04	3024.0	4.7141	0.12	1

Additional Unit Conversions for Power

$$\begin{aligned}
 1 \frac{\text{J}}{\text{s}} = V \cdot A = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3} &= 1 \text{ W} &= 3.412 \frac{\text{Btu}}{\text{hr}} \\
 1 \frac{\text{kgf} \cdot \text{m}}{\text{s}} &= 9.8067 \text{ W} &= 33.461 \frac{\text{Btu}}{\text{hr}} \\
 1 \frac{\text{atm} \cdot \text{m}^3}{\text{hr}} &= 28.15 \text{ W} &= 96.049 \frac{\text{Btu}}{\text{hr}} \\
 1 \text{ PS (metric)} &= 735.48 \text{ W} &= 2509.5 \frac{\text{Btu}}{\text{hr}} \\
 1 \frac{\text{erg}}{\text{s}} &= 1 \cdot 10^{-7} \text{ W} &= 3.412 \cdot 10^{-7} \frac{\text{Btu}}{\text{hr}} \\
 1 \frac{\text{CHU}}{\text{hr}} &= 0.5275 \text{ W} &= 1.8 \frac{\text{Btu}}{\text{hr}} \\
 1 \frac{\text{lbf} \cdot \text{ft}}{\text{min}} &= 0.0226 \text{ W} &= 0.0771 \frac{\text{Btu}}{\text{hr}} \\
 1 \frac{\text{lbf} \cdot \text{ft}}{\text{sec}} &= 1.3558 \text{ W} &= 4.626 \frac{\text{Btu}}{\text{hr}} \\
 1 \frac{\text{pdl} \cdot \text{ft}}{\text{sec}} &= 0.0421 \text{ W} &= 0.14378 \frac{\text{Btu}}{\text{hr}} \\
 1 \text{ hp (British)} &= 756.7 \text{ W} &= 2581.9 \frac{\text{Btu}}{\text{hr}} \\
 1 \text{ hp (Boiler)} &= 9809.5 \text{ W} &= 3.347 \cdot 10^4 \frac{\text{Btu}}{\text{hr}} \\
 1 \text{ hp} &= 550 \frac{\text{lbf} \cdot \text{ft}}{\text{sec}}
 \end{aligned}$$

1.2.3.21 Heat Flux

Conversion Table for Common Units of Heat Flux

Heat Flux	$\frac{\text{W}}{\text{m}^2}$	$\frac{\text{Btu}}{\text{ft}^2 \cdot \text{hr}}$	$\frac{\text{kcal}}{\text{m}^2 \cdot \text{hr}}$	$\frac{\text{cal}}{\text{cm}^2 \cdot \text{s}}$	$\frac{\text{kcal}}{\text{ft}^2 \cdot \text{hr}}$	$\frac{\text{CHU}}{\text{ft}^2 \cdot \text{hr}}$
$1 \frac{\text{W}}{\text{m}^2} =$	1	0.317	0.85985	2.3885E-05	0.07989	0.17611
$1 \frac{\text{Btu}}{\text{ft}^2 \cdot \text{hr}} =$	3.1546	1	2.7125	7.5346E-05	0.25201	0.55554
$1 \frac{\text{kcal}}{\text{m}^2 \cdot \text{hr}} =$	1.163	0.36867	1	2.7778E-05	0.09291	0.20481
$1 \frac{\text{cal}}{\text{cm}^2 \cdot \text{s}} =$	4.1868E+04	1.3272E+04	3.6000E+04	1	3344.6	7372.2
$1 \frac{\text{kcal}}{\text{ft}^2 \cdot \text{hr}} =$	12.518	3.9682	10.764	2.9899E-04	1	2.2045
$1 \frac{\text{CHU}}{\text{ft}^2 \cdot \text{hr}} =$	5.6784	1.8	4.8825	1.3563E-04	0.45362	1

1.2.3.22 Dynamic Viscosity

Conversion Table for Common Units of Dynamic Viscosity

Viscosity (dynamic)	Pa • s	cP	Poise	$\frac{\text{lbf} \cdot \text{sec}}{\text{ft}^2}$	$\frac{\text{lbf} \cdot \text{sec}}{\text{in}^2}$	$\frac{\text{lbm}}{\text{ft} \cdot \text{sec}}$
1 Pa • s =	1	1000	10	0.02089	1.4504E-04	0.67195
1 cP =	0.001	1	0.01	2.0885E-05	1.4504E-07	6.7195E-04
1 Poise =	0.1	100	1	2.0885E-03	1.4504E-05	0.06720
$1 \frac{\text{lbf} \cdot \text{sec}}{\text{ft}^2} =$	47.88	4.7880E+04	478.8	1	6.9444E-03	32.173
$1 \frac{\text{lbf} \cdot \text{sec}}{\text{in}^2} =$	6894.8	6.8948E+06	6.8948E+04	144	1	4633
$1 \frac{\text{lbm}}{\text{ft} \cdot \text{sec}} =$	1.4882	1488.2	14.882	0.03108	2.1585E-04	1

Additional Unit Conversions for Dynamic Viscosity

$$\begin{aligned}
 1 \text{ Poise} &= 1 \frac{\text{g}}{\text{cm} \cdot \text{s}} = 1 \frac{\text{dyne} \cdot \text{s}}{\text{cm}^2} = 0.1 \text{ Pa} \cdot \text{s} = 2.0885 \cdot 10^{-3} \frac{\text{lbf} \cdot \text{sec}}{\text{ft}^2} \\
 1 \frac{\text{lbf} \cdot \text{sec}}{\text{ft}^2} &= 1 \frac{\text{slug}}{\text{ft} \cdot \text{s}} = 47.8803 \text{ Pa} \cdot \text{s} = 1 \frac{\text{lbf} \cdot \text{sec}}{\text{ft}^2} \\
 1 \frac{\text{lbm}}{\text{ft} \cdot \text{s}} &= \frac{\text{pdl} \cdot \text{s}}{\text{ft}^2} = 1.4882 \text{ Pa} \cdot \text{s} = 0.03108 \frac{\text{lbf} \cdot \text{sec}}{\text{ft}^2} \\
 1 \frac{\text{lbm}}{\text{ft} \cdot \text{hr}} &= 4.1338 \cdot 10^{-4} \text{ Pa} \cdot \text{s} = 8.6336 \cdot 10^{-6} \frac{\text{lbf} \cdot \text{sec}}{\text{ft}^2} \\
 1 \frac{\text{kgf} \cdot \text{s}}{\text{m}^2} &= 9.8067 \text{ Pa} \cdot \text{s} = 0.20482 \frac{\text{lbf} \cdot \text{sec}}{\text{ft}^2} \\
 1 \frac{\text{kgf} \cdot \text{hr}}{\text{m}^2} &= 3.5320 \cdot 10^{-4} \text{ Pa} \cdot \text{s} = 7.3767 \cdot 10^{-6} \frac{\text{lbf} \cdot \text{sec}}{\text{ft}^2} \\
 1 \frac{\text{kg}}{\text{ft} \cdot \text{hr}} &= 9.1134 \cdot 10^{-4} \text{ Pa} \cdot \text{s} = 1.9034 \cdot 10^{-5} \frac{\text{lbf} \cdot \text{sec}}{\text{ft}^2}
 \end{aligned}$$

1.2.3.23 Diffusion Coefficient, Thermal Diffusivity, and Kinematic Viscosity

Conversion Table for Common Units of the Diffusion Coefficient, Thermal Diffusivity, and Kinematic Viscosity

Diffusivity	$\frac{\text{m}^2}{\text{s}}$	$St = \frac{\text{cm}^2}{\text{s}}^*$	$\frac{\text{ft}^2}{\text{sec}}$	$\frac{\text{ft}^2}{\text{hr}}$	$\frac{\text{in}^2}{\text{sec}}$	$\frac{\text{liter}}{\text{in.-hr}}$
$1 \frac{\text{m}^2}{\text{s}} =$	1	1.0000E+04	10.764	3.8751E+04	1550	9.1441E+04
$1 St = \frac{\text{cm}^2}{\text{s}}^* =$	1.0000E-04	1	1.0764E-03	3.8751	0.155	9.1441
$1 \frac{\text{ft}^2}{\text{sec}} =$	0.09290	929.03	1	3600	144	8495.2
$1 \frac{\text{ft}^2}{\text{hr}} =$	2.5806E-05	0.25806	2.7777E-04	1	0.04	2.3597
$1 \frac{\text{in}^2}{\text{sec}} =$	6.4516E-04	6.4516	6.9444E-03	25	1	58.994
$1 \frac{\text{liter}}{\text{in.-hr}} =$	1.0936E-05	0.10936	1.1771E-04	0.42378	0.01695	1

* St = Stokes

1.2.3.24 Heat Capacity and Specific Entropy

Conversion Table for Common Units of Heat Capacity and Specific Entropy

Heat Capacity	$\frac{\text{W}}{\text{m} \cdot \text{K}}$	$\frac{\text{Btu}}{\text{lbm} \cdot \text{F}}$	$\frac{\text{lbf-ft}}{\text{lbm} \cdot \text{R}} =$
$1 \frac{\text{W}}{\text{m} \cdot \text{K}} =$	1	2.3885E-04	0.18586
$1 \frac{\text{Btu}}{\text{lbm} \cdot \text{F}} =$	4186.8	1	778.17
$1 \frac{\text{lbf-ft}}{\text{lbm} \cdot \text{R}} =$	5.3803	1.2851E-03	1

$$1 \frac{\text{Btu}}{\text{lbm} \cdot \text{F}} = 1 \frac{\text{kcal}}{\text{kg} \cdot ^\circ\text{C}} = 1 \frac{\text{cal}}{\text{g} \cdot ^\circ\text{C}} = 1 \frac{\text{CHU}}{\text{lbm} \cdot ^\circ\text{C}}$$

1.2.3.25 Thermal Conductivity

Conversion Table for Common Units of Thermal Conductivity

Thermal Conductivity	$\frac{W}{m \cdot K}$	$\frac{Btu}{hr \cdot ft \cdot ^\circ F}$	$\frac{Btu \cdot in}{hr \cdot ft^2 \cdot ^\circ F}$	$\frac{kcal}{hr \cdot m \cdot ^\circ C}$	$\frac{cal}{s \cdot cm \cdot ^\circ C}$
$1 \frac{W}{m \cdot K} =$	1	0.57777	6.9334	0.85985	2.3885E-03
$1 \frac{Btu}{hr \cdot ft \cdot ^\circ F} =$	1.7308	1	12	1.4882	4.1339E-03
$1 \frac{Btu \cdot in}{hr \cdot ft^2 \cdot ^\circ F} =$	0.14423	0.08333	1	0.12402	3.4449E-04
$1 \frac{kcal}{hr \cdot m \cdot ^\circ C} =$	1.163	0.67194	8.0635	1	2.7778E-03
$1 \frac{cal}{s \cdot cm \cdot ^\circ C} =$	418.68	241.9	2902.9	360	1

$$1 \frac{Btu}{hr \cdot ft \cdot ^\circ F} = 1 \frac{CHU}{hr \cdot ft \cdot ^\circ C}$$

1.2.3.26 Heat-Transfer Coefficient

Conversion Table for Common Units of the Heat-Transfer Coefficient

Heat-Transfer Coefficient	$\frac{W}{m^2 \cdot K}$	$\frac{Btu}{hr \cdot ft^2 \cdot ^\circ F}$	$\frac{Btu}{sec \cdot ft^2 \cdot ^\circ F}$	$\frac{kcal}{hr \cdot m^2 \cdot ^\circ C}$	$\frac{cal}{s \cdot cm^2 \cdot ^\circ C}$	$\frac{kcal}{hr \cdot ft^2 \cdot ^\circ C}$
$1 \frac{W}{m^2 \cdot K} =$	1	0.1761	4.8919E-05	0.85985	2.3885E-05	0.07989
$1 \frac{Btu}{hr \cdot ft^2 \cdot ^\circ F} =$	5.6785	1	2.7779E-04	4.8826	1.3563E-04	0.45363
$1 \frac{Btu}{sec \cdot ft^2 \cdot ^\circ F} =$	2.0442E+04	3599.8	1	1.7577E+04	0.48824	1633
$1 \frac{kcal}{hr \cdot m^2 \cdot ^\circ C} =$	1.1630	0.20481	5.6893E-05	1	2.7778E-05	0.09291
$1 \frac{cal}{s \cdot m^2 \cdot ^\circ C} =$	4.1868E+04	7373.1	2.0482	3.6000E+04	1	3344.6
$1 \frac{kcal}{hr \cdot ft^2 \cdot ^\circ C} =$	12.518	2.2045	6.1237E-04	10.764	2.9899E-04	1

$$1 \frac{Btu}{hr \cdot ft^2 \cdot ^\circ F} = 1 \frac{CHU}{hr \cdot ft^2 \cdot ^\circ C}$$

1.2.3.27 Surface Tension

Conversion Table for Common Units of Surface Tension

Surface Tension	$\frac{\text{N}}{\text{m}}$	$\frac{\text{lbf}}{\text{in.}}$	$\frac{\text{lbf}}{\text{ft}}$	$\frac{\text{g}}{\text{cm}}$	$\frac{\text{dyne}}{\text{cm}}$	$\frac{\text{pdl}}{\text{in.}}$
$1 \frac{\text{N}}{\text{m}} =$	1	5.7101E-03	4.7585E-04	1.0197	1000	0.18372
$1 \frac{\text{lbf}}{\text{in.}} =$	175.13	1	0.08333	178.58	1.7513E+05	32.174
$1 \frac{\text{lbf}}{\text{ft}} =$	2101.5	12	1	2143	2.1015E+06	386.09
$1 \frac{\text{g}}{\text{cm}} =$	0.98067	5.5997E-03	4.6665E-04	1	980.67	0.18017
$1 \frac{\text{dyne}}{\text{cm}} =$	0.001	5.7101E-06	4.7585E-07	1.0197E-03	1	1.8372E-04
$1 \frac{\text{pdl}}{\text{in.}} =$	5.4431	0.03108	2.5901E-03	5.5504	5443.1	1

1.2.3.28 Cubic Expansion Coefficient

Conversion Table for Common Units of Cubic Expansion

Cubic Expansion	$\frac{\text{kg}}{\text{m}^3 \cdot \text{K}}$	$\frac{\text{lbm}}{\text{m}^3 \cdot ^\circ\text{F}}$	$\frac{\text{g}}{\text{cm}^3 \cdot ^\circ\text{C}}$
$1 \frac{\text{kg}}{\text{m}^3 \cdot \text{K}} =$	1	0.03468	0.001
$1 \frac{\text{lbm}}{\text{m}^3 \cdot ^\circ\text{F}} =$	28.833	1	0.02883
$1 \frac{\text{g}}{\text{cm}^3 \cdot ^\circ\text{C}} =$	1000	34.682	1

1.2.3.29 Temperature

Conversion Table for Temperature Units

	Kelvin (K)	Celsius (°C)	Rankine (°R)	Fahrenheit (°F)
$T(\text{K}) =$	$T(\text{K})$	$T(^{\circ}\text{C}) + 273.15$	$\frac{5}{9} T(^{\circ}\text{R})$	$\frac{5}{9} T(^{\circ}\text{F}) + 255.37$
$T(^{\circ}\text{C}) =$	$T(\text{K}) - 273.15$	$T(^{\circ}\text{C})$	$\frac{5}{9} T(^{\circ}\text{R}) - 273.15$	$\frac{5}{9} T(^{\circ}\text{F}) - 17.78$
$T(^{\circ}\text{R}) =$	$\frac{9}{5} T(\text{K})$	$\frac{9}{5} T(^{\circ}\text{C}) + 491.67^{\circ}\text{R}$	$T(^{\circ}\text{R})$	$T(^{\circ}\text{F}) + 459.67$
$T(^{\circ}\text{F}) =$	$\frac{9}{5} T(\text{K}) - 459.67$	$\frac{9}{5} T(^{\circ}\text{C}) + 32$	$T(^{\circ}\text{R}) - 459.67$	$T(^{\circ}\text{F})$

1.3 Mathematics

1.3.1 Algebra

1.3.1.1 Linear Algebra

Straight Line

General form: $Ax + By + C = 0$

Standard form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Two-point form: $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

Intercept form: $\frac{x}{x_0} + \frac{y}{y_0} - 1 = 0$, where intercepts $x_0 \neq 0$, $y_0 \neq 0$

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Angle between lines with slopes m_1 and m_2 : $\alpha = \arctan\left(\frac{m_2 - m_1}{1 + m_2 m_1}\right)$

Distance between two points (two-dimensional space): $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

Intersection of two straight lines: $x_i = \frac{b_2 - b_1}{m_1 - m_2}$ $y_i = \frac{m_1 b_2 - m_2 b_1}{m_1 - m_2}$

1.3.1.2 Polynomials

Quadratic Equation

Standard form: $ax^2 + bx + c = 0$

Normal form: $x^2 + px + q = 0$

Roots: $x_{1,2} = \frac{-b}{2a} \left(1 \pm \sqrt{1 - \frac{4ac}{b^2}}\right) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x_{1,2} = \frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$$

Vieta's Rule: $p = -(x_1 + x_2)$ $q = x_1 x_2$

If $(b^2 - 4ac) > 0$, the roots are real and unequal.

If $(b^2 - 4ac) = 0$, the roots are real and equal.

If $(b^2 - 4ac) < 0$, the roots are imaginary and unequal.

If $(b^2 - 4ac) = n^2$ (perfect square), the roots are rational and unequal.

Expansion of General Algebraic Expressions

$$(a \pm b)^2 = a^2 \pm 2 a b + b^2$$

$$(a \pm b)^3 = a^3 \pm 3 a^2 b + 3 a \cdot b^2 \pm b^3$$

$$(a \pm b)^4 = a^4 \pm 4 a^3 b + 6 a^2 b^2 \pm 4 a b^3 + b^4$$

$$a^2 - b^2 = (a + b) (a - b)$$

$$a^3 + b^3 = (a + b) (a^2 - a b + b^2)$$

$$a^3 - b^3 = (a - b) (a^2 + a b + b^2)$$

$$a^4 + b^4 = (a^2 + a b \sqrt{2} + b^2) (a^2 - a b \sqrt{2} + b^2)$$

$$a^4 - b^4 = (a^2 + b^2) (a^2 - b^2)$$

Quadratic Surface (Sphere)

Standard form: $(x - h)^2 + (y - k)^2 + (z - m)^2 = r^2$

Distance between two points in three-dimensional space: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

1.3.1.3 Logarithms, Exponents, and Roots

Logarithms

General definition: $\log_b(x) = c$ where: $x = b^c$

Natural logarithm: $\ln(x) = c$ where: $x = e^c$ (base: $e = 2.71828$)

Base 10 logarithm: $\log(x) = c$ where: $x = 10^c$ (base: 10)

To change from one base to another:

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

$$\ln(x) = \frac{\log_{10}(x)}{\log_{10}(e)} = 2.302585 \log_{10}(x)$$

$$\log(x) = \frac{\ln(x)}{\ln(10)} = 0.4343 \ln(x)$$

Identities:

$$\log_b(1) = 0$$

$$\log_b(b) = 1$$

$$\log_b(b^n) = n$$

$$\log_b(x^c) = c \log_b(x)$$

$$\log_b\left(\frac{1}{x^c}\right) = \log_b(x^{-c}) = -c \log_b(x) = c \log_b\left(\frac{1}{x}\right)$$

$$\log_b\left({}^c\sqrt{x}\right) = \log_b\left[(x)^{\frac{1}{c}}\right] = \frac{1}{c} \log_b(x)$$

$$\log_b(x y) = \log_b(x) + \log_b(y)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

$$b^{n \log_b(x)} = x^n$$

$$b^{\frac{\log_b(x)}{n}} = x^{\frac{1}{n}}$$

Rules for Exponents and Radicals

$$a^0 = 1$$

$$a^1 = a$$

Identities:

$$p a^n \pm q a^n = (p \pm q) a^n$$

$$a^n a^m = a^{n+m}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$(a^m)^n = (a^n)^m = a^{n m}$$

$$a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$$

$$a^n b^n = (a b)^n$$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

$$(a)^{\frac{1}{n}} = \sqrt[n]{a}$$

$$\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m = (a)^{\frac{m}{n}}$$

$$\sqrt[n]{a^m} = \sqrt[n]{a^m}$$

$$p(\sqrt[n]{a}) + q(\sqrt[n]{a}) = (p + q)(\sqrt[n]{a})$$

$$(\sqrt[m]{a})(\sqrt[n]{a}) = \sqrt[n+m]{a}$$

$$\sqrt[n]{a b} = \sqrt[n]{a} \sqrt[n]{b}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} = \left(\frac{a}{b}\right)^{\frac{1}{n}}$$

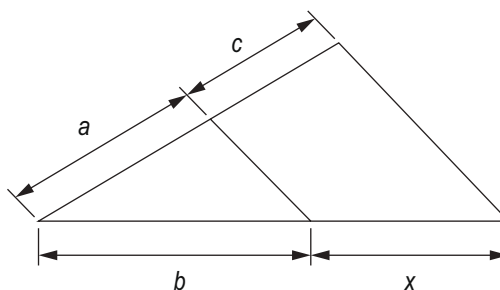
1.3.1.4 Proportions

Directly proportional (4th proportional):

$$x \propto c$$

$$a : b = c : x$$

$$x = \frac{bc}{a}$$



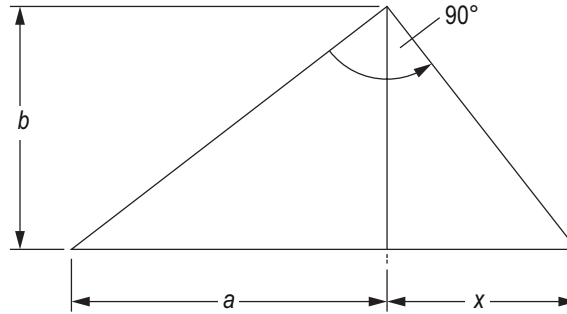
If $\frac{a}{b} = \frac{c}{x}$ then: $\frac{a+b}{b} = \frac{c+x}{x}$ and $\frac{a-b}{b} = \frac{c-x}{x}$ and $\frac{a-b}{a+b} = \frac{c-x}{c+x}$

Square proportional (3rd proportional):

$$x \propto b^2$$

$$a : b = b : x$$

$$x = \frac{b^2}{a}$$

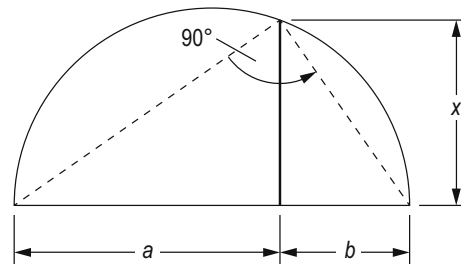


Mean proportional:

$$x \propto \sqrt{b}$$

$$a : x = x : b$$

$$x = \sqrt{ab}$$



Inversely proportional:

$$x \propto \frac{1}{b}$$

Inversely square proportional:

$$x \propto \frac{1}{b^2}$$

1.3.1.5 Complex Numbers

Rectangular form: $z = a + ib$

where $i = \sqrt{-1}$

a = real component

b = imaginary component

Polar form: $z = c \angle \theta = c(\cos \theta + i \sin \theta) = c e^{i\theta}$

$$c = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1} \left(\frac{b}{a} \right)$$

$$a = c \cos \theta$$

$$b = c \sin \theta$$

Addition and Subtraction (in rectangular form): $z_1 \pm z_2 = (a_1 \pm a_2) + i(b_1 \pm b_2)$

Multiplication and Division (in polar form):

$$z_1 z_2 = (c_1 c_2) \angle (\theta_1 + \theta_2)$$

$$\frac{z_1}{z_2} = \left(\frac{c_1}{c_2}\right) \angle (\theta_1 - \theta_2)$$

$$z^n = (a + i b)^n = c^n \angle (n\theta)$$

Complex Conjugate:

$$z^* = a - i b$$

$$z z^* = a^2 + b^2$$

Euler's Identity:

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$e^{-i\theta} = \cos \theta - i \sin \theta$$

$$\cos \theta = \frac{1}{2} (e^{i\theta} + e^{-i\theta})$$

$$\sin \theta = \frac{1}{2i} (e^{i\theta} - e^{-i\theta})$$

1.3.2 Geometry and Trigonometry

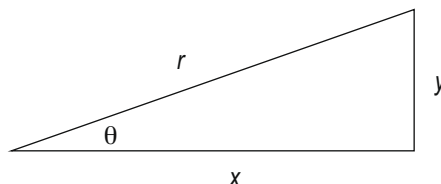
1.3.2.1 Circular Transcendental Functions

Trigonometric functions are defined using a right triangle:

$$\sin \theta = \frac{y}{r}, \cos \theta = \frac{x}{r}$$

$$\sec \theta = \frac{r}{x}, \csc \theta = \frac{r}{y}$$

$$\tan \theta = \frac{y}{x}, \cot \theta = \frac{x}{y}$$



$$\arcsin\left(\frac{y}{r}\right) = \theta, \operatorname{arccsc}\left(\frac{r}{y}\right) = \theta$$

$$\arccos\left(\frac{x}{r}\right) = \theta, \operatorname{arcsec}\left(\frac{r}{x}\right) = \theta$$

$$\arctan\left(\frac{y}{x}\right) = \theta, \operatorname{arccot}\left(\frac{x}{y}\right) = \theta$$

Law of Sines

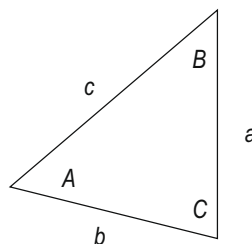
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



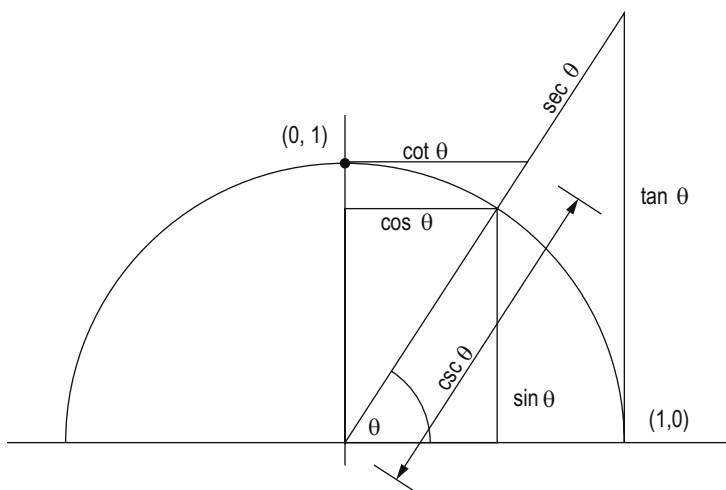
Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(A-B)}{\tan \frac{1}{2}(A+B)}$$

$$\frac{b-c}{b+c} = \frac{\tan \frac{1}{2}(B-C)}{\tan \frac{1}{2}(B+C)}$$

$$\frac{a-c}{a+c} = \frac{\tan \frac{1}{2}(A-C)}{\tan \frac{1}{2}(A+C)}$$

Trigonometric Functions in a Unit Circle



Trigonometric Identities

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\cos \theta = \sin\left(\theta + \frac{\pi}{2}\right) = -\sin\left(\theta - \frac{\pi}{2}\right)$$

$$\sin \theta = \cos\left(\theta - \frac{\pi}{2}\right) = -\cos\left(\theta + \frac{\pi}{2}\right)$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

Double-Angle Formulas

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 1 - 2 \sin^2 \alpha = 2 \cos^2 \alpha - 1$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\cot 2\alpha = \frac{\cot^2 \alpha - 1}{2 \cot \alpha}$$

Two-Angle Formulas

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{(\tan \alpha + \tan \beta)}{(1 - \tan \alpha \tan \beta)}$$

$$\cot(\alpha + \beta) = \frac{(\cot \alpha \cot \beta - 1)}{(\cot \alpha + \cot \beta)}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{(\tan \alpha - \tan \beta)}{(1 + \tan \alpha \tan \beta)}$$

$$\cot(\alpha - \beta) = \frac{(\cot \alpha \cot \beta + 1)}{(\cot \beta - \cot \alpha)}$$

Half-Angle Formulas

$$\sin\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{(1 - \cos \alpha)}{2}}$$

$$\cos\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{(1 + \cos \alpha)}{2}}$$

$$\tan\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{(1 - \cos \alpha)}{(1 + \cos \alpha)}}$$

$$\cot\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{(1 + \cos \alpha)}{(1 - \cos \alpha)}}$$

Combination of the Trigonometric Functions of Different Angles

$$\sin \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

$$\sin \alpha + \sin \beta = 2 \sin \left[\left(\frac{1}{2} \right) (\alpha + \beta) \right] \cos \left[\left(\frac{1}{2} \right) (\alpha - \beta) \right]$$

$$\sin \alpha - \sin \beta = 2 \cos \left[\left(\frac{1}{2} \right) (\alpha + \beta) \right] \sin \left[\left(\frac{1}{2} \right) (\alpha - \beta) \right]$$

$$\cos \alpha + \cos \beta = 2 \cos \left[\left(\frac{1}{2} \right) (\alpha + \beta) \right] \cos \left[\left(\frac{1}{2} \right) (\alpha - \beta) \right]$$

$$\cos \alpha - \cos \beta = -2 \sin \left[\left(\frac{1}{2} \right) (\alpha + \beta) \right] \sin \left[\left(\frac{1}{2} \right) (\alpha - \beta) \right]$$

Miscellaneous Formulas

$$\sin \theta = 2 \sin \left(\frac{1}{2} \theta \right) \cos \left(\frac{1}{2} \theta \right) = \sqrt{\frac{1 - \cos 2\theta}{2}}$$

$$\cos \theta = \cos^2 \left(\frac{1}{2} \theta \right) - \sin^2 \left(\frac{1}{2} \theta \right) = \sqrt{\frac{1 + \cos 2\theta}{2}}$$

$$\tan \theta = \frac{2 \tan \left(\frac{1}{2} \theta \right)}{1 - \tan^2 \left(\frac{1}{2} \theta \right)} = \frac{2 \left(\frac{1}{2} \theta \right) \cos \left(\frac{1}{2} \theta \right)}{\cos^2 \left(\frac{1}{2} \theta \right) - \sin^2 \left(\frac{1}{2} \theta \right)}$$

$$\tan \theta = \sqrt{\frac{1 - \cos 2\theta}{1 + \cos 2\theta}} = \frac{\sin 2\theta}{1 + \cos 2\theta} = \frac{1 - \cos 2\theta}{\sin 2\theta}$$

$$\cot \theta = \frac{\cot^2 \left(\frac{1}{2} \theta \right) - 1}{2 \cot \left(\frac{1}{2} \theta \right)} = \frac{\cos^2 \left(\frac{1}{2} \theta \right) - \sin^2 \left(\frac{1}{2} \theta \right)}{2 \left(\frac{1}{2} \theta \right) \cos \left(\frac{1}{2} \theta \right)}$$

$$\cot \theta = \sqrt{\frac{1 + \cos 2\theta}{1 - \cos 2\theta}} = \frac{1 + \cos 2\theta}{\sin 2\theta} = \frac{\sin 2\theta}{1 - \cos 2\theta}$$

1.3.2.2 Planar Geometry—Area and Perimeter

A = area

P = perimeter (circumference)

a, b, c = lengths of sides

d = diagonal(s) or diameter

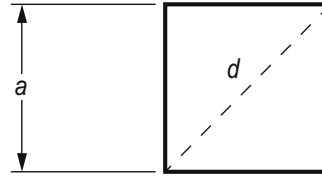
h = height

Square

$$A = a^2$$

$$P = 4a$$

$$d = a\sqrt{2}$$

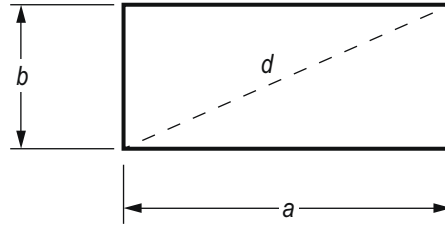


Rectangle

$$A = ab$$

$$P = 2(a + b)$$

$$d = \sqrt{a^2 + b^2}$$



Parallelogram

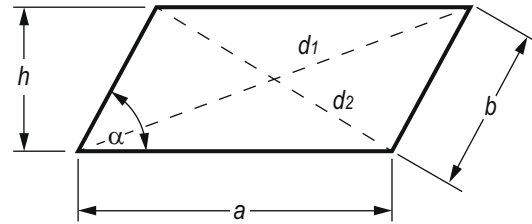
$$A = ah = ab \sin \alpha$$

$$P = 2(a + b) = 2\left(a + \frac{h}{\sin \alpha}\right)$$

$$d_1 = \sqrt{(a + h \cot \alpha)^2 + h^2}$$

$$d_2 = \sqrt{(a - h \cot \alpha)^2 + h^2}$$

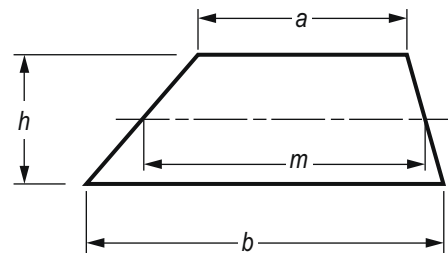
$$d_1^2 + d_2^2 = 2(a^2 + b^2)$$



Trapezoid

$$A = \frac{a+b}{2}h = mh$$

$$m = \frac{a+b}{2}$$



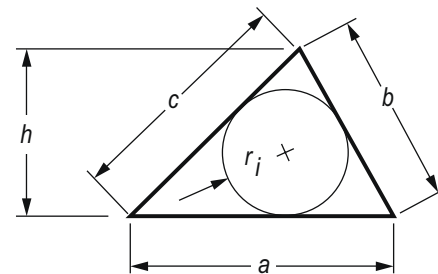
Triangle (oblique)

$$P = a + b + c$$

$$r_i = \sqrt{\left(1 - \frac{2a}{P}\right)\left(1 - \frac{2b}{P}\right)\left(1 - \frac{2c}{P}\right)}$$

$$A = \frac{1}{2}ah = \frac{1}{2}r_i P$$

$$A = \frac{1}{2}\sqrt{P(P-2a)(P-2b)(P-2c)}$$

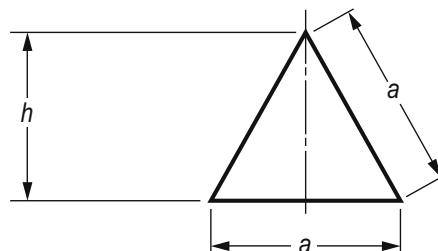


Triangle (equilateral)

$$P = 3a$$

$$A = \frac{a^2\sqrt{3}}{4} = \frac{1}{2}ah$$

$$h = \frac{a\sqrt{3}}{2}$$

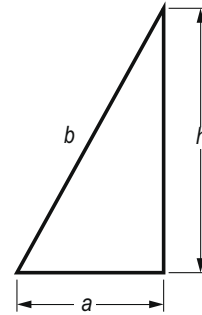


Triangle (right)

$$P = a + b + h$$

$$A = \frac{1}{2} a h$$

$$b = \sqrt{a^2 + h^2}$$



Regular Polygon (*n* equal sides)

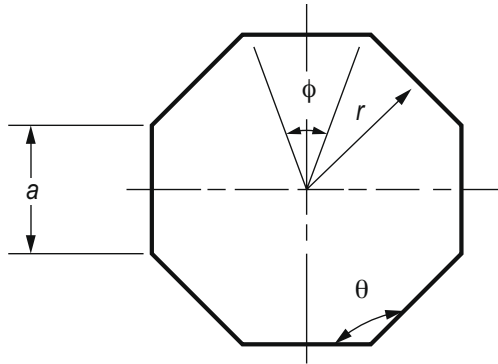
$$P = n a$$

$$A = \frac{n r a}{2}$$

$$a = 2 r \left[\tan \left(\frac{\phi}{2} \right) \right]$$

$$\phi = \frac{2\pi}{n}$$

$$\theta = \frac{\pi(n-2)}{n}$$

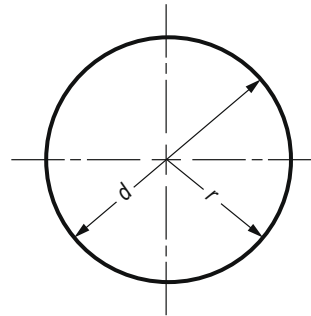


Circle

$$P = 2\pi r = \pi d$$

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

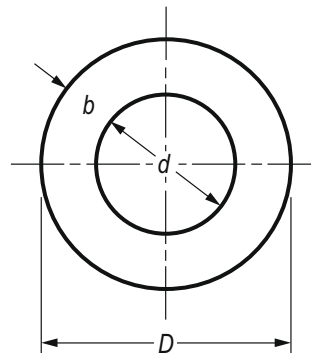
$$d = 2r$$



Annulus

$$A = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} (D + d)(D - d) = \pi b(b + d)$$

$$b = \frac{D - d}{2}$$

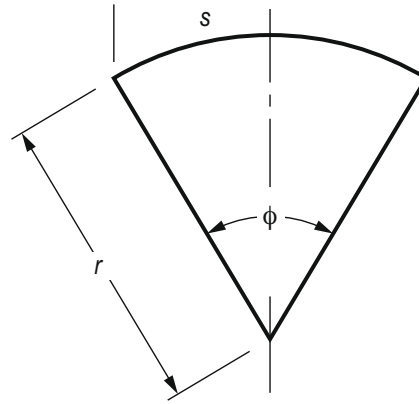


Sector of a Circle

$$A = \frac{r^2 \phi}{2} = \frac{s r}{2}$$

$$s = r \phi$$

$$P = 2r + s = r(2 + \phi)$$



Segment of a Circle

$$b = 2r \sin\left(\frac{\phi}{2}\right)$$

$$\phi = \frac{s}{r} = 2 \arccos\left(\frac{r-d}{r}\right)$$

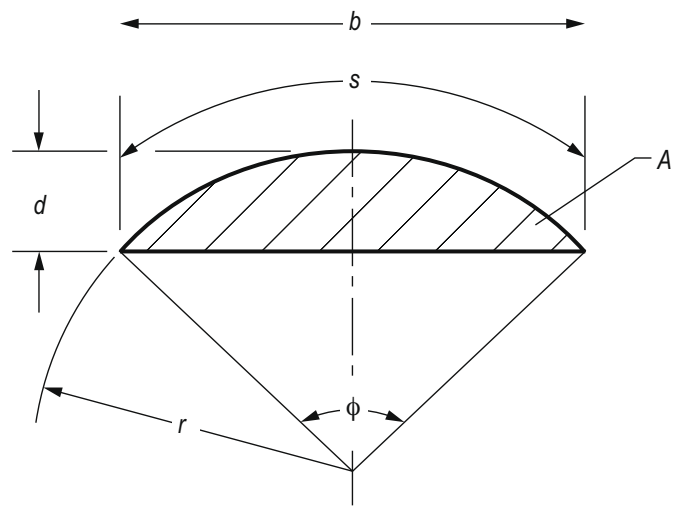
$$A = \frac{r^2 (\phi - \sin \phi)}{2}$$

$$A = \frac{d}{6b} (3d^2 + 4b^2)$$

$$r = \frac{d}{2} + \frac{b^2}{8d}$$

$$d = r \left[1 - \cos\left(\frac{\phi}{2}\right) \right] = \frac{b}{2} \tan\left(\frac{\phi}{4}\right)$$

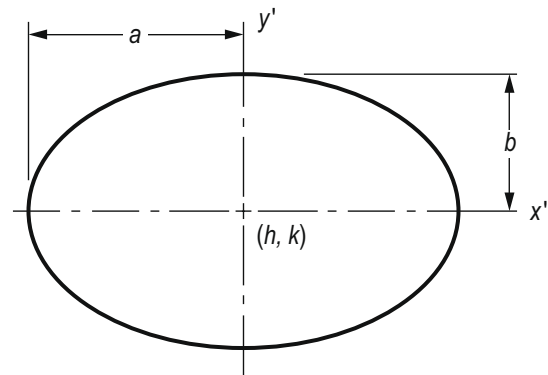
$$P = s + b = r \left[\phi + 2 \sin\left(\frac{\phi}{2}\right) \right]$$



Ellipse

$$P_{\text{approx}} = 2\pi \sqrt{\frac{a^2 + b^2}{2}}$$

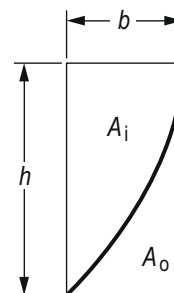
$$A = \pi a b$$



Parabola

$$A_i = \frac{2bh}{3}$$

$$A_o = \frac{bh}{3}$$



1.3.2.3 Cubic Geometry—Volume and Surface Area

A = surface area

V = volume

a, b, c = lengths of sides

d = diagonal(s) or diameter

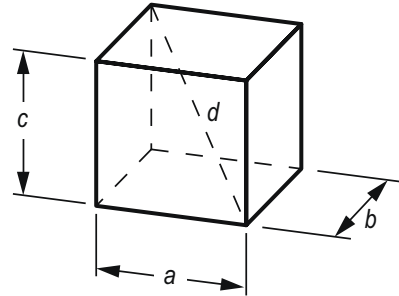
h = height

Cube

$$V = a^3$$

$$A = 6a^2$$

$$d = a\sqrt{3}$$

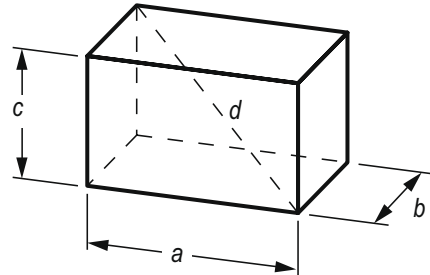


Cuboid

$$V = abc$$

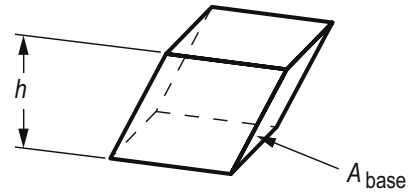
$$A = 2(ab + ac + bc)$$

$$d = \sqrt{a^2 + b^2 + c^2}$$



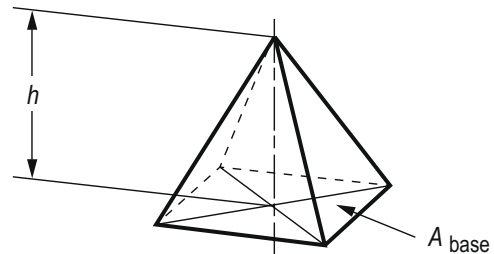
Parallelepiped

$$V = A_{\text{base}} h$$



Pyramid

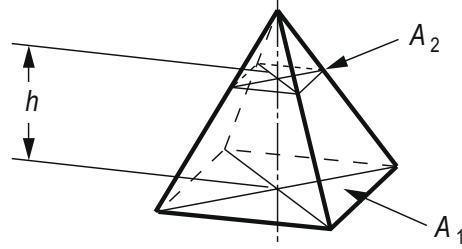
$$V = \frac{A_{\text{base}} h}{3}$$



Frustum of Pyramid

$$V = \frac{h}{3} (A_1 + A_2 + \sqrt{A_1 A_2})$$

$$V \approx h \frac{A_1 + A_2}{2} \text{ for } A_1 \approx A_2$$

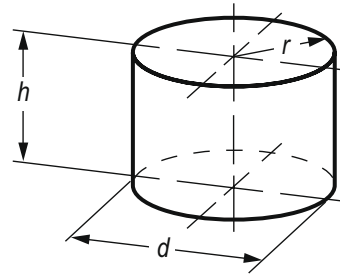


Right Circular Cylinder

$$V = \frac{\pi}{4} d^2 h$$

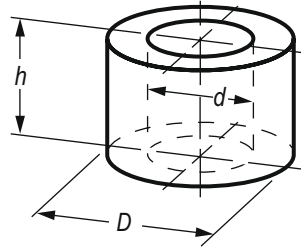
$$A_{\text{mantle}} = 2\pi r h$$

$$A = 2\pi r(r + h)$$



Hollow Cylinder

$$V = \frac{\pi}{4} h (D^2 - d^2)$$



Right Circular Cone

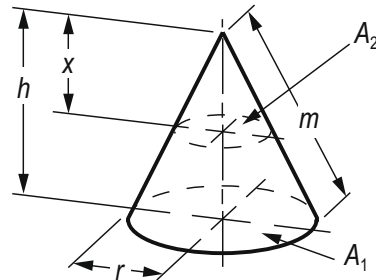
$$V = \frac{\pi}{3} r^2 h = \frac{\pi}{12} d^2 h$$

$$A_{\text{mantle}} = \pi r m$$

$$A = \pi r(r + m)$$

$$m = \sqrt{h^2 + r^2}$$

$$A_1 : A_2 = x^2 : h^2$$

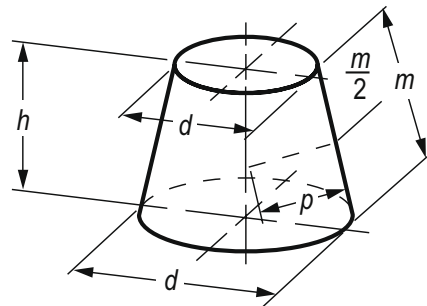


Frustum of Cone

$$V = \frac{\pi}{12} h (D^2 + Dd + d^2)$$

$$A_{\text{mantle}} = \frac{\pi}{2} m (D + d) = 2\pi p m$$

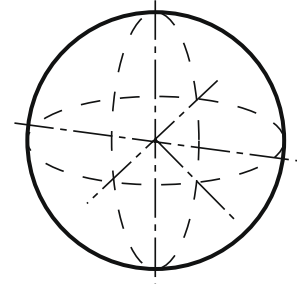
$$m = \sqrt{h^2 + \left(\frac{D-d}{2}\right)^2}$$



Sphere

$$V = \frac{4}{3} \pi r^3 = \frac{1}{6} \pi d^3$$

$$A = 4 \pi r^2 = \pi d^2$$

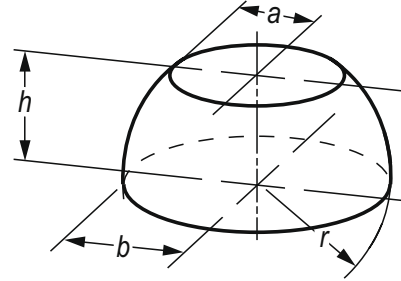


Zone of a Sphere

$$V = \frac{\pi}{6} h (3a^2 + 3b^2 + h^2)$$

$$A_{\text{mantle}} = 2 \pi r h$$

$$A = \pi (2 r h + a^2 + b^2)$$



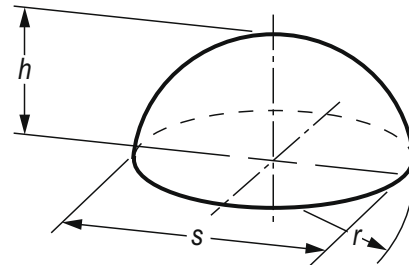
Segment of a Sphere (Spherical Cap)

$$V = \frac{\pi}{6} h \left(\frac{3}{4} s^2 + h^2 \right) = \pi h^2 \left(r - \frac{h}{3} \right)$$

$$A_{\text{cap}} = 2 \pi r h = \frac{\pi}{4} (s^2 + 4 h^2)$$

$$A_{\text{cap}} = 2 \pi r^2 (1 - \cos \theta_0)$$

$$s = 2 \sqrt{h(2r - h)}$$

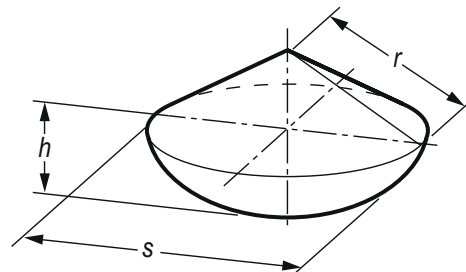


θ_0 is the angle of the cutout, rotated from the center of the radius.

Sector of a Sphere

$$V = \frac{2}{3} \pi r^2 h$$

$$A = \frac{\pi}{2} r (4h + s)$$

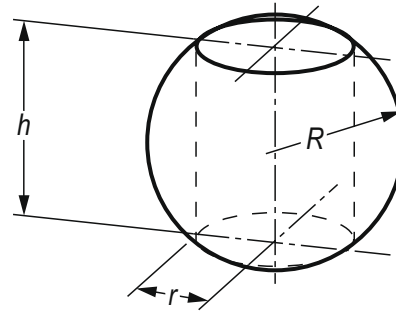


Sphere with Cylindrical Boring

$$V = \frac{\pi}{6} h^3$$

$$A = 2\pi h(R + r)$$

$$h = \sqrt{R^2 - r^2}$$

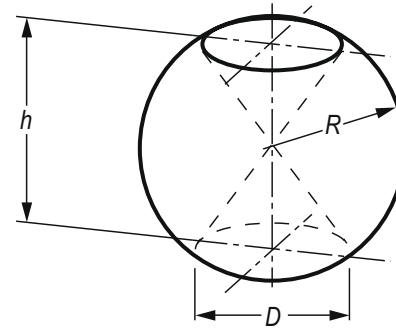


Sphere with Conical Boring

$$V = \frac{2}{3} \pi R^2 h$$

$$A = 2\pi h(R + \frac{D}{2})$$

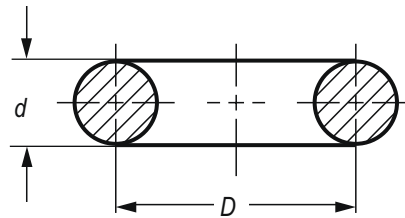
$$D = 2\sqrt{R^2 - h^2}$$



Torus

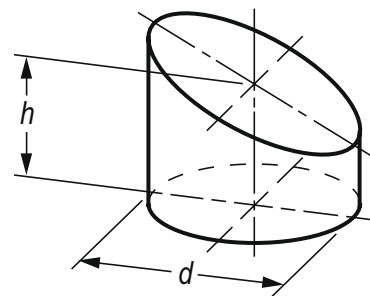
$$V = \frac{\pi^2}{4} D d^2$$

$$A = \pi^2 D d$$



Sliced Cylinder

$$V = \frac{\pi}{4} d^2 h$$

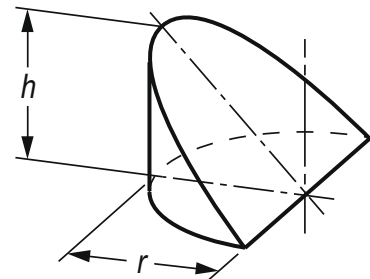


Ungula

$$V = \frac{2}{3} r^2 h$$

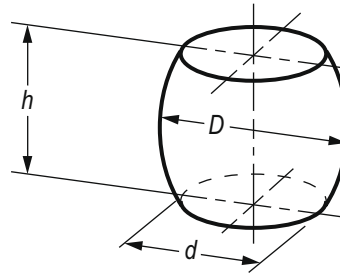
$$A_{\text{mantle}} = 2r h$$

$$A = r^2 \left[2\frac{h}{r} + \frac{\pi}{2} \left(1 + \sqrt{1 + \frac{h^2}{r^2}} \right) \right]$$



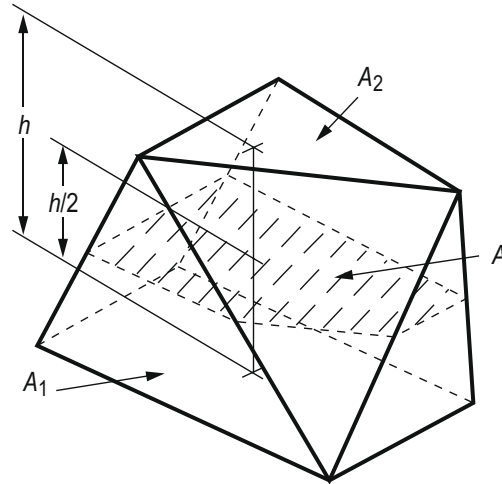
Barrel

$$V \approx \frac{\pi}{12} h (2D^2 + d^2)$$



Prismoid

$$V = \frac{h}{6} (A_1 + A_2 + 4A)$$



Regular Polyhedra

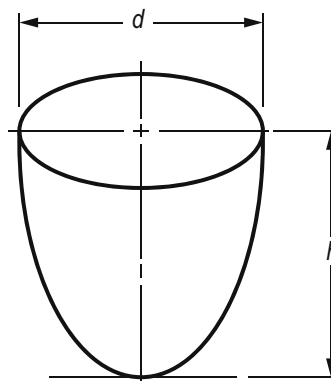
Name	No. of Faces	Form of Faces	Total Surface Area	Volume
Tetrahedron	4	Equilateral triangle	$1.7321a^2$	$0.1179a^3$
Cube	6	Square	$6a^2$	a^3
Octahedron	8	Equilateral triangle	$3.4641a^2$	$0.4714a^3$
Dodecahedron	12	Regular pentagon	$20.6457a^2$	$7.6631a^3$
Icosahedron	20	Equilateral Triangle	$8.6603a^2$	$2.1817a^3$

The radius of a sphere inscribed within a regular polyhedron is:

$$r = \frac{3V}{A}$$

Paraboloid of Revolution

$$V = \frac{\pi}{8} h d^2$$



1.3.3 Calculus

1.3.3.1 Differentiation

For any function $y = f(x)$, the derivative $= D_x y = \frac{dy}{dx} = y'$

$$y' = \lim_{\Delta x \rightarrow 0} \left(\frac{\Delta y}{\Delta x} \right)$$

$$= \lim_{\Delta x \rightarrow 0} \left\{ \frac{[f(x + \Delta x) - f(x)]}{(\Delta x)} \right\}$$

where $y' =$ the slope of the curve $f(x)$

Test for a Maximum

$y = f(x)$ is a maximum for $x = a$, if $f'(a) = 0$ and $f''(a) < 0$

Test for a Minimum

$y = f(x)$ is a minimum for $x = a$, if $f'(a) = 0$ and $f''(a) > 0$

Test for a Point of Inflection

$y = f(x)$ has a point of inflection at $x = a$, if $f''(a) = 0$, and if $f''(x)$ changes sign as x increases through $x = a$

L'Hôpital's Rule

If the fractional function $\frac{f(x)}{g(x)}$ assumes one of the indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$ (where α is finite or infinite), then:

$$\lim_{x \rightarrow \alpha} \frac{f(x)}{g(x)}$$

is equal to the first of the expressions

$$\lim_{x \rightarrow \alpha} \frac{f'(x)}{g'(x)} \quad \lim_{x \rightarrow \alpha} \frac{f''(x)}{g''(x)} \quad \lim_{x \rightarrow \alpha} \frac{f'''(x)}{g'''(x)}$$

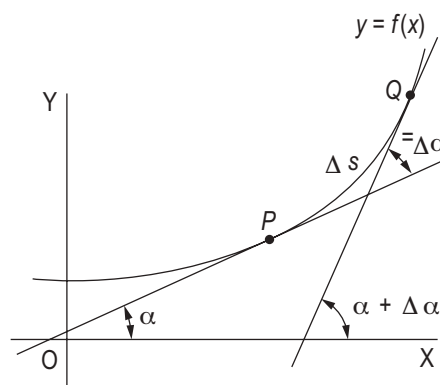
which is not indeterminate, provided such first indicated limit exists.

Curvature K of a Function

The curvature K of a curve at point P is the limit of its average curvature for the arc PQ as Q approaches P . This is also expressed as:

The curvature of a curve at a given point is the rate of change of its inclination with respect to its arc length.

$$K = \lim_{\Delta s \rightarrow 0} \frac{\Delta \alpha}{\Delta s} = \frac{d\alpha}{ds}$$



Curvature in Rectangular Coordinates

$$K = \frac{y''}{\left[1 + (y')^2\right]^{\frac{3}{2}}}$$

When it may be easier to differentiate the function with respect to y rather than x , the notation x' will be used for the derivative.

$$x' = \frac{dx}{dy}$$

$$K = \frac{-x''}{\left[1 + (x')^2\right]^{\frac{3}{2}}}$$

Radius of Curvature

The *radius of curvature* R at any point on a curve is defined as the absolute value of the reciprocal of the curvature K at that point.

$$R = \frac{1}{|K|} \quad (K \neq 0)$$

$$R = \left| \frac{\left[1 + (y')^2\right]^{\frac{3}{2}}}{|y''|} \right| \quad (y'' \neq 0)$$

List of Derivatives

u , v , and w represent functions of x .

a , c , and n represent constants.

Arguments of trigonometric functions are in radians. The following definitions are used:

$$\arcsin u = \sin^{-1}(u), \quad (\sin u)^{-1} = \frac{1}{\sin u}$$

$$1. \quad \frac{dc}{dx} = 0$$

$$2. \quad \frac{dx}{dx} = 1$$

$$3. \quad \frac{d(cu)}{dx} = c \frac{du}{dx}$$

$$4. \quad \frac{d(u + v - w)}{dx} = \frac{du}{dx} + \frac{dv}{dx} - \frac{dw}{dx}$$

$$5. \quad \frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$6. \quad \frac{d(uvw)}{dx} = u v \frac{dw}{dx} + u w \frac{dv}{dx} + v w \frac{du}{dx}$$

$$7. \quad \frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$8. \quad \frac{d(u^n)}{dx} = n u^{n-1} \frac{du}{dx}$$

$$9. \frac{d[f(u)]}{dx} = \left\{ \frac{d[f(u)]}{du} \right\} \frac{du}{dx}$$

$$10. \frac{du}{dx} = \frac{1}{\left(\frac{dx}{du}\right)}$$

$$11. \frac{d(\log_a u)}{dx} = (\log_a e) \frac{1}{u} \frac{du}{dx}$$

$$12. \frac{d(\ln u)}{dx} = \frac{1}{u} \frac{du}{dx}$$

$$13. \frac{d(a^u)}{dx} = (\ln a) a^u \frac{du}{dx}$$

$$14. \frac{d(e^u)}{dx} = e^u \frac{du}{dx}$$

$$15. \frac{d(u^v)}{dx} = v u^{v-1} \frac{du}{dx} + (\ln u) u^v \frac{dv}{dx}$$

$$16. \frac{d(\sin u)}{dx} = \cos u \frac{du}{dx}$$

$$17. \frac{d(\cos u)}{dx} = -\sin u \frac{du}{dx}$$

$$18. \frac{d(\tan u)}{dx} = \sec^2 u \frac{du}{dx}$$

$$19. \frac{d(\cot u)}{dx} = -\csc^2 u \frac{du}{dx}$$

$$20. \frac{d(\sec u)}{dx} = \sec u \tan u \frac{du}{dx}$$

$$21. \frac{d(\csc u)}{dx} = -\csc u \cot u \frac{du}{dx}$$

$$22. \frac{d(\sin^{-1} u)}{dx} = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx} \quad \left(-\frac{\pi}{2} \leq \sin^{-1} u \leq \frac{\pi}{2} \right)$$

$$23. \frac{d(\cos^{-1} u)}{dx} = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx} \quad (0 \leq \cos^{-1} u \leq \pi)$$

$$24. \frac{d(\tan^{-1} u)}{dx} = \frac{1}{1+u^2} \frac{du}{dx} \quad \left(-\frac{\pi}{2} < \tan^{-1} u < \frac{\pi}{2} \right)$$

$$25. \frac{d(\cot^{-1} u)}{dx} = -\frac{1}{1+u^2} \frac{du}{dx} \quad (0 < \cot^{-1} u < \pi)$$

$$26. \frac{d(\sec^{-1} u)}{dx} = \frac{1}{u\sqrt{u^2-1}} \frac{du}{dx} \quad \left(0 < \sec^{-1} u \leq \frac{\pi}{2} \right) \text{ and } \left(-\pi \leq \sec^{-1} u < -\frac{\pi}{2} \right)$$

$$27. \frac{d(\csc^{-1} u)}{dx} = -\frac{1}{u\sqrt{u^2-1}} \frac{du}{dx} \quad \left(0 < \csc^{-1} u \leq \frac{\pi}{2} \right) \text{ and } \left(-\pi < \csc^{-1} u \leq -\frac{\pi}{2} \right)$$

Parametric Form of the Derivative

$$y'(x(t)) = \frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = \frac{\dot{y}}{\dot{x}}$$

$$y''(x(t)) = \frac{d^2y}{(dx)^2} = \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{\dot{x}^3}$$

where

$$\dot{y} = \frac{dy}{dt}$$

$$\ddot{y} = \frac{d^2y}{dt^2}$$

Derivative of Inverse Functions

The equation $y = f(x)$ solved for x gives the inverse function $x = \varphi(y)$.

$$f'(x) = \frac{1}{\varphi'(y)}$$

1.3.3.2 Integration

The indefinite integral $F(x)$ is a function such that $F'(x) = f(x)$.

$$\int f(x)dx = F(x) + C$$

C is an unknown constant which disappears on differentiation.

The definite integral:

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x_i = \int_a^b f(x)dx = F(x) \Big|_a^b = F(b) - F(a)$$

Also, $\Delta x_i \rightarrow 0$ for all i .

To find the integral: Use the list of indefinite integrals (below), integration by parts (equation #6 in the list), integration by substitution, and separation of rational fractions into partial fractions.

List of Indefinite Integrals

u , v , and w represent functions of x .

a , c , and n represent constants.

Arguments of trigonometric functions are in radians. The following definitions are used:

$$\arcsin u = \sin^{-1}(u), \quad (\sin u)^{-1} = \frac{1}{\sin u}$$

Note: A constant of integration should be added to the integrals.

1. $\int d f(x) = f(x)$
2. $\int dx = x$
3. $\int a f(x)dx = a \int f(x)dx$
4. $\int [u(x) \pm v(x)]dx = \int u(x)dx \pm \int v(x)dx$

5. $\int x^m dx = \frac{x^{m+1}}{m+1}$ ($m \neq -1$)
6. $\int u(x) dv(x) = u(x)v(x) - \int v(x) du(x)$
7. $\int \frac{dx}{ax+b} = \frac{1}{a} \ln|ax+b|$ for $a = 1$ and $b = 0$: $\int \frac{dx}{x} = \ln x$
8. $\int \frac{dx}{\sqrt{x}} = 2\sqrt{x}$
9. $\int a^x dx = \frac{a^x}{\ln a}$
10. $\int \sin x dx = -\cos x$
11. $\int \cos x dx = \sin x$
12. $\int \sin^2 x dx = \frac{x}{2} - \frac{\sin 2x}{4}$
13. $\int \cos^2 x dx = \frac{x}{2} + \frac{\sin 2x}{4}$
14. $\int x \sin x dx = \sin x - x \cos x$
15. $\int x \cos x dx = \cos x + x \sin x$
16. $\int \sin x \cos x dx = \frac{\sin^2 x}{2}$
17. $\int \sin ax \cos bx dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}$ ($a^2 \neq b^2$)
18. $\int \tan x dx = -\ln|\cos x| = \ln|\sec x|$
19. $\int \cot x dx = -\ln|\csc x| = \ln|\sin x|$
20. $\int \tan^2 x dx = \tan x - x$
21. $\int \cot^2 x dx = -\cot x - x$
22. $\int e^{ax} dx = \left(\frac{1}{a}\right)e^{ax}$
23. $\int xe^{ax} dx = \left(\frac{e^{ax}}{a^2}\right)(ax-1)$
24. $\int \ln x dx = x[\ln(x)-1]$ ($x > 0$)
25. $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$ ($a \neq 0$)
26. $\int \frac{dx}{ax^2+c} = \frac{1}{\sqrt{ac}} \tan^{-1}\left(x\sqrt{\frac{a}{c}}\right)$ ($a > 0, c > 0$)
- 27a. $\int \frac{dx}{ax^2+bx+c} = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}}$ ($4ac-b^2 > 0$)
- 27b. $\int \frac{dx}{ax^2+bx+c} = \frac{1}{\sqrt{b^2-4ac}} \ln \left| \frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}} \right|$ ($b^2-4ac > 0$)
- 27c. $\int \frac{dx}{ax^2+bx+c} = -\frac{2}{2ax+b}$ ($b^2-4ac = 0$)

1.3.3.3 Multivariable Calculus

Partial Derivatives

In a function of two independent variables x and y , a derivative with respect to one of the variables may be found if the other variable is *assumed* to remain constant. If y is *kept fixed*, the function

$$z = f(x, y)$$

becomes a function of the *single variable* x , and its derivative (if it exists) can be found. This derivative is called the *partial derivative of z with respect to x* . The partial derivative with respect to x is denoted as follows:

$$\frac{\partial z}{\partial x} = \frac{\partial f(x, y)}{\partial x}$$

Total Derivative

Given $f(x, y)$, then the total derivative df is

$$df = \left(\frac{\partial f}{\partial x} \right)_y dx + \left(\frac{\partial f}{\partial y} \right)_x dy$$

Chain Rule

Given $f(x, y)$ where $x = g(t)$ and $y = h(t)$, then

$$\frac{df}{dt} = \left(\frac{\partial f}{\partial x} \right)_y \frac{dx}{dt} + \left(\frac{\partial f}{\partial y} \right)_x \frac{dy}{dt}$$

Identities in Partial Derivatives

$$\left(\frac{\partial x}{\partial x} \right)_z = 1$$

$$\left(\frac{\partial x}{\partial z} \right)_x = 0$$

Implicit Differentiation

If $f(x, y, z)$ cannot be converted to an explicit expression in the form of $z = f^*(x, y)$, then

$$\left(\frac{\partial z}{\partial x} \right)_y = \frac{-\left(\frac{\partial f}{\partial x} \right)_{y,z}}{\left(\frac{\partial f}{\partial z} \right)_{x,y}} \quad \text{and} \quad \left(\frac{\partial z}{\partial y} \right)_x = \frac{-\left(\frac{\partial f}{\partial y} \right)_{x,z}}{\left(\frac{\partial f}{\partial z} \right)_{x,y}}$$

Rules for changing the constant or the variable on a partial derivative:

Given $f(x,y,z) = \text{constant}$, then

$$\left(\frac{\partial f}{\partial x}\right)_z = \left(\frac{\partial f}{\partial x}\right)_y + \left(\frac{\partial f}{\partial y}\right)_x \left(\frac{\partial y}{\partial x}\right)_z$$

$$\left(\frac{\partial f}{\partial z}\right)_x = \left(\frac{\partial f}{\partial y}\right)_x \left(\frac{\partial y}{\partial z}\right)_x$$

Cyclic relation:

$$\left(\frac{\partial z}{\partial x}\right)_y \cdot \left(\frac{\partial x}{\partial y}\right)_z \cdot \left(\frac{\partial y}{\partial z}\right)_x = -1$$

Reciprocity relation:

$$\left(\frac{\partial z}{\partial x}\right)_y = \frac{1}{\left(\frac{\partial x}{\partial z}\right)_y}$$

1.3.3.4 Differential Equations

A common class of ordinary linear differential equations is

$$b_n \frac{d^n y(x)}{dx^n} + \dots + b_1 \frac{dy(x)}{dx} + b_0 y(x) = f(x)$$

where $b_n, \dots, b_p, \dots, b_1, b_0$ are constants.

When the equation is a homogeneous differential equation, $f(x) = 0$, the solution is

$$y_h(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x} + \dots + C_i e^{r_i x} + C_n e^{r_n x}$$

where r_n is the n th distinct root of the characteristic polynomial $P(x)$ with

$$P(r) = b_n r^n + b_{n-1} r^{n-1} + b_1 r + b_0$$

If the root $r_1 = r_2$, then $C_2 e^{r_2 x}$ is replaced with $C_2 x e^{r_1 x}$.

Higher orders of multiplicity imply higher powers of x . The complete solution for the differential equation is

$$y(x) = y_h(x) + y_p(x)$$

where $y_p(x)$ is any particular solution with $f(x)$ present. If $f(x)$ has $e^{r_n x}$ terms, then resonance is manifested. Furthermore, specific $f(x)$ forms result in specific $y_p(x)$ forms, some of which are

$f(x)$	$y_p(x)$
A	B
$Ae^{\alpha x}$	$Be^{\alpha x}, \alpha \neq r_n$
$A_1 \sin \omega x + A_2 \cos \omega x$	$B_1 \sin \omega x + B_2 \cos \omega x$

Common First-Order Differential Equations and Their Solutions

Form	Solution	Substitution/Conditions
Linear, homogeneous ODE with constant coefficients $y' + a y = 0$	$y(x) = C e^{-ax}$	C is a constant that satisfies the initial condition.
Linear, homogeneous ODE $y' + p(x) y = 0$	$y(x) = C e^{(-\int p(x) dx)}$	C is a constant that satisfies the initial condition.

Common First-Order Differential Equations and Their Solutions (cont'd)

Form	Solution	Substitution/Conditions
Linear, inhomogeneous ODE with constant coefficients $\tau y' + y = K p(t)$	$y(t) = KA + (KB - KA)\left(1 - e^{-\frac{t}{\tau}}\right)$ $\frac{t}{\tau} = \ln\left(\frac{KB - KA}{KB - y}\right)$	$p(t) = \begin{cases} A & t < 0 \\ B & t > 0 \end{cases} \quad y(0) = K A$ τ = time constant, K = gain
	Comment: Solution is for a step function.	
Implicit ODE, no y term $x = f(y')$	$x = f(p)$ $y = \int p f(p) dp + C$	Substitution: $y' = p$
	Comment: Elimination of p leads to a solution in parametric form.	
Implicit ODE, no x term $y = f(y')$	$x = \int \frac{f(p)}{p} dp + C$ $y = f(p)$	$y' = p$
	Comment: Elimination of p leads to a solution in parametric form.	
Separable ODE $y' = \frac{dy}{dx} = \frac{f(x)}{g(y)}$	$\int g(y) dy = \int f(x) dx + C$	
	Comment: The variables x and y can be separated into the left and right sides of the equation.	
Similarity ODE $y' = f\left(\frac{y}{x}\right)$	$\int \frac{dx}{x} = \int \frac{du}{f(u) - u} + C$	Substitution: $u = \frac{y}{x}$ $y' = u + x \frac{du}{dx}$
	Comment: Check whether it is possible to transform to $f\left(\frac{y}{x}\right)$.	

Common Second-Order Differential Equations and Their Solutions

Form	Solution	Substitution
ODE, y and y' terms missing $y'' = f(x)$	$y(x) = C_1 + C_2 x + \int \left[\int f(x) dx \right] dx$	
	Comment: Start the calculation with the inner integral.	
ODE, y term missing $y'' + p_1(x) f(y') = 0$	$\int \frac{du}{f(u)} = - \int p_1(x) dx + C_1$ $y = \int u dx + C_2$	Substitution: $u = y'$ $y'' = \frac{du}{dx} \quad f(y') = f(u)$
ODE, x term missing $y'' = f(y, y')$	$u \frac{du}{dy} = f(y, u)$ $x = \int \frac{dy}{u(y)} + C$	Substitution: $u = y'$ $y'' = \frac{du}{dx} = u \frac{du}{dy} = f(y, u)$ Then substitute $y' = \frac{dy}{dx}$ for u . where $u = u(y)$ and $y = y(x)$

Common Second-Order Differential Equations and Their Solutions

Form	Solution	Substitution
Linear, homogeneous ODE with constant coefficients $y'' + a y' + b y = 0$	Solution depends on the values of a and b .	$r_{1,2} = \frac{1}{2}(-a \pm \sqrt{a^2 - 4b})$
	$y(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x}$	$a^2 > 4b$ (overdamped)
	$y(x) = (C_1 + C_2 x) e^{r_1 x}$	$a^2 = 4b$ (critically damped)
	$y(x) = e^{\alpha x}$ $[C_1 \cos(\beta x) + C_2 \sin(\beta x)]$ $\alpha = -\frac{1}{2}a \quad \beta = \frac{1}{2}\sqrt{4b - a^2}$	$a^2 < 4b$ (underdamped)

1.3.3.5 The Fourier Transform and Its Inverse

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi f t} dt$$

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{j2\pi f t} df$$

We say that $x(t)$ and $X(f)$ form a *Fourier transform pair*:

$$x(t) \longleftrightarrow X(f)$$

Fourier Transform Pairs

Fourier Transform Pairs	
$x(t)$	$X(f)$
1	$\delta(f)$
$\delta(t)$	1
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\Pi\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}(\tau f)$
$\operatorname{sinc}(Bt)$	$\frac{1}{B}\Pi\left(\frac{f}{B}\right)$
$\Lambda\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}^2(\tau f)$
$e^{-at}u(t)$	$\frac{1}{a + j2\pi f} \quad a > 0$
$te^{-at}u(t)$	$\frac{2a}{a^2 + (2\pi f)^2} \quad a > 0$
$e^{-a t }$	$\frac{2a}{a^2 + (2\pi f)^2} \quad a > 0$
$e^{-(at)^2}$	$\frac{\sqrt{\pi}}{a} e^{-\left(\frac{\pi f}{a}\right)^2}$

Fourier Transform Pairs (cont'd)

$x(t)$	$X(f)$
$\cos(2\pi f_0 t + \theta)$	$\frac{1}{2} [e^{j\theta} \delta(f - f_0) + e^{-j\theta} \delta(f + f_0)]$
$\sin(2\pi f_0 t + \theta)$	$\frac{1}{2j} [e^{j\theta} \delta(f - f_0) - e^{-j\theta} \delta(f + f_0)]$
$\sum_{n=-\infty}^{n=+\infty} \delta(t - nT_s)$	$f_s \sum_{k=-\infty}^{k=+\infty} \delta(f - kf_s) \quad f_s = \frac{1}{T_s}$

Fourier Transform Theorems

Fourier Transform Theorems

Linearity	$ax(t) + by(t)$	$aX(f) + bY(f)$
Scale change	$x(at)$	$\frac{1}{ a } X\left(\frac{f}{a}\right)$
Time reversal	$x(-t)$	$X(-f)$
Duality	$X(t)$	$x(-f)$
Time shift	$x(t - t_0)$	$X(f) e^{-j2\pi f t_0}$
Frequency shift	$x(t) e^{-j2\pi f_0 t}$	$X(f - f_0)$
Modulation	$x(t) \cos 2\pi f_0 t$	$\frac{1}{2} X(f - f_0) + \frac{1}{2} X(f + f_0)$
Multiplication	$x(t) \cdot y(t)$	$X(f) * Y(f)$
Convolution	$x(t) * y(t)$	$X(f) \cdot Y(f)$
Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
Integration	$\int_{-\infty}^t x(\lambda) d\lambda$	$\frac{1}{j2\pi f} X(f) + \frac{1}{2} X(0) \delta(f)$

1.3.3.6 Laplace Transforms

The unilateral Laplace transform pair:

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

$$f(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} F(s) e^{st} ds$$

where $s = \sigma + j\omega$

represents a powerful tool for the transient and frequency response of linear time invariant systems. Some useful Laplace transform pairs are

Laplace Transform Pairs

$f(t)$	$F(s)$
$\delta(t)$, Impulse at $t = 0$	1
$u(t)$, Step at $t = 0$	$\frac{1}{s}$
$t[u(t)]$, Ramp at $t = 0$	$\frac{1}{s^2}$
e^{-at}	$\frac{1}{(s+a)}$
te^{-at}	$\frac{1}{(s+a)^2}$
$e^{-\alpha t} \sin \beta t$	$\frac{\beta}{[(s+\alpha)^2 + \beta^2]}$
$e^{-\alpha t} \cos \beta t$	$\frac{(s+\alpha)}{[(s+\alpha)^2 + \beta^2]}$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - \sum_{m=0}^{n-1} s^{n-m-1} \frac{d^m f(0)}{dt^m}$
$\int_0^t f(\tau) d\tau$	$\left(\frac{1}{s}\right) F(s)$
$\int_0^t x(t-\tau) h(\tau) d\tau$	$H(s) X(s)$
$f(t-\tau) u(t-\tau)$	$e^{-\tau s} F(s)$
$\lim_{t \rightarrow \infty} f(t)$	$\lim_{s \rightarrow 0} sF(s)$
$\lim_{t \rightarrow 0} f(t)$	$\lim_{s \rightarrow \infty} sF(s)$

The last two transforms represent the Final Value Theorem (F.V.T.) and Initial Value Theorem (I.V.T.), respectively. It is assumed that the limits exist.

1.3.4 Statistics and Probability

1.3.4.1 Mean, Median, and Mode

If $X_1, X_2, \dots, X_n$ represents the values of a discrete random sample of n items or observations, the *arithmetic mean* of these items or observations, denoted $\bar{X}$, is defined as

$$\bar{X} = \left(\frac{1}{n}\right)(X_1 + X_2 + \dots + X_n) = \left(\frac{1}{n}\right)\sum_{i=1}^n X_i$$

$\bar{X} \rightarrow \mu$ for sufficiently large values of n .

The *weighted arithmetic mean* is

$$\bar{X}_w = \frac{\sum w_i X_i}{\sum w_i}$$

where X_i = the value of the i th observation and w_i = the weight applied to X_i .

The variance of the population is the *arithmetic mean* of the *squared deviations from the population mean*. If μ is the arithmetic mean of a discrete population of size N , the *population variance* is defined by

$$\begin{aligned}\sigma^2 &= \left(\frac{1}{N}\right)\left[(X_1 - \mu)^2 + (X_2 - \mu)^2 + \dots + (X_N - \mu)^2\right] \\ &= \left(\frac{1}{N}\right)\sum_{i=1}^N (X_i - \mu)^2\end{aligned}$$

Standard deviation formulas are

$$\begin{aligned}\sigma_{\text{population}} &= \sqrt{\left(\frac{1}{N}\right)\sum (X_i - \mu)^2} \\ \sigma_{\text{sum}} &= \sqrt{\sigma_1^2 + \sigma_2^2 + \dots + \sigma_n^2} \\ \sigma_{\text{series}} &= \sigma\sqrt{n} \\ \sigma_{\text{mean}} &= \frac{\sigma}{\sqrt{n}} \\ \sigma_{\text{product}} &= \sqrt{A^2 \sigma_b^2 + B^2 \sigma_a^2}\end{aligned}$$

The *sample variance* is $s^2 = \left[\frac{1}{(n-1)}\right]\sum_{i=1}^n (X_i - \bar{X})^2$

The *sample standard deviation* is $= \sqrt{\left(\frac{1}{n-1}\right)\sum_{i=1}^n (X_i - \bar{X})^2}$

The *sample coefficient of variation* is $CV = \frac{s}{\bar{X}}$

The *sample geometric mean* is $\sqrt[n]{X_1 X_2 X_3 \dots X_n}$

The *sample root-mean-square value* is $\sqrt{\left(\frac{1}{n}\right)\sum X_i^2}$

When the discrete data are rearranged in increasing order and n is odd, the median is the value of the $\left(\frac{n+1}{2}\right)^{\text{th}}$ item.

When n is even, the median is the average of the $\left(\frac{n}{2}\right)^{\text{th}}$ and $\left(\frac{n}{2} + 1\right)^{\text{th}}$ items.

The *mode* of a set of data is the value that occurs with greatest frequency.

The *sample range* R is the largest sample value minus the smallest sample value.

1.3.4.2 Permutations and Combinations

A *permutation* is a particular sequence of a given set of objects. A *combination* is the set itself without reference to order.

The number of different *permutations* of n distinct objects *taken* r *at a time* is:

$$P(n, r) = \frac{n!}{(n-r)!}$$

An alternative notation for $P(n, r)$ is nPr .

The number of different *combinations* of n distinct objects *taken* r *at a time* is:

$$C(n, r) = \frac{P(n, r)}{r!} = \frac{n!}{r!(n-r)!}$$

nCr and $\binom{n}{r}$ are alternative notations for $C(n, r)$.

The number of different *permutations* of n objects *taken* n *at a time*, given that n_i are of type i , where $i = 1, 2, \dots, k$ and $\sum n_i = n$, is

$$P(n; n_1, n_2, \dots, n_k) = \frac{n!}{n_1! n_2! \dots n_k!}$$

1.3.4.3 Probabilities

Property 1. General Character of Probability

The probability $P(E)$ of an event E is a real number in the range of 0 to 1. The probability of an impossible event is 0 and that of an event certain to occur is 1.

Property 2. Law of Total Probability

$$P(A + B) = P(A) + P(B) - P(A, B)$$

where $P(A+B)$ = the probability that either A or B occurs alone or that both occur together

$P(A)$ = the probability that A occurs

$P(B)$ = the probability that B occurs

$P(A, B)$ = the probability that both A and B occur simultaneously

Property 3. Law of Compound or Joint Probability

If neither $P(A)$ nor $P(B)$ is zero, then

$$P(A, B) = P(A) P(B | A) = P(B) P(A | B)$$

where

$P(B | A)$ = the probability that B occurs given the fact that A has occurred

$P(A | B)$ = the probability that A occurs given the fact that B has occurred

If either $P(A)$ or $P(B)$ is zero, then $P(A, B) = 0$.

Bayes' Theorem:

$$P(B_j | A) = \frac{P(B_j) P(A | B_j)}{\sum_{i=1}^n P(A | B_i) P(B_i)}$$

where $P(A_j)$ is the probability of event A_j within the population of A

$P(B_j)$ is the probability of event B_j within the population of B

1.3.4.4 Distributions and Expected Values

A random variable X has a probability associated with each of its possible values. The probability is termed a *discrete probability* if X can assume only discrete values, or

$$X = x_1, x_2, x_3, \dots, x_n$$

The discrete probability of any single event $X = x_i$ occurring is defined as $P(x_i)$ while the *probability mass function* of the random variable X is defined by

$$f(x_k) = P(X = x_k), \quad k = 1, 2, \dots, n$$

Probability Density Function

If X is continuous, the *probability density function*, f , is defined such that

$$P(a \leq X \leq b) = \int_a^b f(x) dx$$

See the table of probability and density functions.

Cumulative Distribution Function

The *cumulative distribution function*, F , of a discrete random variable, X , that has a probability distribution described by $P(x_i)$ is defined as

$$F(x_m) = \sum_{k=1}^m P(x_k) = P(X \leq x_m), \quad m = 1, 2, \dots, n$$

If X is continuous, the cumulative distribution function F is defined by

$$F(x) = \int_{-\infty}^x f(t) dt$$

which implies that $F(a)$ is the probability that $X \leq a$.

Expected Values

Let X be a *discrete random variable* having probability mass function:

$$f(x_k), \quad k = 1, 2, \dots, n$$

The expected value of X is defined as

$$\mu = E[X] = \sum_{k=1}^n x_k f(x_k)$$

The variance of X is defined as

$$\sigma^2 = V[X] = \sum_{k=1}^n (x_k - \mu)^2 f(x_k)$$

Let X be a continuous random variable having a density function $f(X)$ and let $Y = g(X)$ be some general function. The expected value of Y is

$$E[Y] = E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x) dx$$

The mean or expected value of the random variable X is now defined as

$$\mu = E[X] = \int_{-\infty}^{\infty} x f(x) dx$$

while the variance is

$$\sigma^2 = V[X] = E[(X - \mu)^2] = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$$

The standard deviation is $\sigma = \sqrt{V[X]}$.

The coefficient of variation is defined as $\frac{\sigma}{\mu}$.

Combinations of Random Variables

$$Y = a_1 X_1 + a_2 X_2 + \dots + a_n X_n$$

The expected value of Y is $\mu_y = E(Y) = a_1 E(X_1) + a_2 E(X_2) + \dots + a_n E(X_n)$.

If the random variables are statistically *independent*, then the variance of Y is

$$\begin{aligned} \sigma_y^2 &= V(Y) = a_1^2 V(X_1) + a_2^2 V(X_2) + \dots + a_n^2 V(X_n) \\ &= a_1^2 \sigma_1^2 + a_2^2 \sigma_2^2 + \dots + a_n^2 \sigma_n^2 \end{aligned}$$

Also, the standard deviation of Y is $\sigma_y = \sqrt{\sigma_y^2}$.

When $Y = f(X_1, X_2, \dots, X_n)$ and X_i are independent, the standard deviation of Y is expressed as

$$\sigma_y = \sqrt{\left(\frac{\partial f}{\partial X_1} \sigma_{x_1}\right)^2 + \left(\frac{\partial f}{\partial X_2} \sigma_{x_2}\right)^2 + \dots + \left(\frac{\partial f}{\partial X_n} \sigma_{x_n}\right)^2}$$

Normal Distribution (Gaussian Distribution)

This is a unimodal distribution, the mode being $x = \mu$, with two points of inflection (each located at a distance σ to either side of the mode). The averages of n observations tend to become normally distributed as n increases. The variate x is said to be normally distributed if its density function $f(x)$ is given by an expression of the form

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where

μ = the population mean

σ = the standard deviation of the population

$$-\infty \leq x \leq \infty$$

When $\mu = 0$ and $\sigma^2 = \sigma = 1$, the distribution is called a *standardized* or *unit normal* distribution. Then

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

where $-\infty \leq x \leq \infty$

It is noted that $Z = \frac{x - \mu}{\sigma}$ follows a standardized normal distribution function.

A unit normal distribution table is included in this section. In the table, the following notations are used:

$F(x)$ = the area under the curve from $-\infty$ to x

$R(x)$ = the area under the curve from x to ∞

$W(x)$ = the area under the curve between $-x$ and x

$F(-x) = 1 - F(x)$

1.3.4.5 Confidence Intervals

Confidence Interval for the Mean μ of a Normal Distribution

When standard deviation σ is known:

$$\bar{X} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

When standard deviation σ is not known:

$$\bar{X} - t_{\alpha/2} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{\alpha/2} \frac{s}{\sqrt{n}}$$

where $t_{\alpha/2}$ corresponds to $n - 1$ degrees of freedom.

Confidence Interval for the Difference Between Two Means μ_1 and μ_2

When standard deviations σ_1 and σ_2 are known:

$$\bar{X}_1 - \bar{X}_2 - Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq \bar{X}_1 - \bar{X}_2 + Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

When standard deviations σ_1 and σ_2 are not known:

$$\begin{aligned} \bar{X}_1 - \bar{X}_2 - t_{\alpha/2} \sqrt{\frac{\left(\frac{1}{n_1} + \frac{1}{n_2}\right) \left[(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2\right]}{n_1 + n_2 - 2}} &\leq \mu_1 - \mu_2 \leq \\ \bar{X}_1 - \bar{X}_2 + t_{\alpha/2} \sqrt{\frac{\left(\frac{1}{n_1} + \frac{1}{n_2}\right) \left[(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2\right]}{n_1 + n_2 - 2}} & \end{aligned}$$

where $t_{\alpha/2}$ corresponds to $n_1 + n_2 - 2$ degrees of freedom.

Confidence Intervals for the Variance σ^2 of a Normal Distribution

$$\frac{(n-1)s^2}{x_{\alpha/2, n-1}^2} \leq \sigma^2 \leq \frac{(n-1)s^2}{x_{1-\alpha/2, n-1}^2}$$

Sample Size

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad n = \left(\frac{\sigma z_{\alpha/2}}{\bar{x} - \mu} \right)^2$$

The Central Limit Theorem

Let $X_1, X_2, \dots, X_n$ be a sequence of independent and identically distributed random variables having mean μ and variance σ^2 . Then for a large n , the Central Limit Theorem asserts that the sum

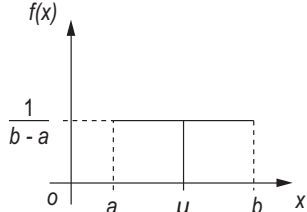
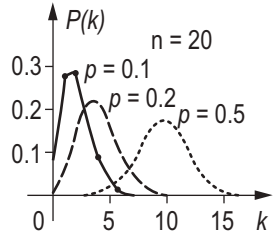
$Y = X_1 + X_2 + \dots + X_n$ is approximately normal

$$\mu_y = \mu$$

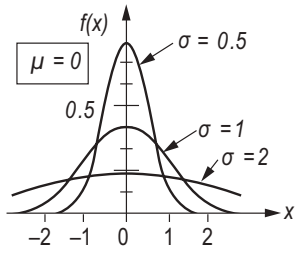
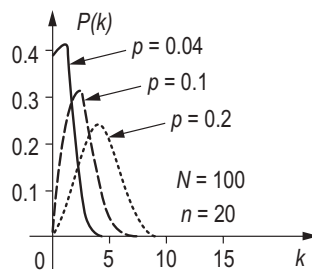
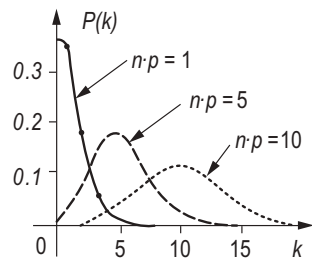
and the standard deviation is $\sigma_y = \frac{\sigma}{\sqrt{n}}$.

1.3.4.6 Probability and Density Functions

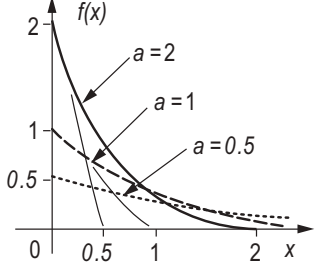
Probability and Density Functions

Kind of Distribution	Probability Density Function $f(x)$ Distribution Function $F(x)$	Expected Mean (μ), Mean ($\bar{x}$), Variance (σ^2)	Form of the Density Function
General (continuous)	Comment: General distribution for continuous values		
	$f(x)$ $F(x) = \int_{-\infty}^x f(t) dt$	$\bar{x} = \int_{-\infty}^{\infty} x f(x) dx$ $\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$	
General (discrete)	Comment: General distribution for discrete values: n is the number in a random sample, x_i is the discrete value of the random variable, and P_i is the probability.		
	P_i $F(x) = \sum_{i < x} P_i$	$\bar{x} = \sum_{i=1}^n (x_i P_i)$ $\sigma^2 = \sum_{i=1}^n (x_i^2 P_i - \mu^2)$	
Uniform	Comment: Random variable $x = 0$ only within the interval $<a, b>$, where each value is of equal probability. Use when only maximum and minimum values are known but no other information about the distribution in between.		
	$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{for outside} \end{cases}$ $F(x) = \begin{cases} 0 & \text{for } -\infty < x < a \\ \frac{x-a}{b-a} & \text{for } a \leq x \leq b \\ 1 & \text{for } b < x < \infty \end{cases}$	$\bar{x} = \frac{a+b}{2}$ $\sigma^2 = \frac{(b-a)^2}{12}$	
Binomial	Comment: If $P(k)$ is the probability that in n random samples exactly k errors will occur, the error probability is p . Lot size is assumed to be ∞ .		
	$P(k) = \binom{n}{k} p^k (1-p)^{n-k}$ $F(x) = \sum_{k < x} \binom{n}{k} p^k (1-p)^{n-k}$	$\bar{x} = n p$ $\sigma^2 = n p (1-p)$	

Probability and Density Functions (cont'd)

Kind of Distribution	Probability Density Function $f(x)$ Distribution Function $F(x)$	Expected Mean (μ), Mean ($\bar{x}$), Variance (σ^2)	Form of the Density Function
Normal (Gaussian)	Comment: Often obtained in practice as measured values with a bell-shaped distribution occurring around a mean value. Special case of the binomial distribution with $n \rightarrow \infty$ and $p = 0.5$.		
	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$ $F(x) = \int_{-\infty}^x \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right] dt$	μ σ^2	
Standardized (unit normal)	Comment: Special case of the Normal (Gaussian) distribution. A unit normal table is included below.		
	$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$ $F(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt$	$\mu = 0$ $\sigma^2 = 1$	---
Hypergeometric	Comment: Sample of dichotomous population (population of two types, e.g., defective/not defective parts) without replacement. N is lot size, pN is number of defective parts in the lot, P is the probability that in n random samples k will be defective.		
	$P(k) = \frac{\binom{pN}{k} \binom{N(1-p)}{n-k}}{\binom{N}{n}}$ $F(x) = \sum_{k \leq x} \frac{\binom{pN}{k} \binom{N(1-p)}{n-k}}{\binom{N}{n}}$	$\bar{x} = np$ $\sigma^2 = np \frac{N-n}{N-1} (1-p)$	
Poisson	Comment: $P(k)$ is the probability that in n random samples k errors will occur. Used for curves in a random sampling valuation. Conditions: large value of random samples with a small value for proportion defective.		
	$P(k) = \frac{(np)^k}{k!} e^{-np}$ $F(x) = \sum_{k \leq x} \frac{(np)^k}{k!} e^{-np}$	$\bar{x} = np$ $\sigma^2 = np$	

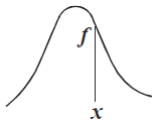
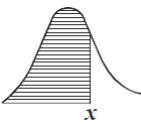
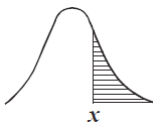
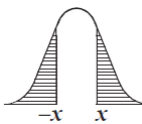
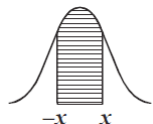
Probability and Density Functions (cont'd)

Kind of Distribution	Probability Density Function $f(x)$ Distribution Function $F(x)$	Expected Mean (μ), Mean ($\bar{x}$), Variance (σ^2)	Form of the Density Function
Exponential	Comment: Special case of the Poisson distribution for $x = 0$ that gives the probability without error. When used for reliability calculations, replace $(a \cdot x)$ with failure rate r multiplied by control time t .		
	$f(x) = a e^{-ax}$ $a > 0$ $x \geq 0$ $F(x) = 1 - e^{-ax}$	$\bar{x} = \frac{1}{a}$ $\sigma^2 = \frac{1}{a^2}$	
Geometric	Comment: Describes the number of trials needed to get the first success, with p as the success parameter.		
	$f(x,p) = p(1-p)^{x-1}$ $F(x) = \sum_{n=1}^x p(1-p)^{n-1}$	$\mu = \frac{1}{p}$ $\sigma^2 = \frac{1-p}{p^2}$	---
Negative Binomial	Comment: Describes the trial number of the k th success, with k as the stopping parameter and p as the success probability.		
	$P(k) = \binom{n-1}{k-1} p^k (1-p)^{n-k}$ $F(x) = \sum_{k < x} \binom{n-1}{k-1} p^k (1-p)^{n-k}$	$\bar{x} = k \frac{1}{p}$ $\sigma^2 = k \frac{(1-p)}{p^2}$	---
Gamma	Comment: Gamma distribution is widely used to model physical quantities that take positive values. The Gamma function is defined as		
	$f(x) = \frac{1}{b^k \Gamma(k)} x^{k-1} e^{-x/b}$ $F(x) = \frac{\Gamma(k, \frac{x}{b})}{\Gamma(k)} \quad \text{where } k > 0, b > 0$	$\bar{x} = bk$ $\sigma^2 = b^2 k$	---
Weibull	Comment: Note that when $k = 1$, the Weibull distribution reduces to the exponential distribution with parameter 1.		
	$f(t) = \frac{k}{b^k} t^{k-1} e^{-\left(\frac{t}{b}\right)^k}$ $F(t) = 1 - e^{-\left(\frac{t}{b}\right)^k}$ where $0 < t < \infty$	$\bar{x} = b \Gamma\left(1 + \frac{1}{k}\right)$ $\sigma^2 = b^2 \left[\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right) \right]$	---

Probability and Density Functions (cont'd)

Kind of Distribution	Probability Density Function $f(x)$ Distribution Function $F(x)$	Expected Mean (μ), Mean ($\bar{x}$), Variance (σ^2)	Form of the Density Function
Triangular	Comment: The triangular distribution is based on a simple geometric shape. The distribution arises naturally when uniformly distributed random variables are transformed in various ways.		
	$f(x) = \begin{cases} \frac{2}{p\omega^2}(x-a), & a \leq x \leq a+p\omega \\ \frac{2}{p\omega^2}(a+\omega-x), & a+p\omega \leq x \leq a+\omega \end{cases}$ $F(x) = \begin{cases} \frac{1}{p\omega^2}(x-a)^2, & a \leq x \leq a+p\omega \\ 1 - \frac{1}{\omega^2(1-p)}(a+\omega-x)^2, & a+p\omega \leq x \leq a+\omega \end{cases}$		$\bar{x} = a + \frac{\omega}{3}(1+p)$ $\sigma^2 = \frac{\omega^2}{18}[1-p(1-p)]$
Semicircle	Comment: The semicircular distribution is based on the shape of a semicircle with center a (location parameter) and radius r (scale parameter).		
	$f(x) = \frac{2}{\pi r^2} \sqrt{r^2 - (x-a)^2}$ $F(x) = \frac{1}{2} + \frac{x-a}{\pi r^2} \sqrt{r^2 - (x-a)^2} + \frac{1}{\pi} \arcsin\left(\frac{x-a}{r}\right)$ where $a-r \leq x \leq a+r$	$\bar{x} = a$ $\sigma^2 = \frac{r^2}{4}$	---
U-Power Distribution	Comment: $f(x)$ is symmetric about m .		
	$f(x) = \frac{2k+1}{2c} \left(\frac{x-\mu}{c}\right)^{2k}$ $F(x) = \frac{1}{2} \left[1 + \left(\frac{x-\mu}{c}\right)^{2k+1} \right]$ where $\mu - c \leq x \leq \mu + c$	$\bar{x} = \mu$ $\sigma^2 = c^2 \frac{2k+1}{2k+3}$	---

Normal Distribution Table

x	f(x)	F(x)	R(x)	2 R(x)	W(x)
					
0.0	0.3989	0.5000	0.5000	1.0000	0.0000
0.1	0.3970	0.5398	0.4602	0.9203	0.0797
0.2	0.3910	0.5793	0.4207	0.8415	0.1585
0.3	0.3814	0.6179	0.3821	0.7642	0.2358
0.4	0.3683	0.6554	0.3446	0.6892	0.3108
0.5	0.3521	0.6915	0.3085	0.6171	0.3829
0.6	0.3332	0.7257	0.2743	0.5485	0.4515
0.7	0.3123	0.7580	0.2420	0.4839	0.5161
0.8	0.2897	0.7881	0.2119	0.4237	0.5763
0.9	0.2661	0.8159	0.1841	0.3681	0.6319
1.0	0.2420	0.8413	0.1587	0.3173	0.6827
1.1	0.2179	0.8643	0.1357	0.2713	0.7287
1.2	0.1942	0.8849	0.1151	0.2301	0.7699
1.3	0.1714	0.9032	0.0968	0.1936	0.8064
1.4	0.1497	0.9192	0.0808	0.1615	0.8385
1.5	0.1295	0.9332	0.0668	0.1336	0.8664
1.6	0.1109	0.9452	0.0548	0.1096	0.8904
1.7	0.0940	0.9554	0.0446	0.0891	0.9109
1.8	0.0790	0.9641	0.0359	0.0719	0.9281
1.9	0.0656	0.9713	0.0287	0.0574	0.9426
2.0	0.0540	0.9772	0.0228	0.0455	0.9545
2.1	0.0440	0.9821	0.0179	0.0357	0.9643
2.2	0.0355	0.9861	0.0139	0.0278	0.9722
2.3	0.0283	0.9893	0.0107	0.0214	0.9786
2.4	0.0224	0.9918	0.0082	0.0164	0.9836
2.5	0.0175	0.9938	0.0062	0.0124	0.9876
2.6	0.0136	0.9953	0.0047	0.0093	0.9907
2.7	0.0104	0.9965	0.0035	0.0069	0.9931
2.8	0.0079	0.9974	0.0026	0.0051	0.9949
2.9	0.0060	0.9981	0.0019	0.0037	0.9963
3.0	0.0044	0.9987	0.0013	0.0027	0.9973

Normal Distribution Table (cont'd)

x	f(x)	F(x)	R(x)	2 R(x)	W(x)
Fractiles					
1.2816	0.1755	0.9000	0.1000	0.2000	0.8000
1.6449	0.1031	0.9500	0.0500	0.1000	0.9000
1.9600	0.0584	0.9750	0.0250	0.0500	0.9500
2.0537	0.0484	0.9800	0.0200	0.0400	0.9600
2.3263	0.0267	0.9900	0.0100	0.0200	0.9800
2.5758	0.0145	0.9950	0.0050	0.0100	0.9900

1.3.4.7 Linear Regression and Goodness of Fit

Least Squares

$$y = \hat{a} + \hat{b}x$$

where

$$\text{y-intercept} = \hat{a} = \bar{y} - \hat{b}\bar{x}$$

$$\text{slope} = \hat{b} = \frac{S_{xy}}{S_{xx}}$$

$$S_{xy} = \sum_{i=1}^n x_i y_i - \left(\frac{1}{n}\right) \left(\sum_{i=1}^n x_i\right) \left(\sum_{i=1}^n y_i\right)$$

$$S_{xx} = \sum_{i=1}^n x_i^2 - \left(\frac{1}{n}\right) \left(\sum_{i=1}^n x_i\right)^2$$

$$\bar{y} = \left(\frac{1}{n}\right) \left(\sum_{i=1}^n y_i\right)$$

$$\bar{x} = \left(\frac{1}{n}\right) \left(\sum_{i=1}^n x_i\right)$$

where

n = sample size

S_{xx} = sum of squares of x

S_{yy} = sum of squares of y

S_{xy} = sum of x - y products

Standard Error Estimate S_e^2

$$S_e^2 = \frac{S_{xx}S_{yy} - S_{xy}^2}{S_{xx}(n-2)} = MSE$$

where

$$S_{yy} = \sum_{i=1}^n y_i^2 - \left(\frac{1}{n}\right) \left(\sum_{i=1}^n y_i\right)^2$$

Confidence Interval for Intercept $\hat{a}$

$$\hat{a} \pm t_{\alpha/2, n-2} \sqrt{\left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right) MSE}$$

Confidence Interval for Slope $\hat{b}$

$$\hat{b} \pm t_{\alpha/2, n-2} \sqrt{\frac{MSE}{S_{xx}}}$$

Sample Correlation Coefficient R and Coefficient of Determination R^2

$$R = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} \quad R^2 = \frac{S_{xy}^2}{S_{xx}S_{yy}}$$

1.3.4.8 Test Statistics

The following definitions apply:

$$Z_{\text{var}} = \frac{\bar{X} - \mu_o}{\frac{\sigma}{\sqrt{n}}} \quad t_{\text{var}} = \frac{\bar{X} - \mu_o}{\frac{s}{\sqrt{n}}}$$

where

Z_{var} = the standard normal Z score

t_{var} = the sample distribution test statistic

σ = known standard deviation

μ_o = population mean

$\bar{X}$ = hypothesized mean or sample mean

n = sample size

s = computed sample standard deviation

The Z score is applicable when the standard deviations are known. The test statistic is applicable when the standard deviations are computed at time of sampling.

Z_{α} corresponds to the appropriate probability under the normal probability curve for a given Z_{var} .

$t_{\alpha, n-1}$ corresponds to the appropriate probability under the t distribution with $n-1$ degrees of freedom for a given t_{var} .

Values of $Z_{\alpha/2}$

Confidence Interval	$Z_{\alpha/2}$
80%	1.2816
90%	1.6449
95%	1.9600
96%	2.0537
98%	2.3263
99%	2.5758

1.4 Chemistry and Physical Properties

1.4.1 Periodic Table of the Elements

I												VIII							
1 H 1.0079												2 He 4.0026							
II												III		IV	V	VI		VII	
3 Li 6.941	4 Be 9.0122											5 B 10.811	6 C 12.011	7 N 14.007	8 O 15.999	9 F 18.998	10 Ne 20.179		
11 Na 22.990	12 Mg 24.305											13 Al 26.981	14 Si 28.086	15 P 30.974	16 S 32.066	17 Cl 35.453	18 Ar 39.948		
19 K 39.098	20 Ca 40.078	21 Sc 44.956	22 Ti 47.88	23 V 50.941	24 Cr 51.996	25 Mn 54.938	26 Fe 55.847	27 Co 58.933	28 Ni 58.69	29 Cu 63.546	30 Zn 65.39	31 Ga 69.723	32 Ge 72.61	33 As 74.921	34 Se 78.96	35 Br 79.904	36 Kr 83.80		
37 Rb 85.468	38 Sr 87.62	39 Y 88.906	40 Zr 91.224	41 Nb 92.906	42 Mo 95.94	43 Tc (98)	44 Ru 101.07	45 Rh 102.91	46 Pd 106.42	47 Ag 107.87	48 Cd 112.41	49 In 114.82	50 Sn 118.71	51 Sb 121.75	52 Te 127.60	53 I 126.90	54 Xe 131.29		
55 Cs 132.91	56 Ba 137.33	57* La 138.91	72 Hf 178.49	73 Ta 180.95	74 W 183.85	75 Re 186.21	76 Os 190.2	77 Ir 192.22	78 Pt 195.08	79 Au 196.97	80 Hg 200.59	81 Tl 204.38	82 Pb 207.2	83 Bi 208.98	84 Po (209)	85 At (210)	86 Rn (222)		
87 Fr (223)	88 Ra 226.02	89** Ac 227.03	104 Rf (261)	105 Ha (262)															
*Lanthanide Series			58 Ce 140.12	59 Pr 140.91	60 Nd 144.24	61 Pm (145)	62 Sm 150.36	63 Eu 151.96	64 Gd 157.25	65 Tb 158.92	66 Dy 162.50	67 Ho 164.93	68 Er 167.26	69 Tm 168.93	70 Yb 173.04	71 Lu 174.97			
**Actinide Series			90 Th 232.04	91 Pa 231.04	92 U 238.03	93 Np 237.05	94 Pu (244)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (251)	99 Es (252)	100 Fm (257)	101 Md (258)	102 No (259)	103 Lr (260)			

Atomic Number

Symbol

Atomic Weight

1.4.1.1 Relative Atomic Mass

Table of Relative Atomic Mass (Atomic Weight)

Name	Symbol	Atomic Number	Atomic Mass
Actinium	Ac	89	---*
Aluminum	Al	13	26.9815
Americium	Am	95	---*
Antimony	Sb	51	121.75
Argon	Ar	18	39.948
Arsenic	As	33	74.9216
Astatine	At	85	---*
Barium	Ba	56	137.34
Berkelium	Bk	97	---*
Beryllium	Be	4	9.0122
Bismuth	Bi	83	208.980
Boron	B	5	10.811
Bromine	Br	35	79.904
Cadmium	Cd	48	112.40
Calcium	Ca	20	40.08
Californium	Cf	98	---*
Carbon	C	6	12.01115
Cerium	Ce	58	140.12
Cesium	Cs	55	132.905
Chlorine	Cl	17	35.453
Chromium	Cr	24	51.996
Cobalt	Co	27	58.9332
Copper	Cu	29	63.546
Curium	Cm	96	---*
Dysprosium	Dy	66	162.50
Einsteinium	Es	99	---*
Erbium	Er	68	167.26
Europium	Eu	63	151.96
Fermium	Fm	100	---*
Fluorine	F	9	18.9984
Francium	Fr	87	---*
Gadolinium	Gd	64	157.25
Gallium	Ga	31	69.72
Germanium	Ge	32	72.59
Gold	Au	79	196.967
Hafnium	Hf	72	178.49
Helium	He	2	4.0026
Rhodium	Rh	45	102.905
Rubidium	Rb	37	85.47

Name	Symbol	Atomic Number	Atomic Mass
Holmium	Ho	67	164.930
Hydrogen	H	1	1.00797
Indium	In	49	114.82
Iodine	I	53	126.9044
Iridium	Ir	77	192.2
Iron	Fe	26	55.847
Krypton	Kr	36	83.80
Lanthanum	La	57	138.91
Lead	Pb	82	207.19
Lithium	Li	3	6.939
Lutetium	Lu	71	174.97
Magnesium	Mg	12	24.312
Manganese	Mn	25	54.9380
Mendelevium	Md	101	---*
Mercury	Hg	80	200.59
Molybdenum	Mo	42	95.94
Neodymium	Nd	60	144.24
Neon	Ne	10	20.183
Neptunium	Np	93	---*
Nickel	Ni	28	58.71
Niobium	Nb	41	92.906
Nitrogen	N	7	14.0067
Nobelium	No	102	---*
Osmium	Os	76	190.2
Oxygen	O	8	15.9994
Palladium	Pd	46	106.4
Phosphorus	P	15	30.9738
Platinum	Pt	78	195.09
Plutonium	Pu	94	---*
Polonium	Po	84	---*
Potassium	K	19	39.102
Praseodymium	Pr	59	140.907
Promethium	Pm	61	---*
Protactinium	Pa	91	---*
Radium	Ra	88	---*
Radon	Rn	86	---*
Rhenium	Re	75	186.2
Terbium	Tb	65	158.924
Thallium	Tl	81	204.37

Table of Relative Atomic Mass (Atomic Weight) (cont'd)

Name	Symbol	Atomic Number	Atomic Mass
Ruthenium	Ru	44	101.07
Samarium	Sm	62	150.35
Scandium	Sc	21	44.956
Selenium	Se	34	78.96
Silicon	Si	14	28.086
Silver	Ag	47	107.868
Sodium	Na	11	22.9898
Strontium	Sr	38	87.62
Sulfur	S	16	32.064
Tantalum	Ta	73	180.948
Technetium	Tc	43	---*
Tellurium	Te	52	127.60

Name	Symbol	Atomic Number	Atomic Mass
Thorium	Th	90	232.038
Thulium	Tm	69	168.934
Tin	Sn	50	118.69
Titanium	Ti	22	47.90
Tungsten	W	74	183.85
Uranium	U	92	238.03
Vanadium	V	23	50.942
Xenon	Xe	54	131.30
Ytterbium	Yb	70	173.04
Yttrium	Y	39	88.905
Zinc	Zn	30	65.37
Zirconium	Zr	40	91.22

* Multiple isotopes

1.4.1.2 Oxidation Number

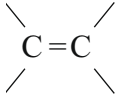
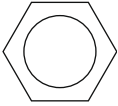
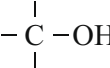
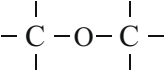
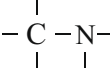
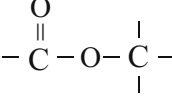
Oxidation Number or Charge Number

Name	Symbol	Charge
Acetate	$\text{C}_2\text{H}_3\text{O}_2$	-1
Aluminum	Al	+3
Ammonium	NH_4	+1
Barium	Ba	+2
Borate	BO_3	-3
Boron	B	+3
Bromine	Br	-1
Calcium	Ca	+2
Carbon	C	+4, -4
Carbonate	CO_3	-2
Chlorate	ClO_3	-1
Chlorine	Cl	-1
Chlorite	ClO_2	-1
Chromate	CrO_4	-2
Chromium	Cr	+2, +3, +6
Copper	Cu	+1, +2
Cyanide	CN	-1
Dichromate	Cr_2O_7	-2
Fluorine	F	-1
Gold	Au	+1, +3
Hydrogen	H	+1
Hydroxide	OH	-1
Hypochlorite	ClO	-1

Name	Symbol	Charge
Iron	Fe	+2, +3
Lead	Pb	+2, +4
Lithium	Li	+1
Magnesium	Mg	+2
Mercury	Hg	+1, +2
Nickel	Ni	+2, +3
Nitrate	NO_3	-1
Nitrite	NO_2	-1
Nitrogen	N	-3, +1, +2, +3, +4, +5
Oxygen	O	-2
Perchlorate	ClO_4	-1
Permanganate	MnO_4	-1
Phosphate	PO_4	-3
Phosphorus	P	-3, +3, +5
Potassium	K	+1
Silicon	Si	+4, -4
Silver	Ag	+1
Sodium	Na	+1
Sulfate	SO_4	-2
Sulfite	SO_3	-2
Sulfur	S	-2, +4, +6
Tin	Sn	+2, +4
Zinc	Zn	+2

1.4.1.3 Organic Compounds

Families of Organic Compounds

FAMILY	Specific Example	IUPAC Name	Common Name	General Formula	Functional Group
Alkane	CH_3CH_3	Ethane	Ethane	RH	C–H and C–C bonds
Alkene	$\text{H}_2\text{C} = \text{CH}_2$	Ethene or ethylene	Ethylene	$\text{RCH} = \text{CH}_2$ $\text{RCH} = \text{CHR}$ $\text{R}_2\text{C} = \text{CHR}$ $\text{R}_2\text{C} = \text{CR}_2$	
Alkyne	$\text{HC} = \text{CH}$	Ethyne or acetylene	Acetylene	$\text{RC} = \text{CH}$ $\text{RC} = \text{CR}$	$-\text{C} \equiv \text{C}-$
Arene		Benzene	Benzene	ArH	Aromatic ring
Haloalkane	$\text{CH}_3\text{CH}_2\text{Cl}$	Chloroethane	Ethyl chloride	RX	
Alcohol	$\text{CH}_3\text{CH}_2\text{OH}$	Ethanol	Ethyl alcohol	ROH	
Ether	CH_3OCH_3	Methoxymethane	Dimethyl ether	ROR	
Amine	CH_3NH_2	Methanamine	Methylamine	RNH_2 R_2NH R_3N	
Aldehyde	$\begin{array}{c} \text{O} \\ \parallel \\ \text{CH}_3\text{CH} \end{array}$	Ethanal	Acetaldehyde	$\begin{array}{c} \text{O} \\ \parallel \\ \text{RCH} \end{array}$	$\begin{array}{c} \text{O} \\ \parallel \\ -\text{C}-\text{H} \end{array}$
Ketone	$\begin{array}{c} \text{O} \\ \parallel \\ \text{CH}_3\text{CCH}_3 \end{array}$	Acetone	Dimethyl ketone	$\begin{array}{c} \text{O} \\ \parallel \\ \text{R}_1\text{CR}_2 \end{array}$	$\begin{array}{c} \text{O} \\ \parallel \\ -\text{C}- \end{array}$
Carboxylic Acid	$\begin{array}{c} \text{O} \\ \parallel \\ \text{CH}_3\text{COH} \end{array}$	Ethanoic acid	Acetic acid	$\begin{array}{c} \text{O} \\ \parallel \\ \text{RCOH} \end{array}$	$\begin{array}{c} \text{O} \\ \parallel \\ -\text{C}-\text{OH} \end{array}$
Ester	$\begin{array}{c} \text{O} \\ \parallel \\ \text{CH}_3\text{COCH}_3 \end{array}$	Methyl ethanoate	Methyl acetate	$\begin{array}{c} \text{O} \\ \parallel \\ \text{RCOR} \end{array}$	

1.4.2 Industrial Chemicals

Common Names of Industrial Chemicals

Common Name	Chemical Name	Molecular Formula
Acetone	Acetone	$(\text{CH}_3)_2\text{CO}$
Acetylene	Acetylene	C_2H_2
Ammonia	Ammonia	NH_3
Ammonium	Ammonium hydroxide	NH_4OH
Anatase/rutile	Titanium dioxide	TiO_2
Aniline	Aminobenzene	$\text{C}_6\text{H}_5\text{NH}_2$
Baking soda	Sodium bicarbonate	NaHCO_3
Battery acid	Sulfuric acid	H_2SO_4
Bauxite	Aluminum oxide	Al_2O_3
	Hydrated aluminum oxide	$\text{Al}_2\text{O}_3 \cdot 2\text{H}_2\text{O}$
Bleach	Calcium hypochloride	$\text{Ca}(\text{ClO})_2$
Bleach	Sodium hypochlorite	NaClO
Borane	Borane	BH_3
Borax	Sodium tetraborate	$\text{Na}_2\text{B}_4\text{O}_7 \cdot 10\text{H}_2\text{O}$
Brine, salt	Sodium chloride (solution)	NaCl
Carbide	Calcium carbide	CaC_2
Carbolic acid	Phenol	$\text{C}_6\text{H}_5\text{OH}$
Carbon dioxide	Carbon dioxide	CO_2
Carborundum	Silicon carbide	SiC
Caustic soda/lye	Sodium hydroxide	NaOH
Chalk	Calcium carbonate	CaCO_3
Chlorite	Chlorite ion	ClO_2^{-1}
Chlorate	Chlorate ion	ClO_3^{-1}
Cinnabar	Mercuric sulfide	HgS
Cumene	Isopropyl benzene	$\text{C}_6\text{H}_5\text{CH}(\text{CH}_3)_2$
Deuterium	Deuterium	^2H
Dichromate	Dichromate ion	$\text{Cr}_2\text{O}_7^{-2}$
Dolomite	Magnesium carbonate	MgCO_3
Epsom salt	Magnesium sulfate	MgSO_4
Ether	Diethyl ether	$(\text{C}_2\text{H}_5)_2\text{O}$
Ethylene oxide	Ethylene oxide	$\text{C}_2\text{H}_4\text{O}$
Eyewash	Boric acid (solution)	H_3BO_3
Formic acid	Methanoic acid	HCOOH
Glauber's salt	Decahydrated sodium sulfate	$\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$
Glycerine	Glycerine	$\text{C}_3\text{H}_5(\text{OH})_3$
Grain alcohol	Ethanol	$\text{C}_2\text{H}_5\text{OH}$
Graphite	Crystalline carbon	C
Gypsum	Calcium sulfate	$\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$

Common Names of Industrial Chemicals (cont'd)

Common Name	Chemical Name	Molecular Formula
Heavy water	Deuterium oxide	$(^2\text{H})_2\text{O}$
Hydronium	Hydronium ion	H_3O^{+1}
Hydroquinone	P-dihydroxybenzene	$\text{C}_6\text{H}_4(\text{OH})_2$
Hypochlorite	Hypochlorite ion	OCl^{-1}
Iron chloride	Ferrous chloride	$\text{FeCl}_2 \cdot 4\text{H}_2\text{O}$
Laughing gas	Nitrous oxide	N_2O
Limestone	Calcium carbonate	CaCO_3
Magnesia	Magnesium oxide	MgO
Magnetite	Ferrous/ferric oxide	Fe_3O_4
Marsh gas	Methane	CH_4
Muriate of potash	Potassium chloride	KCl
Muriatic acid	Hydrochloric acid	HCl
Neopentane	2,2-dimethylpropane	$\text{CH}_3\text{C}(\text{CH}_3)_2\text{CH}_3$
Niter	Sodium nitrate	NaNO_3
Niter cake	Sodium bisulfate	NaHSO_4
Oleum	Fuming sulfuric acid	SO_3 in H_2SO_4
Ozone	Ozone	O_3
Perchlorate	Perchlorate ion	ClO_4^{-1}
Permanganate	Permanganate ion	MnO_4^{-1}
Phosgene	Phosgene	COCl_2
Potash	Potassium carbonate	K_2CO_3
Potash	Potassium hydroxide	KOH
Prussic acid	Hydrogen cyanide	HCN
Pyrite, Fool's Gold	Ferrous sulfide	FeS
Pyrolusite	Manganese dioxide	MnO_2
Quicklime	Calcium oxide	CaO
Quicksilver	Mercury	Hg
Sal soda/washing soda	Decahydrated sodium carbonate	$\text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O}$
Salammoniac	Ammonium chloride	NH_4Cl
Salt/halite	Sodium chloride	NaCl
Salt cake	Sodium sulfate (crude)	Na_2SO_4
Sand/silica	Silicon dioxide	SiO_2
Silane	Silane	SiH_4
Slaked lime	Calcium hydroxide	$\text{Ca}(\text{OH})_2$
Soda ash	Sodium carbonate	Na_2CO_3
Styrene	Vinyl benzene	$\text{C}_6\text{H}_5\text{CH}=\text{CH}_2$
Sugar	Sucrose	$\text{C}_{12}\text{H}_{22}\text{O}_{11}$
Stannous chloride	Stannous chloride	$\text{SnCl}_2 \cdot 2\text{H}_2\text{O}$
Superphosphate	Monohydrated primary calcium phosphate	$\text{Ca}(\text{H}_2\text{PO}_4)_2 \cdot \text{H}_2\text{O}$
Toluene	Methyl benzene	$\text{C}_6\text{H}_5\text{CH}_3$

Common Names of Industrial Chemicals (cont'd)

Common Name	Chemical Name	Molecular Formula
Trilene	Trichloromethylene	C_2HCl_3
Tritium	Tritium	^3H
Urea	Urea	$(NH_2)_2CO$
Vinegar (acetic acid)	Ethanoic acid	CH_3COOH
Vinyl alcohol	Vinyl alcohol	$CH_2=CHOH$
Vinyl chloride	Vinyl chloride	$CH_2=CHCl$
Wood alcohol	Methanol	CH_3OH
Wolfram	Tungsten	W
Xylene	Dimethyl benzene	$C_6H_4(CH_3)_2$
Zinc blende	Zinc sulfide	ZnS



2 MASS AND ENERGY BALANCES

2.1 Symbols and Definitions

Symbols

Symbol	Description	Units (U.S.)	Units (SI)
H	Enthalpy	Btu	J
h	Specific enthalpy	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}} = \frac{\text{m}^2}{\text{s}^2}$
MW	Molecular weight (molar mass)	$\frac{\text{lbm}}{\text{lb mole}}$	$\frac{\text{kg}}{\text{kg} \cdot \text{mol}}$
m	Mass	lbm	kg
n	Number of moles	lb mole	kg • mol
P	Pressure	$\frac{\text{lbf}}{\text{in}^2}$ or psi	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
P^{sat}	Saturation pressure, or vapor pressure	$\frac{\text{lbf}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
p	Partial pressure	$\frac{\text{lbf}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
Q	Heat	Btu	J
S	Entropy	$\frac{\text{Btu}}{^\circ\text{R}}$	$\frac{\text{J}}{\text{K}}$
SG	Specific gravity	dimensionless	
s	Specific entropy [†]	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{R}}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}}$
T	Temperature	$^\circ\text{R}$ or $^\circ\text{F}$	K or $^\circ\text{C}$
t	Time	hr	s
U	Internal energy	Btu	J

Symbols (cont'd)

Symbol	Description	Units (U.S.)	Units (SI)
u	Specific internal energy [†]	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}}$
V	Volume	ft^3	m^3
v	Specific volume [†]	$\frac{\text{ft}^3}{\text{lbm}}$	$\frac{\text{m}^3}{\text{kg}}$
w	Weight fraction	dimensionless	
x	Mole fraction	dimensionless	
x	Quality	dimensionless	
y	Mole fraction	dimensionless	
γ_B	Mass concentration	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{kg}}{\text{m}^3}$
ϕ	Volume fraction	dimensionless	
ρ	Density	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{kg}}{\text{m}^3}$

[†] Property values on molar basis are denoted by $\hat{\cdot}$. For example, molar volume is $\hat{v}$.

2.2 Composition and Density

2.2.1 Measures of Composition

2.2.1.1 Mole, Mass, and Volume Fractions

Mole Fraction (or mole%): x_i

$$x_A = \frac{n_A}{n}$$

$$n = \sum_i n_i$$

$$\sum_i x_i = 1$$

For binary systems:

$$x_A = \frac{n_A}{n_A + n_B} = \frac{1}{1 + \frac{n_B}{n_A}}$$

$$n_A = \left(\frac{1}{x_B} - 1 \right) n_B$$

$$x_A + x_B = 1$$

Mass Fractions (Weight Fraction or wt%): w_i

$$w_A = \frac{m_A}{m}$$

$$m = \sum_i m_i$$

$$\sum_i w_i = 1$$

For binary systems:

$$w_A = \frac{m_A}{m_A + m_B} = \frac{1}{1 + \frac{m_B}{m_A}}$$

$$m_A = \left(\frac{1}{w_B} - 1 \right) m_B$$

$$w_A + w_B = 1$$

Conversion Between Mole Fraction and Mass Fraction

$$MW_A = \frac{m_A}{n_A}$$

$$m_A = n_A MW_A$$

$$n_A = \frac{m_A}{MW_A}$$

$$x_A = \frac{\frac{m_A}{MW_A}}{\sum_i \frac{m_i}{MW_i}} = \frac{w_A}{\sum_i w_i \frac{MW_A}{MW_i}}$$

$$w_A = \frac{\frac{n_A}{MW_A}}{\sum_i \frac{n_i}{MW_i}} = \frac{x_A}{\sum_i x_i \frac{MW_i}{MW_A}}$$

For binary systems:

$$x_A = \frac{m_A}{m_A + m_B} = \frac{1}{1 + \left(\frac{1}{w_A} - 1\right) \frac{MW_A}{MW_B}} \quad w_A = \frac{n_A}{n_A + n_B} = \frac{1}{1 + \left(\frac{1}{x_A} - 1\right) \frac{MW_B}{MW_A}}$$

Volume Fraction (%vol): ϕ_i

$$\phi_i = \frac{V_i^*}{V^*} \quad \text{where } V^* = \sum_i V_i^* \quad \sum_i \phi_i = 1$$

Volume fraction is the volume of a constituent of a mixture prior to mixing (V_i^*) divided by the sum of volumes of all constituents prior to mixing (V^*).

For mixtures of ideal gases: $\phi_i = x_i$

For ideal solutions (no volume change due to mixing): $\phi_i = w_i \frac{\rho}{\rho_i}$

Density and Average Molecular Weight ($\overline{MW}$) of a Mixture:

$$\overline{MW} = \sum_i x_i MW_i$$

For ideal solutions (no change in volume due to mixing): $\frac{1}{\rho} = \sum_i \frac{w_i}{\rho_i}$

For solutions of components with similar densities (assume volume of the solution is proportional to the mass):

$$\rho = \sum_i w_i \rho_i$$

Mass Fraction on a Dry Basis

For mixtures containing water, the mass fraction can be expressed on a dry basis, i.e., excluding the water.

$$W_{\text{dry}} = \frac{W_{\text{wet}}}{(1 - W_{\text{H}_2\text{O}})}$$

where $W_{\text{H}_2\text{O}}$ is the mass fraction of water in the mixture.

2.2.1.2 Ratios or Loading

Mole Ratio: X_i

Ratios are used primarily for dilute solution or when one component is not affected by the process. For solutions with a solvent it is also called "solute-free basis" and for combustion gases "dry basis."

Note: Component A is the basis (the solvent, the inert, or the predominant component).

$$X_i = \frac{n_i}{n_A} = \frac{x_i}{x_A} \quad \sum_{i \neq A} X_i = \frac{n}{n_A} - 1 \quad X_A = 1$$

For binary systems (A: Solvent, B: Solute):

$$X_B = \frac{x_B}{1 - x_B} = \frac{1}{\frac{1}{x_B} - 1} = \frac{1}{x_A} - 1 \quad x_B = \frac{1}{1 + \frac{1}{X_B}} \quad x_A = \frac{1}{1 + X_B}$$

For dilute systems with $x_A \rightarrow 1$: $X_i \rightarrow x_i$

Mass Ratio: W_i

$$W_i = \frac{m_i}{m_A} = \frac{w_i}{w_A} \qquad \sum_{i \neq A} W_i = \frac{m}{m_A} - 1 \qquad W_A = 1$$

For binary systems (A: Solvent, B: Solute):

$$W_B = \frac{w_B}{1 - w_B} = \frac{1}{\frac{1}{w_B} - 1} = \frac{1}{w_A} - 1 \qquad w_B = \frac{1}{1 + W_B} \qquad w_A = \frac{1}{1 + W_B}$$

For dilute systems with $w_A \rightarrow 1$: $W_i \rightarrow w_i$

Conversion Between Mole Ratio and Mass Ratio

$$W_i = X_i \frac{MW_i}{MW_A} \qquad X_i = W_i \frac{MW_A}{MW_i}$$

2.2.1.3 Concentrations

Molar Concentration: c_i or $[i]$

$$c_i = \frac{n_i}{V}$$

For ideal gases: $c_i = x_i \frac{P}{RT}$

Mass Concentration: γ_i

$$\gamma_i = \frac{m_i}{V} = w_i \rho$$

Volume Concentration: ϕ_i

$$\phi_i = \frac{V_i^*}{V}$$

Volume fraction is the volume of a constituent of a mixture prior to mixing V_i^* divided by the volume of the mixture (V).

For mixtures in which volume decreases on mixing:

$$\sum_i V_i^* > V_{\text{mix}} \qquad \sum_i \phi_i > 1$$

Ideal solution (no volume change due to mixing):

$$\phi_i = \varphi_i = w_i \frac{\rho}{\rho_i}$$

$$\sum_i \phi_i = 1 \quad (\text{ideal solutions only})$$

2.2.1.4 Molarity and Molality

Molarity (M)

$$\text{Molarity} = \frac{\text{gram} \cdot \text{moles of solute}}{\text{liters of solution}}$$

Note that molarity is temperature-dependent.

Molality (m)

$$\text{Molality} = \frac{\text{gram} \cdot \text{moles of solute}}{\text{kg of solvent}}$$

Note that molality is temperature-independent.

2.2.1.5 Special Measures of Composition

Normality (N)

$$\text{Normality} = \frac{\text{equivalent grams of the solute}}{\text{liters of solution}}$$

Gram equivalent weight is a measure of the reactive capacity of a given molecule and thus is reaction-dependent.

Note that normality is temperature-dependent.

pH and pOH

$$\text{pH} = -\log_{10}[\text{H}^+] \quad \text{or} \quad \text{pH} = -\log_{10}[\text{H}_3\text{O}^+]$$

$$\text{pOH} = -\log_{10}[\text{OH}^-]$$

and

$$\text{pK} = \text{pH} + \text{pOH} = -\log_{10} K \quad \text{where} \quad K = [\text{H}_3\text{O}^+][\text{OH}^-]$$

$$\text{For water at } 20^\circ\text{C: } K = 10^{-14} \text{ and } \text{pK} = 14$$

Note that all concentrations are in gram • moles/liter.

Proof (for Alcohol Content)

$$\text{Proof} = 2 \text{ abv} = 200 \frac{\text{mL of pure ethanol}}{\text{mL of solution}}$$

abv = alcohol % by volume (volume concentration)

For Dilute Solution (Can Be Based on Mass, Molar, or Volume)

$$\text{ppm} = \text{parts per million} = 10^{-6}$$

$$\text{ppb} = \text{parts per billion} = 10^{-9}$$

$$\text{ppt} = \text{parts per trillion} = 10^{-12}$$

$$\text{Percent: } 1\% = 10,000 \text{ ppm}$$

$$\text{Permil: } 1\text{‰} = 1000 \text{ ppm}$$

2.2.1.6 Conversion Table Between Different Measures of Concentration

Multicomponent Systems

	Mole Fraction x_i	Mass Fraction w_i	Mole Ratio X_i	Mass Ratio W_i	Molar concentration c_i	Mass concentration γ_i
Mole Fraction $x_i =$	x_i	$\frac{w_i}{\sum_j w_j \frac{MW_i}{MW_j}}$	$\frac{X_i}{1 + \sum_{j \neq A} X_j}$	$\frac{W_i}{\frac{MW_i}{MW_A} + \sum_{j \neq A} W_j \frac{MW_i}{MW_j}}$	$\frac{c_i \overline{MW}}{\rho}$	$\frac{\gamma_i \overline{MW}}{\rho \overline{MW_i}}$
Mass Fraction $w_i =$	$\frac{x_i}{\sum_j x_j \frac{MW_j}{MW_i}}$	w_i	$\frac{X_i MW_i}{MW_A + \sum_{j \neq A} X_j MW_j}$	$\frac{W_i}{1 + \sum_{j \neq A} W_j}$	$\frac{c_i MW_i}{\rho}$	$\frac{\gamma_i}{\rho}$
Mole Ratio $X_i =$	$\frac{x_i}{x_A}$	$\frac{w_i}{w_A} \frac{MW_i}{MW_A}$	X_i	$W_i \frac{MW_A}{MW_i}$	$\frac{c_i}{c_A}$	$\frac{\gamma_i MW_A}{\gamma_A MW_i}$
Mass Ratio $W_i =$	$\frac{x_i}{x_A} \frac{MW_A}{MW_i}$	$\frac{w_i}{w_A}$	$X_i \frac{MW_i}{MW_A}$	W_i	$\frac{c_i MW_i}{c_A MW_A}$	$\frac{\gamma_i}{\gamma_A}$
Molar concentration $c_i =$	$\frac{x_i \rho}{\overline{MW}}$	$\frac{w_i \rho}{MW_i}$	$X_i c_A$	$\frac{W_i \gamma_A}{MW_i}$	c_i	$\frac{\gamma_i}{MW_i}$
Mass concentration $\gamma_i =$	$\frac{x_i MW_i \rho}{\overline{MW}}$	$w_i \rho$	$X_i MW_i c_A$	$w_i \gamma_A$	$c_i MW_i$	γ_i
Avg. MW $\overline{MW} =$	$\sum_j x_j MW_j$	$\frac{1}{\sum_j \frac{w_j}{MW_j}}$	$\frac{MW_A + \sum_{j \neq A} X_j MW_j}{1 + \sum_{j \neq A} X_j}$	$\frac{\sum_{j \neq A} W_j}{\frac{1}{MW_A} + \sum_{j \neq A} X_j \frac{W_j}{MW_j}}$		
Avg. * Density $\rho =$	$\frac{\overline{MW}}{\sum_j \frac{x_j MW_j}{\rho_j}}$	$\frac{1}{\sum_j \frac{w_j}{\rho_j}}$				

*Ideal solutions only

Chapter 2: Mass and Energy Balances

Binary Systems

	Mole Fraction x_B	Mass Fraction w_B	Mole Ratio X_B	Mass Ratio W_B	Molar concentration c_B	Mass concentration γ_B
Mole Fraction $x_B =$	x_B	$\frac{w_B}{w_B + (1 - w_B) \frac{MW_B}{MW_A}}$	$\frac{X_B}{1 + X_B}$	$\frac{W_B}{\frac{MW_B}{MW_A} + W_B}$	$\frac{c_B \overline{MW}}{\rho}$	$\frac{\gamma_B \overline{MW}}{\rho MW_B}$
Mass Fraction $w_B =$	$\frac{x_B}{x_B + (1 - x_B) \frac{MW_A}{MW_B}}$	w_B	$\frac{X_B}{\frac{MW_A}{MW_B} + X_B}$	$\frac{W_B}{1 + W_B}$	$\frac{c_B MW_B}{\rho}$	$\frac{\gamma_B}{\rho}$
Mole Ratio $X_B =$	$\frac{x_B}{1 - x_B}$	$\frac{w_B}{1 - w_B} \frac{MW_A}{MW_B}$	X_B	$W_B \frac{MW_A}{MW_B}$	$\frac{c_B}{c_A}$	$\frac{\gamma_B MW_A}{\gamma_A MW_B}$
Mass Ratio $W_B =$	$\frac{x_B}{1 - x_B} \frac{MW_B}{MW_A}$	$\frac{w_B}{1 - w_B}$	$X_B \frac{MW_B}{MW_A}$	W_B	$\frac{c_B MW_B}{c_A MW_A}$	$\frac{\gamma_B}{\gamma_A}$
Molar concentration $c_B =$	$\frac{x_B \rho}{\overline{MW}}$	$\frac{w_B \rho}{MW_B}$	$X_B c_A$	$\frac{W_B \gamma_A}{MW_B}$	c_B	$\frac{\gamma_B}{MW_B}$
Mass concentration $\gamma_B =$	$\frac{x_B MW_B \rho}{\overline{MW}}$	$w_B \rho$	$X_B c_A MW_B$	$w_B \gamma_A$	$c_B MW_B$	γ_B
Avg. MW $\overline{MW} =$	$x_B MW_B + (1 - x_B) MW_A$	$\frac{MW_B}{w_B + (1 - w_B) \frac{MW_B}{MW_A}}$	$\frac{MW_A + X_B MW_B}{1 + X_B}$	$\frac{1 + W_B}{\frac{1}{MW_A} + \frac{W_B}{MW_B}}$		
Avg. * Density $\rho =$	$\frac{\rho_B \overline{MW}}{x_B + (1 - x_B) \frac{MW_A}{MW_B} \frac{\rho_B}{\rho_A}}$	$\frac{\rho_B}{w_B + (1 - w_B) \frac{\rho_B}{\rho_A}}$	$\frac{\rho_B \left(\frac{\overline{MW}}{MW_B} + X_B \right)}{X_B + \frac{MW_A}{MW_B} \frac{\rho_B}{\rho_A}}$	$\frac{(1 + W_B) \rho_B}{W_B + \frac{\rho_B}{\rho_A}}$		

*Ideal solutions only

Note: For mole and mass ratios, "A" is the basis component (e.g., the solvent).

2.2.2 Density

2.2.2.1 Density and Relative Density

Density is

$$\rho = \frac{m}{V}$$

Relative density is

$$RD = \frac{\rho}{\rho_{\text{ref}}}$$

where ρ_{ref} = density of a reference material

2.2.2.2 Specific Gravity

Specific Gravity (Relative Density) of Gas

$$SG = \frac{\rho_{\text{gas}}}{\rho_{\text{air at ref temp, press}}}$$

The reference temperatures are commonly either 0°C or 60°F and the reference pressure is commonly 14.696 psia (101,325 Pa).

For ideal gas:

$$SG = \frac{MW_{\text{gas}}}{MW_{\text{air}}} = \frac{MW_{\text{gas}}}{28.96 \frac{\text{g}}{\text{mol}}}$$

Specific Gravity (Relative Density) of Liquid

$$SG = \frac{\rho}{\rho_{\text{H}_2\text{O at ref temp}}}$$

where $\rho_{\text{H}_2\text{O}, 4^\circ\text{C}} = 62.4 \frac{\text{lbm}}{\text{ft}^3} = 1000 \frac{\text{kg}}{\text{m}^3}$

The reference temperatures are commonly either 4°C or 60°F.

Specific Gravity (Relative Density) in Baumé

For liquids lighter than water, using degrees Baumé or B°: $SG = \frac{140}{130 + B^\circ}$

For liquids heavier than water, using degrees Baumé or B°: $SG = \frac{145}{145 - B^\circ}$

Specific Gravity (Relative Density) for Hydrocarbon Liquid

$$SG_{60^\circ\text{F}} = \frac{141.5}{131.5 + \text{API}} \quad \text{API} = \frac{141.5}{SG_{60^\circ\text{F}}} - 131.5$$

where API = American Petroleum Institute gravity or API gravity

Specific Gravity (Relative Density) for Slurries

Bulk density and specific gravity of solids and liquid mixtures (slurries) are

$$\frac{1}{\rho_{\text{bulk}}} = \frac{1}{\rho_{\text{liquid}}} + x_{\text{solids}} \left(\frac{1}{\rho_{\text{solids}}} - \frac{1}{\rho_{\text{liquid}}} \right)$$

$$\frac{1}{SG_{\text{bulk}}} = \frac{1}{SG_{\text{liquid}}} + x_{\text{solids}} \left(\frac{1}{SG_{\text{solids}}} - \frac{1}{SG_{\text{liquid}}} \right)$$

where x_{solids} is the mass fraction of the solid in the slurry.

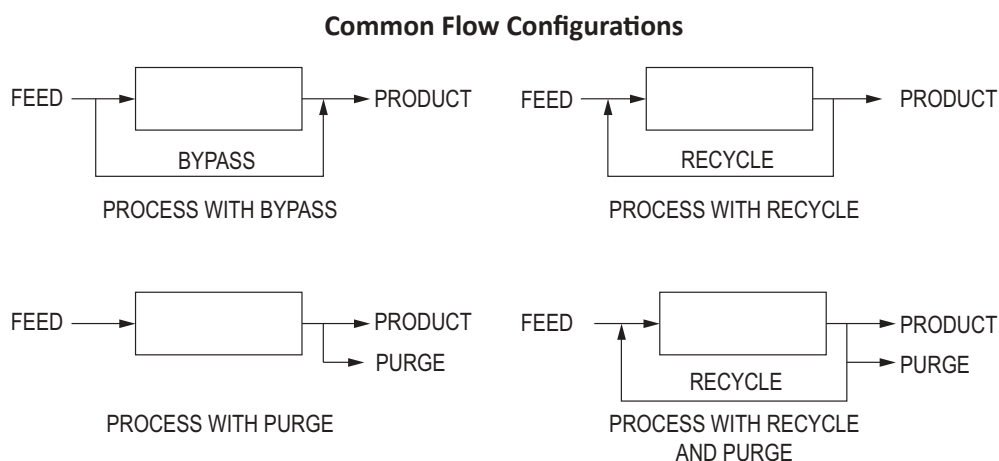
2.3 Mass Balance

General balance equation: $Accumulation = Input - Output + Generation - Consumption$

2.3.1 Mass Balances Without Reaction

Balanced equation at steady state with no reaction: $Input = Output$

2.3.1.1 Purge, Bypass and Recycle Configurations



2.3.2 Mass Balances With Reaction

Balanced equation at steady state with reaction:

$$Input + Generation = Output + Consumption$$

2.3.2.1 Stoichiometry

The stoichiometric amount of each reactant is the exact amount needed for the reaction to go to 100% conversion with no reactants left over. Such a reaction makes the stoichiometric amount of each product.

2.3.2.2 Combustion Reactions

Theoretical (stoichiometric) air is the minimum theoretical air required for complete combustion.

$$\text{Molar air-fuel ratio } \left(\frac{A}{F}\right) = \frac{\text{Moles of air}}{\text{Moles of fuel}}$$

$$\text{Percent theoretical air} = \frac{\left(\frac{A}{F}\right)_{\text{Actual}}}{\left(\frac{A}{F}\right)_{\text{Theoretical}}} \times 100$$

$$\text{Percent excess air} = \frac{\left(\frac{A}{F}\right)_{\text{Actual}} - \left(\frac{A}{F}\right)_{\text{Theoretical}}}{\left(\frac{A}{F}\right)_{\text{Theoretical}}} \times 100$$

Gross or higher heating value (HHV) is the heat of combustion assuming all water generated is condensed as a liquid.

Net or lower heating value (LHV) is the heat of combustion assuming that no water is condensed.

Major Components of Air

Element	Volume, %
Nitrogen	78.09
Oxygen	20.94
Argon	0.93

The dry adiabatic lapse rate Γ_{AD} is 0.98°C per 100 m (5.4°F per 1000 ft). This is the rate at which dry air cools adiabatically with altitude. The actual (environmental) lapse rate Γ is compared to Γ_{AD} to determine stability.

Stability of Adiabatic Lapse Rate

Lapse Rate	Stability Condition
$\Gamma > \Gamma_{AD}$	Unstable
$\Gamma = \Gamma_{AD}$	Neutral
$\Gamma < \Gamma_{AD}$	Stable

2.4 Energy Balances

2.4.1 Energy Balances without Reaction

2.4.1.1 Sensible Heat

$$\dot{Q} = m c_p \frac{dT}{dt}$$

$$c_p = \frac{\Delta H}{m \Delta T}$$

Heat transferred in or out of a flowing material:

$$\dot{Q} = \dot{m} c_p \Delta T$$

Note: The dot superscript (e.g., $\dot{Q}$ and $\dot{m}$) indicates a rate.

2.4.1.2 Heat of Solution

Ideal mixing applies to gases at low pressures; liquids and high-pressure gases involve nonideal mixing. In these cases, make calculations on a mole or mass basis instead of on a mole-fraction or mass-fraction basis. For the heat of a solution for a binary mixture on a molar basis:

$$n \hat{h}_{\text{mix, actual}} = n \Delta \hat{h} + n_1 \hat{h}_1 + n_2 \hat{h}_2$$

where

n = total moles of solution

n_1 = moles of Component 1

n_2 = moles of Component 2

This equation also applies to solids or gases dissolving into liquids. The $\Delta \hat{h}$ value must be known.

Heats of solutions often appear in charts, and enthalpies of mixing are presented as a function of composition. For evolved or absorbed heat:

$$n \Delta \hat{h} = n \hat{h}_{\text{mix, final}} - (n_1 \hat{h}_{\text{mix1}} + n_2 \hat{h}_{\text{mix2}})$$

where h_{mix1} and h_{mix2} can be either mixtures or pure components. This is calculated on a mass basis if the data are on a mass basis.

2.4.1.3 Vapor-Liquid Systems

Quality x (for liquid-vapor systems at saturation) is defined as the mass fraction of the vapor phase:

$$x = \frac{m_v}{m_v + m_l}$$

where

m_v = mass of vapor

m_l = mass of liquid

Note: Quality for steam might be expressed as a percentage. Moisture is the fraction of mass in a liquid phase.

$$\text{Moisture} = 1 - x$$

Specific volume of a two-phase system can be represented as

$$v = xv_v + (1 - x) v_l \quad \text{or} \quad v = v_l + x\Delta v_{\text{vap}}$$

where

v_v = specific volume of saturated vapor

v_l = specific volume of saturated liquid

Δv_{vap} = specific volume change upon vaporization

$$= v_v - v_l$$

Similar expressions exist for u , h , and s :

$$u = xu_v + (1 - x) u_l \quad \text{or} \quad u = u_l + x\Delta u_{\text{vap}}$$

$$h = xh_v + (1 - x) h_l \quad \text{or} \quad h = h_l + x\Delta h_{\text{vap}}$$

$$s = xs_v + (1 - x) s_l \quad \text{or} \quad s = s_l + x\Delta s_{\text{vap}}$$

The energy difference between two phases in equilibrium at a given temperature (or pressure) is the latent heat. The three types of latent heat are

$$\text{Latent heat of fusion (melting):} \quad \Delta h_{\text{fusion}} = h_l - h_s$$

$$\text{Latent heat of sublimation:} \quad \Delta h_{\text{subl}} = h_v - h_s$$

$$\text{Latent heat of vaporization:} \quad \Delta h_{\text{vap}} = h_v - h_l$$

2.4.2 Energy Balances with Reaction

2.4.2.1 Heat of Reaction

Calculate standard state heat of reaction $\Delta \hat{h}_R^0$ from standard heat of formation $\Delta \hat{h}_f^0$ at 298 K (25°C) and 1 atm, using

$$\Delta \hat{h}_R^0 = \sum_{\text{products}} \Delta \hat{h}_f^0 - \sum_{\text{reactants}} \Delta \hat{h}_f^0$$

Calculate $\Delta \hat{h}_R$ at temperature T using

$$\Delta \hat{h}_R = \Delta \hat{h}_R^0 + \sum_{\text{products}} \Delta \hat{h}_f + \sum_{\text{reactants}} \Delta \hat{h}_f$$

where $\Delta \hat{h}_f$ includes the sensible and latent heat changes between T and 298K

2.4.2.2 Heat of Formation and Heat of Combustion

The standard heats of formation and combustion at 25°C are shown in the tables below. The products of combustion are H₂O (l) and CO₂ (g). Solids are listed as s in the tables below.

Heats of Formation and Heats of Combustion for Alkanes

Name	Formula	Phase	$\Delta \hat{h}_f^0$		$-\Delta \hat{h}_c^0$ HHV	
			$\frac{\text{kJ}}{\text{mol}}$	$\frac{\text{Btu}}{\text{lb mol}}$	$\frac{\text{kJ}}{\text{mol}}$	$\frac{\text{Btu}}{\text{lb mol}}$
Methane	CH ₄	g	-74.6	-32,070	890.7	382,900
Ethane	C ₂ H ₆	g	-84.00	-36,110	1560	670,700
<i>n</i> -Propane	C ₃ H ₈	g	-104.6	-44,970	2219	954,100
Isobutane	C ₄ H ₁₀	g	-134.3	-57,740	2868	1,233,000
<i>n</i> -Butane	C ₄ H ₁₀	g	-125.5	-53,960	2877	1,237,000
<i>n</i> -Pentane	C ₅ H ₁₂	g	-146.9	-63,160	3535	1,520,000
<i>n</i> -Pentane	C ₅ H ₁₂	l	-173.5	-74,600	3509	1,507,000
Cyclohexane	C ₆ H ₁₂	g	-124.0	-53,310	—	—
Cyclohexane	C ₆ H ₁₂	l	-157.0	-67,500	3930	1,690,000
<i>n</i> -Hexane	C ₆ H ₁₄	g	-167.2	-71,890	4199	1,805,000
<i>n</i> -Hexane	C ₆ H ₁₄	l	-198.8	-85,470	4163	1,790,000
Methylcyclohexane	C ₇ H ₁₄	g	-154.78	-66,540	4601	1,978,000
Methylcyclohexane	C ₇ H ₁₄	l	-190.2	-81,760	4565	1,963,000
<i>n</i> -Heptane	C ₇ H ₁₆	g	-187.9	-80,790	—	—
<i>n</i> -Heptane	C ₇ H ₁₆	l	-225.0	-96,740	4817	2,071,000
<i>n</i> -Octane	C ₈ H ₁₈	g	-208.4	-89,600	—	—
<i>n</i> -Octane	C ₈ H ₁₈	l	-250.0	-107,500	5430	2,335,000
<i>n</i> -Nonane	C ₉ H ₂₀	g	-228.3	-98,160	—	—
<i>n</i> -Nonane	C ₉ H ₂₀	l	-274.7	-118,100	6125	2,633,000
<i>n</i> -Decane	C ₁₀ H ₂₂	g	-249.7	-107,400	—	—
<i>n</i> -Decane	C ₁₀ H ₂₂	l	-301.0	-129,400	6779	2,915,000

Heats of Formation and Heats of Combustion for Alkenes and Alkynes

Name	Formula	Phase	$\Delta \hat{h}_f^0$		$-\Delta \hat{h}_c^0$ HHV	
			$\frac{\text{kJ}}{\text{mol}}$	$\frac{\text{Btu}}{\text{lb mol}}$	$\frac{\text{kJ}}{\text{mol}}$	$\frac{\text{Btu}}{\text{lb mol}}$
Acetylene	C ₂ H ₂	g	226.8	97,510	—	—
Ethylene	C ₂ H ₄	g	52.3	22,500	1411	606,600
Propylene	C ₃ H ₆	g	20.4	8770	2058	884,800
1,3-Butadiene	C ₄ H ₆	g	109	46,900	2540	1,092,000
1,3-Butadiene	C ₄ H ₆	l	91	39,100	2522	1,084,000
1-Butene	C ₄ H ₈	g	-0.630	-270	2717	1,168,000
1-Pentene	C ₅ H ₁₀	g	-22	-9500	—	—
1-Pentene	C ₅ H ₁₀	l	-49	-21,000	3350	1,440,000
1-Hexene	C ₆ H ₁₂	g	-42	-18,000	—	—
1-Hexene	C ₆ H ₁₂	l	-73	-31,000	—	—

Heats of Formation and Heats of Combustion for Aromatics

Name	Formula	Phase	$\Delta \hat{h}_f^0$		$-\Delta \hat{h}_c^0$ HHV	
			$\frac{\text{kJ}}{\text{mol}}$	$\frac{\text{Btu}}{\text{lb mol}}$	$\frac{\text{kJ}}{\text{mol}}$	$\frac{\text{Btu}}{\text{lb mol}}$
Benzene	C ₆ H ₆	g	82.9	35,600	—	—
Benzene	C ₆ H ₆	l	49	21,000	3270	1,406,000
Toluene	C ₇ H ₈	g	50	21,000	—	—
Toluene	C ₇ H ₈	l	12	5200	3920	1,685,000
Styrene	C ₈ H ₈	g	147	63,200	—	—
Styrene	C ₈ H ₈	l	103	44,300	4390	1,887,000
Ethylbenzene	C ₈ H ₁₀	g	49	21,000	—	—
Ethylbenzene	C ₈ H ₁₀	l	6.8	2900	4567	1,964,000
<i>p</i> -Xylene	C ₈ H ₁₂	g	17.9	7700	—	—
<i>p</i> -Xylene	C ₈ H ₁₂	l	-24.4	-10,500	4552	1,957,000
<i>o</i> -Xylene	C ₈ H ₁₂	g	19	8200	—	—
<i>o</i> -Xylene	C ₈ H ₁₂	l	-24.4	-10,500	4552	1,957,000

Heats of Formation and Heats of Combustion for Other Organic Compounds

Name	Formula	Phase	$\Delta \hat{h}_f^0$		$-\Delta \hat{h}_c^0$ HHV	
			$\frac{\text{kJ}}{\text{mol}}$	$\frac{\text{Btu}}{\text{lb mol}}$	$\frac{\text{kJ}}{\text{mol}}$	$\frac{\text{Btu}}{\text{lb mol}}$
Methanol	CH ₄ O	g	-205	-88,100	764	328,500
Methanol	CH ₄ O	l	-239	-103,000	726	312,100
Acetaldehyde	C ₂ H ₄ O	g	-171	-73,500	—	—
Acetaldehyde	C ₂ H ₄ O	l	-196	-84,300	—	—
Ethylene oxide	C ₂ H ₄ O	g	-53	-22,700	1306	561,500
Ethylene oxide	C ₂ H ₄ O	l	-96	-41,200	1263	543,000
Acetic Acid	C ₂ H ₄ O ₂	l	-484	-208,000	875	376,000
Ethanol	C ₂ H ₆ O	g	-234	-100,600	1366	587,300
Ethanol	C ₂ H ₆ O	l	-276	-119,000	1367	587,700
Ethylene glycol	C ₂ H ₆ O ₂	l	-460	-197,800	1190	511,600

Heats of Formation and Heats of Combustion for Inorganic Compounds

Name	Formula	Phase	$\Delta \hat{h}_f^0$		$-\Delta \hat{h}_c^0$ HHV	
			$\frac{\text{kJ}}{\text{mol}}$	$\frac{\text{Btu}}{\text{lb mol}}$	$\frac{\text{kJ}}{\text{mol}}$	$\frac{\text{Btu}}{\text{lb mol}}$
Ammonia	NH ₃	g	-45.9	-19,700	383.0	164,700
Calcium carbide	CaC ₂	s	-62.8	-27,000	—	—
Calcium carbonate	CaCO ₃	s	-1207	-518,900	—	—
Calcium chloride	CaCl ₂	s	-795.0	-342,000	—	—
Calcium chloride	CaCl ₂ 6H ₂ O	s	-2607	-1,121,000	—	—
Calcium hydroxide	Ca(OH) ₂	s	-986.6	-424,200	—	—
Calcium oxide	CaO	s	-635.6	-273,200	—	—
Carbon	C, graphite	s	0	0	393.5	169,200
Carbon monoxide	CO	g	-110.5	-47,510	283.0	121,700
Carbon dioxide	CO ₂	g	-393.5	-169,200	—	—
Hydrochloric acid	HCl	g	-92.31	-39,690	—	—
Hydrogen	H ₂	g	0	0	286.0	123,000
Hydrogen sulfide	H ₂ S	g	-20.6	-8860	546.3	234,900
Iron oxide	FeO	s	-269.0	-115,700	—	—
Iron oxide	Fe ₂ O ₃	s	-822.2	-353,500	—	—
Iron oxide	Fe ₃ O ₄	s	-1117	-480,300	—	—
Nitric acid	HNO ₃	g	-134.3	-57,740	—	—
Nitric oxide	NO	g	90.29	38,820	—	—
Nitrogen dioxide	NO ₂	g	33.10	14,200	—	—
Nitrogen trioxide	NO ₃	g	71.13	30,580	—	—
Sodium carbonate	NaCO ₃	s	-1131	-486,300	—	—
Sodium carbonate	NaCO ₃ 10H ₂ O	s	-4082	-1,755,000	—	—

Heats of Formation and Heats of Combustion for Inorganic Compounds (cont'd)

Name	Formula	Phase	$\Delta \hat{h}_f^0$		$-\Delta \hat{h}_c^0$ HHV	
			$\frac{\text{kJ}}{\text{mol}}$	$\frac{\text{Btu}}{\text{lb mol}}$	$\frac{\text{kJ}}{\text{mol}}$	$\frac{\text{Btu}}{\text{lb mol}}$
Sodium chloride	NaCl	s	-411.0	-176,700	—	—
Sodium hydroxide	NaOH	s	-426.7	-183,500	—	—
Sulfur oxide	SO	g	5.01	2150	—	—
Sulfur dioxide	SO ₂	g	-296.8	-127,600	—	—
Sulfur trioxide	SO ₃	g	-395.8	-170,200	—	—
Sulfur trioxide	SO ₃	l	-442.5	-190,300	—	—
Water	H ₂ O	g	-241.83	-103,970	—	—
Water	H ₂ O	l	-285.83	-122,890	—	—

2.4.2.3 Temperature change without Phase Change

$$Q = mc_p \Delta T$$

2.4.2.4 Temperature change with Phase Change

$$Q = mc_p \Delta T \text{ for each phase} + m\Delta h \text{ for each phase change}$$



3 THERMODYNAMICS

3.1 Symbols and Definitions

Symbols

Symbol	Description	Units (U.S.)	Units (SI)
C	Concentration	$\frac{\text{lb mole}}{\text{ft}^3}$	$\frac{\text{mol}}{\text{m}^3}$
c_p, c_v	Specific heat capacity at constant pressure or constant volume [†]	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}} = \frac{\text{m}^2}{\text{s}^2 \cdot \text{K}}$
f	Ratio of vapor phase flow to feed flow	dimensionless	
f	Fugacity of a pure component	$\frac{\text{lb f}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
$\hat{f}_i$	Fugacity of component i in a mixture	$\frac{\text{lb f}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
G	Gibbs free energy	Btu	$\text{J} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$
g	Specific Gibbs free energy [†]	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}} = \frac{\text{m}^2}{\text{s}^2}$
g	Gravitational acceleration	$\frac{\text{ft}}{\text{sec}^2}$	$\frac{\text{m}}{\text{s}^2}$
H	Enthalpy	Btu	J
H_i	Henry's Law constant of component i	$\frac{\text{lb f}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
h	Specific enthalpy	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}} = \frac{\text{m}^2}{\text{s}^2}$
K	Equilibrium constant	varies	
K	Distribution coefficient	dimensionless	

Symbols (cont'd)

Symbol	Description	Units (U.S.)	Units (SI)
k	Isentropic coefficient	dimensionless	
k	Reaction rate constant	$\frac{(\text{lb mole/ft}^3)^{1-n}}{\text{sec}}$	$\frac{(\text{mol/m}^3)^{1-n}}{\text{s}}$
MW	Molecular weight (molar mass)	$\frac{\text{lbm}}{\text{lb mole}}$	$\frac{\text{kg}}{\text{mol}}$
m	Mass	lbm	kg
n	Number of moles	lb mole	mol
n	Polytropic coefficient	dimensionless	
P	Pressure	$\frac{\text{lbf}}{\text{in}^2}$ or psi	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
P_c	Critical pressure	$\frac{\text{lbf}}{\text{in}^2} = \text{psia}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
P_r	Reduced pressure	dimensionless	
P^{sat}	Saturation pressure, or vapor pressure	$\frac{\text{lbf}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
p	Partial pressure	$\frac{\text{lbf}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
Q	Heat	Btu	J
q	Specific heat (heat per unit mass)	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}} = \frac{\text{m}^2}{\text{s}^2}$
q	Ratio of liquid phase flow to feed flow	dimensionless	
R	Universal Gas Constant	$\frac{\text{ft} \cdot \text{lbf}}{\text{lb mole} \cdot ^\circ\text{R}}$	$\frac{\text{J}}{\text{mol} \cdot \text{K}} = \frac{\text{kg} \cdot \text{m}^2}{\text{mol} \cdot \text{K} \cdot \text{s}^2}$
r	Rate of reaction	$\frac{\text{lb mole}}{\text{ft}^3 \cdot \text{sec}}$	$\frac{\text{mol}}{\text{m}^3 \cdot \text{s}}$
r	Compression ratio	dimensionless	
r_c	Cut-off ratio (Diesel)	dimensionless	
r_p	Pressure ratio (Brayton)	dimensionless	
S	Entropy	$\frac{\text{Btu}}{^\circ\text{R}}$	$\frac{\text{J}}{\text{K}}$
s	Specific entropy [†]	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{R}}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}}$
SG	Specific gravity	dimensionless	
T	Temperature	$^\circ\text{R}$ or $^\circ\text{F}$	K or $^\circ\text{C}$
T_c	Critical temperature	$^\circ\text{R}$ or $^\circ\text{F}$	K or $^\circ\text{C}$
T_r	Reduced temperature	dimensionless	
t	Time	hr	s
U	Internal energy	Btu	J
u	Specific internal energy [†]	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}}$

Symbols (cont'd)

Symbol	Description	Units (U.S.)	Units (SI)
u	Velocity	$\frac{\text{ft}}{\text{sec}}$	$\frac{\text{m}}{\text{s}}$
V	Volume	ft^3	m^3
v	Specific volume [†]	$\frac{\text{ft}^3}{\text{lbm}}$	$\frac{\text{m}^3}{\text{kg}}$
W	Work	Btu or lbf-ft	J
w	Specific work (work per unit mass)	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}} = \frac{\text{m}^2}{\text{s}^2}$
w	Weight fraction	dimensionless	
x	Mole fraction	dimensionless	
y	Mole fraction	dimensionless	
z	Mole fraction (typically of the feed)	dimensionless	
Z	Compressibility factor	dimensionless	
z	Elevation, height	ft	m
α	Isentropic compressibility	$\frac{\text{in}^2}{\text{lbf}}$	$\frac{1}{\text{Pa}} = \frac{\text{m} \cdot \text{s}^2}{\text{kg}}$
α_{ij}	Relative volatility for components i and j	dimensionless	
β	Coefficient of expansion	$\frac{1}{^\circ\text{R}}$	$\frac{1}{\text{K}}$
γ	Activity coefficient	dimensionless	
γ	Surface tension	$\frac{\text{lbf}}{\text{in}}$	$\frac{\text{N}}{\text{m}} = \frac{\text{kg}}{\text{s}^2}$
η	Efficiency	dimensionless	
κ	Isothermal compressibility	$\frac{\text{in}^2}{\text{lbf}}$	$\frac{1}{\text{Pa}} = \frac{\text{m} \cdot \text{s}^2}{\text{kg}}$
μ_j	Joule-Thompson coefficient	$\frac{^\circ\text{R} \cdot \text{in}^2}{\text{lbf}}$	$\frac{\text{K}}{\text{Pa}} = \frac{\text{K} \cdot \text{m} \cdot \text{s}^2}{\text{kg}}$
ρ	Density	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{kg}}{\text{m}^3}$
χ	Vapor quality of a 2-phase mixture	dimensionless	
ϕ	Fugacity coefficient	dimensionless	
ϕ	Poynting correction factor	dimensionless	

[†] Property values on molar basis are denoted by $\hat{\cdot}$. For example, molar volume is $\hat{v}$.

3.2 Basic Thermodynamics

3.2.1 State Functions

Intensive properties are independent of mass.

Extensive properties are proportional to mass.

For a single-phase pure component, specifying any two intensive properties specifies the remaining intensive properties.

State Functions

Component	Property	U.S. Units	SI Units
Absolute pressure	P	psia	Pa
Absolute temperature	T	°R	K
Specific volume	$v = \frac{V}{m}$	$\frac{\text{ft}^3}{\text{lbm}}$	$\frac{\text{m}^3}{\text{kg}}$
Specific internal energy	$u = \frac{U}{m}$	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}}$
Specific enthalpy	$h = u + P v = \frac{H}{m}$	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}}$
Specific entropy	$s = \frac{S}{m}$	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{R}}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}}$
Specific Gibbs free energy	$g = h - T s = \frac{G}{m}$	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}}$
Specific Helmholtz free energy	$a = u - T s = g - P v = \frac{A}{m}$	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}}$

Maxwell Relations

$$dU = TdS - PdV \quad \left(\frac{\partial T}{\partial V} \right)_S = - \left(\frac{\partial P}{\partial S} \right)_V$$

$$dH = TdS + VdP \quad \left(\frac{\partial T}{\partial P} \right)_S = \left(\frac{\partial V}{\partial S} \right)_P$$

$$dG = -SdT + VdP \quad \left(\frac{\partial S}{\partial P} \right)_T = - \left(\frac{\partial V}{\partial T} \right)_P$$

$$dA = -SdT - PdV \quad \left(\frac{\partial S}{\partial V} \right)_T = \left(\frac{\partial P}{\partial T} \right)_V$$

Work and Heat

Definition: Work is considered positive if it is directed outward, i.e. done by the system on the surroundings (or subtracted from the system).

Work from various sources:

$$dW = \underset{\substack{\downarrow \\ \text{compression} \\ \downarrow \\ \text{mechanical}}}{PdV} + \underset{\substack{\downarrow \\ \text{potential} \\ \downarrow \\ \text{kinetic}}}{Fds} + \underset{\substack{\downarrow \\ \text{kinetic}}}{mgdz} + \underset{\substack{\downarrow \\ \text{kinetic}}}{mudu} + \underset{\substack{\downarrow \\ \text{surface}}}{\gamma dA_s}$$

where

F = force

s = distance

g = gravitational acceleration

z = elevation, height

u = velocity

γ = surface tension

A_s = surface area

Reversible work: moving along equilibrium path

$$W_{\text{rev}} = \int PdV = \int (PV)d \ln V$$

Irreversible Work: moving along actual path. Example: expansion against constant external pressure.

$$W_{\text{irr}} = P_{\text{ext}}(V_2 - V_1)$$

Useful work (i.e., work that can be extracted from the system)

$$W_f = W_{\text{rev}} - W_{\text{irr}}$$

Heat and Energy: Heat is considered positive if it is inward (added to the system).

$$dU = dQ - dW \quad dH = dQ - dW_f$$

$$dQ = C_v dT \quad \text{at constant volume}$$

$$dQ = C_p dT \quad \text{at constant pressure}$$

Heat Capacity at Constant Pressure

$$c_p = \left(\frac{\partial h}{\partial T} \right)_p$$

$$\text{Dependence on Pressure: } \left(\frac{\partial c_p}{\partial P} \right)_T = -T \left(\frac{\partial^2 v}{(\partial T)^2} \right)_p$$

Heat Capacity at Constant Volume

$$c_v = \left(\frac{\partial u}{\partial T} \right)_v$$

$$\text{Dependence on Volume: } \left(\frac{\partial c_v}{\partial v} \right)_T = T \left(\frac{\partial^2 p}{(\partial T)^2} \right)$$

Common Coefficients

$$\text{Joule-Thompson coefficient} \quad \mu_J = \left(\frac{\partial T}{\partial P} \right)_h$$

$$\text{Coefficient of Expansion} \quad \beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p$$

$$\text{Isothermal Compressibility} \quad \kappa = -\frac{1}{v} \left(\frac{\partial v}{\partial P} \right)_T$$

Isentropic Compressibility $\alpha = -\frac{1}{v} \left(\frac{\partial v}{\partial P} \right)_s$

Isentropic Coefficient $k = \frac{c_p}{c_v} = \frac{\kappa}{\alpha}$

Derived Relations

$$\begin{aligned} c_p - c_v &= Tv \frac{\beta^2}{\kappa} \\ \mu_J &= -\frac{1}{c_p} \left(v - T \left(\frac{\partial v}{\partial T} \right)_p \right) = -\frac{1}{c_p} (v - Tv\beta) \\ \left(\frac{\partial h}{\partial P} \right)_T &= -\mu_J c_p \\ \left(\frac{\partial h}{\partial P} \right)_v &= c_p \left(1 - \frac{\mu_J \beta}{\kappa} \right) \\ \left(\frac{\partial P}{\partial T} \right)_v &= \frac{\beta}{\kappa} \end{aligned}$$

3.2.1.1 Ideal Gases

For an ideal gas,

$$P \hat{v} = RT \quad \text{and} \quad \frac{P_1}{P_2} \frac{\hat{v}_1}{\hat{v}_2} = \frac{T_1}{T_2}$$

where 1 and 2 indicate separate system states.

Alternatively,

$$PV = nRT = \frac{mRT}{MW} \quad \text{and} \quad \frac{P_1}{P_2} \frac{V_1}{V_2} = \frac{n_1}{n_2} \frac{T_1}{T_2}$$

Common Coefficients for Ideal Gases

Joule-Thompson coefficient $\mu_J = 0$

Coefficient of Expansion $\beta = -\frac{1}{T}$

Isothermal Compressibility $\kappa = \frac{1}{P}$

Isentropic Compressibility $\alpha = \frac{1}{P\kappa}$

$$c_p - c_v = R$$

For constant heat capacity:

$$\Delta u = c_v \Delta T$$

$$\Delta h = c_p \Delta T$$

$$\begin{aligned} \Delta s &= c_p \ln \left(\frac{T_2}{T_1} \right) - \frac{R}{MW} \ln \left(\frac{P_2}{P_1} \right) \\ &= c_v \ln \left(\frac{T_2}{T_1} \right) + \frac{R}{MW} \ln \left(\frac{v_2}{v_1} \right) \end{aligned}$$

3.2.1.2 Incompressible Fluids

$$u = u(T) \text{ only}$$

$$c_v = c_p = c$$

For constant heat capacity

$$\Delta u = c \Delta T$$

$$\Delta s = c \ln \frac{T_2}{T_1}$$

$$\Delta h = c \Delta T + v \Delta P$$

For real liquids, assume that κ and β are independent of pressure and temperature:

$$\frac{dv}{v} = \beta dT - \kappa dP$$

$$\ln\left(\frac{v_2}{v_1}\right) = \beta(T_2 - T_1) - \kappa(P_2 - P_1)$$

For incompressible liquids: $dv = 0$

$$\left(\frac{dP}{dT}\right)_v = \frac{\beta}{\kappa}$$

Isothermal Compression (with constant κ)

$$w = \frac{\kappa \cdot v}{2} (P_2^2 - P_1^2)$$

Adiabatic Compression

$$w = v(P_2 - P_1)$$

3.2.1.3 Ideal Gas Mixtures

Dalton's Law of Partial Pressures

$$p_i V = n_i R T$$

$$\text{results in } P = \sum_{i=1}^n p_i \quad \text{or} \quad P = p_1 + \dots + p_n \quad \text{and} \quad y_i = \frac{p_i}{P} = \frac{n_i}{n}$$

where

p_i = partial pressure of component i

n_i = moles of component i

y_i = mole fraction of component i in gas phase

Amagat's Law of Partial Volumes

$$P V_i = n_i R T$$

$$\text{results in: } V = \sum_{i=1}^n V_i \quad \text{or} \quad V = V_1 + \dots + V_n \quad \text{and} \quad y_i = \frac{V_i}{V} = \frac{n_i}{n}$$

where V_i = partial volume of component i

Molar Properties of Ideal Gas Mixtures

$$\hat{u}_{\text{mix}} = \sum_n (y_i \hat{u}_i)$$

$$\hat{h}_{\text{mix}} = \sum_n (y_i \hat{h}_i)$$

$$\hat{c}_{v, \text{mix}} = \sum_n (y_i \hat{c}_{v,i})$$

$$\hat{c}_{P, \text{mix}} = \sum_n (y_i \hat{c}_{P,i})$$

where

$\hat{u}_i$ and $\hat{h}_i$ are evaluated at T

To calculate the molar volume of an ideal gas or liquid mixture:

$$\hat{v}_{\text{mix}} = \sum_n (x_i \hat{v}_i)$$

Note that this equation does not apply to density.

When mixing pure components, the entropy of the mixture is

$$\hat{s}_{\text{mix}} = \sum_n (y_i \hat{s}_i) + R \sum_n y_i \ln \left(\frac{1}{y_i} \right)$$

where $\hat{s}_i$ is evaluated at T and p_i

The entropy of mixing is $\Delta \hat{s}_{\text{mixing}} = R \sum_n y_i \ln \left(\frac{1}{y_i} \right)$

The Gibbs free energy for the mixture is

$$\hat{g}_{\text{mix}} = \sum_n (y_i \hat{g}_i) - RT \sum_n y_i \ln \left(\frac{1}{y_i} \right)$$

3.2.1.4 Equations of State for Nonideal Gases

Van der Waals Equation of State

$$\left(P + \frac{a}{v^2} \right) (v - b) = RT$$

$$a = \frac{27}{64} \frac{R^2 T_c^2}{P_c}$$

$$b = \frac{RT_c}{8P_c}$$

where T_c and P_c are the temperature and pressure at the critical point.

Redlich-Kwong Equation of State

$$P = \frac{RT}{v - b} - \frac{a}{T^{1/2} v(v + b)}$$

$$a = \frac{0.42748 R^2 T_c^{2.5}}{P_c}$$

$$b = \frac{0.08664 RT_c}{P_c}$$

Virial Equation of State

$$Z = \frac{P \hat{v}}{RT} = 1 + \frac{B}{\hat{v}} + \frac{C}{\hat{v}^2} + \frac{D}{\hat{v}^3} + \dots$$

where B, C, D = virial equation coefficients, accounting for two-body, three-body, and four-body interactions, respectively

Alternatively,

$$Z = \frac{P \hat{v}}{RT} = 1 + B' P + C' P^2 + D' P^3 \dots$$

where B', C', D' = virial equation coefficients

The two sets of virial coefficients are related by:

$$B' = \frac{B}{RT}$$

$$C' = \frac{C - B^2}{(RT)^2}$$

$$D' = \frac{D - 3BC + 2B^3}{(RT)^3}$$

Generic Cubic Equation of State

$$P = \frac{RT}{\hat{v} - b} - \frac{a(T)}{(\hat{v} + \epsilon b)(\hat{v} + \sigma b)}$$

where

$a(T)$ = substance-dependent constant

b = substance-dependent constant

ϵ = constant for generic cubic equation of state

σ = constant for generic cubic equation of state

3.2.1.5 Compressibility

Compressibility and Expansivity

The *compressibility factor* is a dimensionless number defined by the equation:

$$Z = \frac{P \hat{v}}{RT}$$

For ideal gas, $Z = 1$

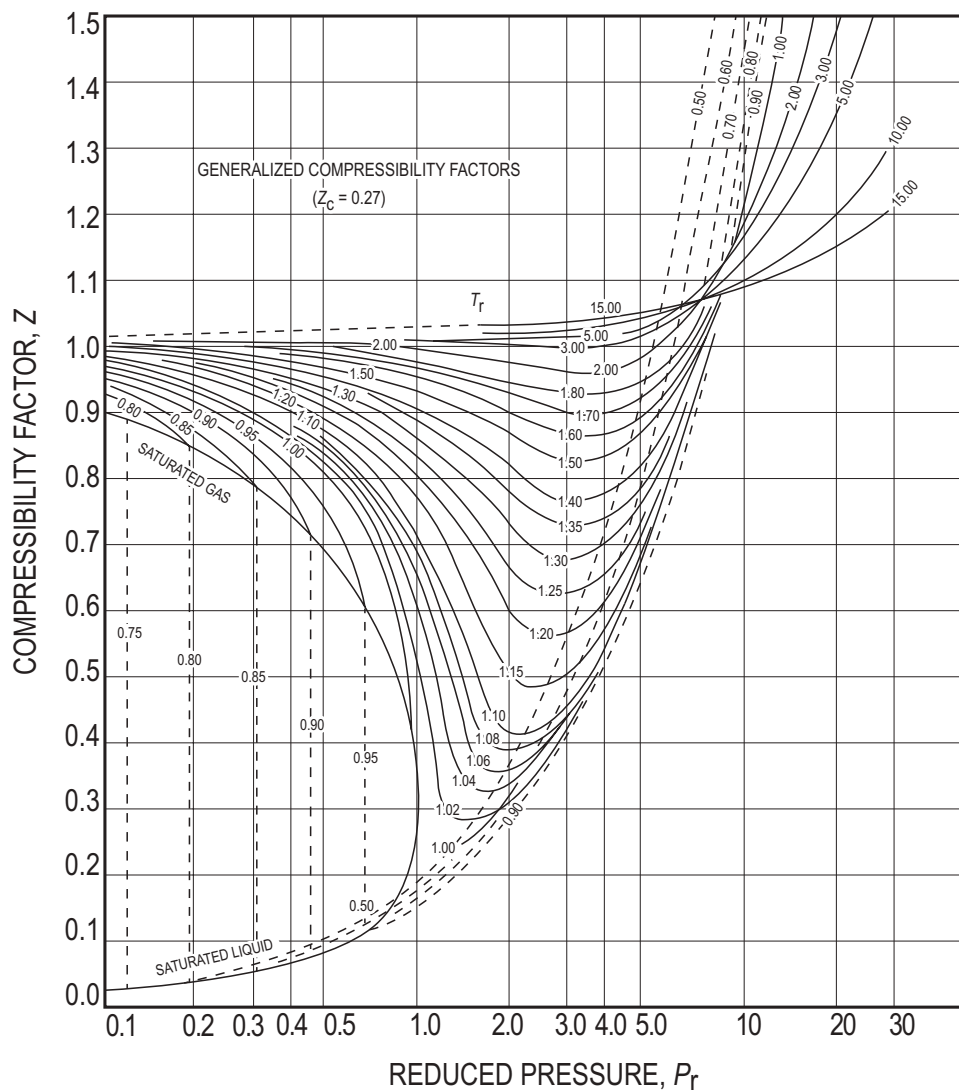
Theorem of Corresponding States

To first approximation, all fluids have the same compressibility factor when compared at the same reduced temperature and reduced pressure.

Reduced temperature (T_r) and *reduced pressure* (P_r) are defined as

$$T_r = \frac{T}{T_c} \quad \text{and} \quad P_r = \frac{P}{P_c}$$

Compressibility Factor Chart



Source: From de Nevers, Noel, *Physical and Chemical Equilibrium for Chemical Engineers*, 2nd ed., New York: Wiley & Sons, 2012.

3.2.1.6 Multicomponent Systems

The properties of a mixture can be estimated using the properties of its pure components, based on either a mass-fraction average or a mole-fraction average. The one exception is entropy, which must be estimated based only on a mole-fraction average.

Mass based mixture properties

$$u_{\text{mix}} = \frac{1}{m} \sum_n m_i u_i = \sum_n w_i u_i$$

$$h_{\text{mix}} = \frac{1}{m} \sum_n m_i h_i = \sum_n w_i h_i$$

$$c_{v,\text{mix}} = \frac{1}{m} \sum_n m_i c_{v,i} = \sum_n w_i c_{v,i}$$

$$c_{P,\text{mix}} = \frac{1}{m} \sum_n m_i c_{P,i} = \sum_n w_i c_{P,i}$$

3.2.1.7 Boiling Point Elevation and Freezing Point Depression

For dilute solutions of nonvolatile solutes in a solvent, the solution has a greater boiling point and lower freezing point than the solvent alone.

Boiling point elevation

$$\Delta T_b = \frac{R T_{bp}^2 x}{\Delta h_{vap}}$$

where

x = solute mole fraction

T_{bp} = boiling point of the pure solvent at the system pressure

Δh_{vap} = latent heat of vaporization of the pure solvent at boiling point T_{bp} and the system pressure

Freezing point depression

$$\Delta T_m = \frac{R T_{mp}^2 x}{\Delta h_{fusion}}$$

where

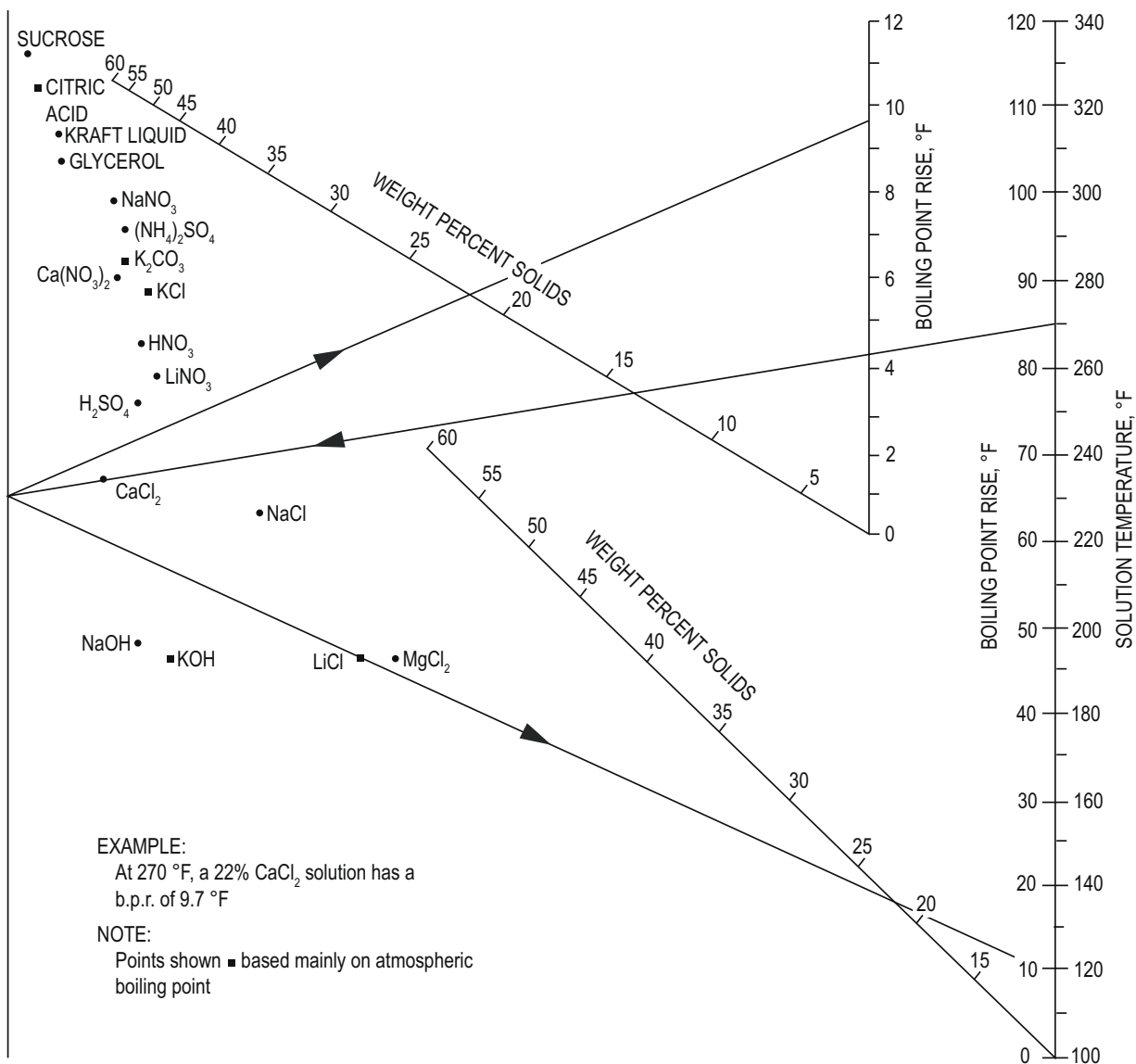
x = solute mole fraction

T_{mp} = melting point of the pure solvent at the system pressure

Δh_{fusion} = latent heat of fusion of the pure solvent at the melting point T_{mp} and the system pressure

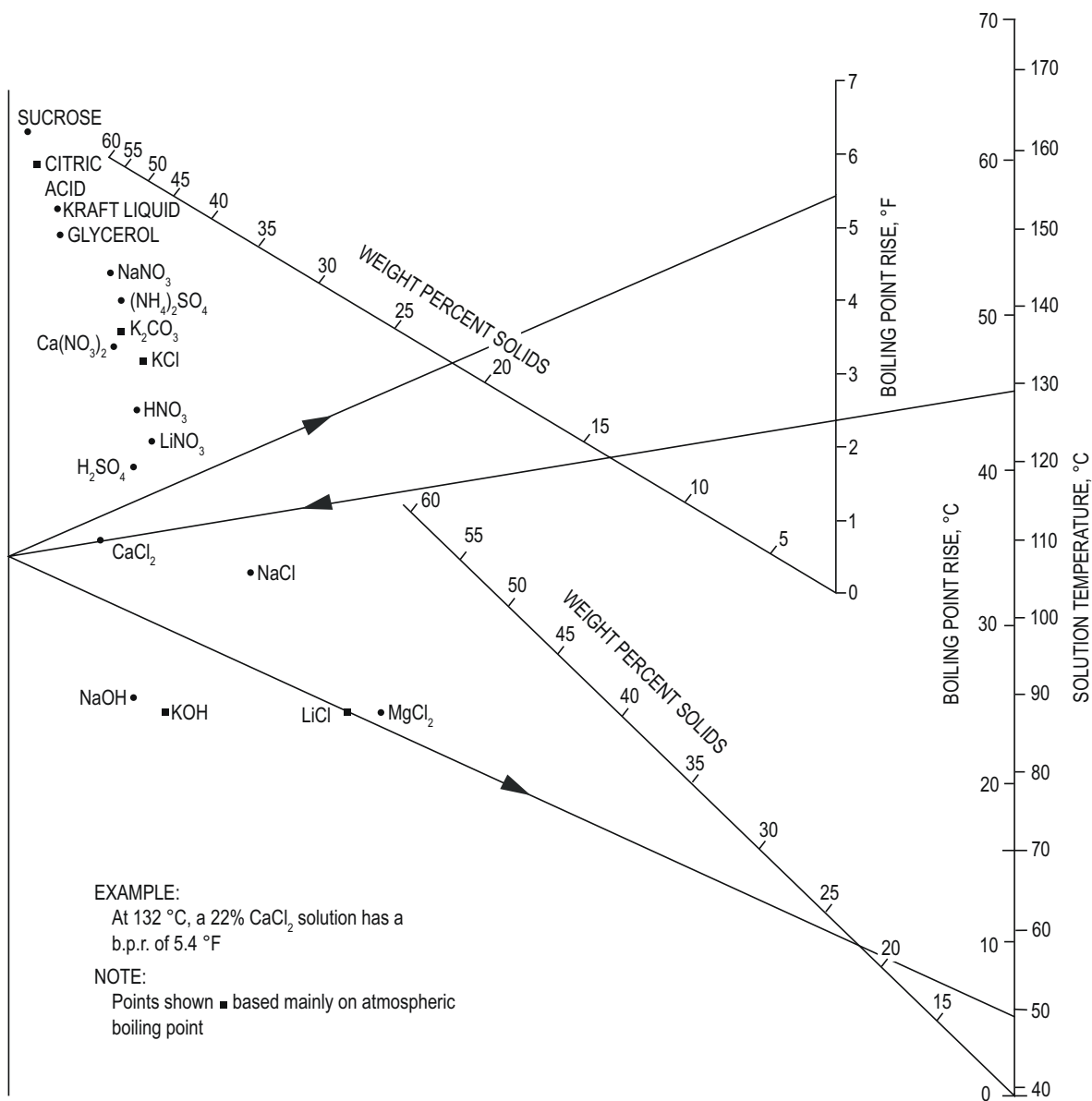
For common aqueous solutions, the boiling point elevation can be determined graphically from the solution temperature and the solute mass fraction.

Boiling-Point Rise of Aqueous Solutions



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Boiling-Point Rise of Aqueous Solutions



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3.2.2 First and Second Laws of Thermodynamics

The *First Law of Thermodynamics* states that energy is neither created nor destroyed but can change from one form into another. The net energy crossing the system boundary is equal to the change in energy inside the system.

Changes in state functions are calculated by changes in Q and W , which are path-dependent. The common paths are

Isobaric	$\Delta P = 0$
Isochoric	$\Delta V = 0$
Isothermal	$\Delta T = 0$
Isenthalpic	$\Delta H = 0$
Adiabatic	$\Delta Q = 0$
Adiabatic and reversible (isentropic)	$\Delta S = 0$
Adiabatic and irreversible (polytropic)	$\Delta S > 0$

Changes in state functions for irreversible processes are calculated by selecting an alternative, reversible process between the same two states.

3.2.2.1 Entropy (S)

The entropy of a system represents the unavailability of the thermal energy of a system for conversion to mechanical work. It is a measure of the microscopic disorder of a system.

$$ds = \frac{1}{T} du + \frac{P}{T} dv \quad (\text{Gibbs equation})$$

$$\begin{aligned} ds &= \frac{dQ_{\text{rev}}}{T} \\ &= c_v d \ln T + \left(\frac{\partial P}{\partial T} \right)_v dv \\ &= c_p d \ln T - \left(\frac{\partial v}{\partial T} \right)_p dP \end{aligned}$$

Pressure dependence of entropy:

$$\left(\frac{\partial s}{\partial p} \right)_T = -\beta v$$

Entropy requires a reference state; absolute entropy cannot be determined.

3.2.2.2 Internal Energy (U)

Energy of a system that is associated with molecular and atomic motions and forces

$$u = u(T, v)$$

Volume dependence of internal energy

$$\left(\frac{\partial u}{\partial v} \right)_T = T \left(\frac{\partial P}{\partial T} \right)_v - P$$

For ideal gases, the internal energy does not depend on volume.

Temperature dependence of internal energy

$$\left(\frac{\partial u}{\partial T} \right)_v = c_v \quad (\text{heat capacity at constant volume})$$

Pressure dependence of internal energy

$$\left(\frac{\partial u}{\partial P} \right)_T = (kP - \beta T) v$$

3.2.2.3 Enthalpy (H)

The enthalpy of a system reflects its ability to do work.

$$h = u + Pv$$

$$dh = Tds + v dP$$

Pressure dependence of enthalpy

$$\left(\frac{\partial h}{\partial P}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_P = v(1 - \beta T)$$

Temperature dependence of enthalpy

$$\left(\frac{\partial h}{\partial T}\right)_P = c_P \quad (\text{heat capacity at constant pressure})$$

Enthalpy requires a reference state; absolute enthalpy cannot be determined.

3.2.2.4 Gibbs Free Energy (G)

Gibbs free energy indicates the potential for reversible work that a system can do at constant pressure and temperature.

$$g = h - Ts$$

$$dg = v dP - s dT$$

$$d\left(\frac{g}{RT}\right) = \frac{v}{RT} dp - \frac{h}{RT^2} dT$$

At equilibrium, the Gibbs free energy attains a minimum value.

3.2.2.5 Helmholtz Free Energy (A)

The Helmholtz free energy is the potential for a system to do work at constant volume and temperature. It is the part of the internal energy which is used for useful work.

$$a = u - Ts$$

$$da = -P dv - s dT$$

3.2.2.6 Closed Thermodynamic Systems

In a *closed thermodynamic system*, no mass crosses the system boundary:

$$Q - W = \Delta U + \Delta KE + \Delta PE$$

where

$$\Delta U = \text{change in internal energy} \quad \Delta U = m(u_2 - u_1)$$

$$\Delta KE = \text{change in kinetic energy} \quad \Delta KE = \frac{1}{2}m(u_2^2 - u_1^2), \text{ where } u \text{ is velocity}$$

$$\Delta PE = \text{change in potential energy} \quad \Delta PE = mg(z_2 - z_1)$$

Energy can cross the system boundary only in the form of heat or work. Work can be shaft work (w_s) or other work forms, such as electrical.

Closed System Energy Changes for 1 Mole of an Ideal Gas

Path	Isochoric ($\Delta V = 0$)	Isobaric ($\Delta P = 0$)	Isothermal ($\Delta T = 0$)	Isentropic ($\Delta S = 0$)	Polytropic
Volume	$\Delta v = 0$	$\frac{v_1}{v_2} = \frac{T_1}{T_2}$	$\frac{v_1}{v_2} = \frac{P_2}{P_1}$	$\frac{v_1}{v_2} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{k-1}} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{k}}$	$\frac{v_1}{v_2} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{n-1}} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$
Pressure	$\frac{P_1}{P_2} = \frac{T_1}{T_2}$	$\Delta P = 0$	$\frac{P_1}{P_2} = \frac{v_2}{v_1}$	$\frac{P_1}{P_2} = \left(\frac{v_2}{v_1}\right)^k = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}$	$\frac{P_1}{P_2} = \left(\frac{v_2}{v_1}\right)^n = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}}$
Temperature	$\frac{T_1}{T_2} = \frac{P_1}{P_2}$	$\frac{v_1}{v_2} = \frac{T_1}{T_2}$	$\Delta T = 0$	$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{k-1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$	$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{n-1} = \left(\frac{P_1}{P_2}\right)^{\frac{n-1}{n}}$
Work (w)	0	$P(v_2 - v_1)$	$RT \ln\left(\frac{P_1}{P_2}\right)$ $RT \ln\left(\frac{v_2}{v_1}\right)$	$\frac{kRT_1}{k-1} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]$	$\left(\frac{P_1 v_1}{n-1}\right) \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right]$
Heat (q)	$\hat{c}_v \Delta T$	$\hat{c}_p \Delta T$	$RT \ln\left(\frac{P_1}{P_2}\right)$ $RT \ln\left(\frac{v_2}{v_1}\right)$	0	$\frac{\hat{c}_v(n-k)\Delta T}{n-1}$
Change in Internal Energy (Δu)	$\hat{c}_v \Delta T$	$\hat{c}_p \Delta T - P(v_2 - v_1)$	0	$-\frac{RT_1}{k-1} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]$	$\hat{c}_v \Delta T$
Change in Entropy (Δs)	$\hat{c}_v \ln\left(\frac{T_2}{T_1}\right)$	$\hat{c}_p \ln\left(\frac{T_2}{T_1}\right)$	$\frac{\hat{q}}{T}$	0	$\frac{\hat{c}_v(n-k)}{n-1} \ln\left(\frac{T_2}{T_1}\right)$
Change in Enthalpy (Δh)	$\hat{c}_p \Delta T$	$\hat{c}_p \Delta T$	0	$\hat{c}_p \Delta T$	$\hat{c}_p \Delta T$

Note: Enthalpy for a closed system does not have a physical meaning, because the pv term does not represent flow work.

where $k = \frac{\hat{c}_p}{\hat{c}_v}$

3.2.2.7 Open Thermodynamic Systems

In an *open thermodynamic system*, mass does cross the system boundary. *Flow work* (Pv) is defined as the work for mass entering and leaving the system.

Reversible flow work = $w_{\text{rev}} = - \int v dP + \Delta KE + \Delta PE$

Open System First Law (energy balance):

$$\sum \dot{m}_i \left[h_i + \frac{u_i^2}{2g_c} + g z_i \right] - \sum \dot{m}_e \left[h_e + \frac{u_e^2}{2g_c} + g z_e \right] + \dot{Q}_{\text{in}} - \dot{W}_{\text{net}} = \frac{d(m_s u_s)}{dt}$$

where

subscripts i and e refer to inlet and exit states of the system

$\dot{W}_{\text{net}}$ = rate of net or shaft work

$\dot{m}$ = mass flow rate

h = enthalpy

g = acceleration of gravity

g_c = gravitational constant

z = elevation

u = velocity

m_s = mass of fluid within the system

u_s = specific internal energy of system

$\dot{Q}_{\text{in}}$ = rate of heat transfer (neglecting kinetic and potential energy of the system)

The table below displays the work, heat, and internal enthalpy changes in open systems for each of the five applicable paths for 1 mole of ideal gas. These changes assume constant heat capacities and neglect kinetic and potential energy changes.

Open System Energy Changes for 1 Mole of Ideal Gas at Steady State

Path	Isochoric ($\Delta V = 0$)	Isobaric ($\Delta P = 0$)	Isothermal ($\Delta T = 0$)	Isentropic ($\Delta S = 0$)	Isenthalpic ($\Delta H = 0$)	Polytropic
Work (w)	$-\hat{v}(P_2 - P_1)$	0	$RT \ln\left(\frac{P_1}{P_2}\right)$ $RT \ln\left(\frac{\hat{v}_2}{\hat{v}_1}\right)$	$\frac{k RT_1}{k-1} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \right]$ $= \frac{P_2 \hat{v}_2 - P_1 \hat{v}_1}{1-k}$ $= -c_v \Delta T$	0	$-\Delta H$
Heat (q)	$\hat{c}_v \Delta T$	$\hat{c}_p \Delta T$	$RT \ln\left(\frac{P_1}{P_2}\right)$ $RT \ln\left(\frac{\hat{v}_2}{\hat{v}_1}\right)$	0	0	$\frac{\hat{c}_v(n-k)\Delta T}{n-1}$
Change in Enthalpy (Δh)	$\hat{c}_v \Delta T + \hat{v}(P_2 - P_1)$	$\hat{c}_p \Delta T$	0	$\frac{-k RT_1}{k-1} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \right]$	0	$\hat{c}_p \Delta T$
Change in Entropy (Δs)				0		$\left[\frac{\hat{c}_v(n-k)}{n-1} \right] \ln\left(\frac{T_2}{T_1}\right)$

3.2.2.8 Steady-Flow Thermodynamics Systems

The system does not change state with time. This assumption is valid for the steady operation of turbines, pumps, compressors, throttling valves, nozzles, and heat exchangers, including boilers and condensers. The letter V denotes velocity in the following three equations:

$$\sum \dot{m}_i \left(h_i + \frac{u_i^2}{2g_c} + g z_i \right) - \sum \dot{m}_e \left(h_e + \frac{u_e^2}{2g_c} + g z_e \right) + \dot{Q} - \dot{W}_s = 0 \quad \sum \dot{m}_i = \sum \dot{m}_e$$

For a single fluid-flow stream at steady state, the equation reduces to:

$$\Delta h + \frac{\Delta u^2}{2g_c} + g \Delta z + \dot{w} - \dot{q} = 0$$

If the fluid is incompressible with negligible friction losses, the equation reduces to:

$$\frac{\Delta P}{\rho} + \frac{\Delta u^2}{2g_c} + g \Delta z + \dot{w}_s - \dot{q} = 0$$

3.3 Work, Heat, and Efficiency

3.3.1 Efficiency

Overall efficiency is the ratio of useful energy output of a process to the energy input

$$\eta_{\text{overall}} = \frac{\text{useful energy out}}{\text{total energy in}}$$

Reversible process efficiency compares the work output of the actual process to that of the reversible process. This efficiency expresses the losses in performance due to friction and other non-reversible contributions.

3.3.2 Compression and Expansion

3.3.2.1 Work in Reversible Compression Processes

Work for compression depends on the type of process (equations below are for an ideal gas):

Isothermal compression ($T = \text{constant}$):

$$Pv = \text{constant}$$

$$w_{\text{rev}, T} = RT \ln \left(\frac{P_2}{P_1} \right)$$

Isotropic compression ($s = \text{constant}$):

$$Pv^k = \text{constant} \quad \text{where } k = \frac{c_p}{c_v}$$

$$w_{\text{rev}, \text{is}} = \frac{k}{1-k} (P_2 v_2 - P_1 v_1) = \frac{k}{1-k} \frac{R(T_2 - T_1)}{MW} = \frac{k}{1-k} \frac{RT_1}{MW} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \right]$$

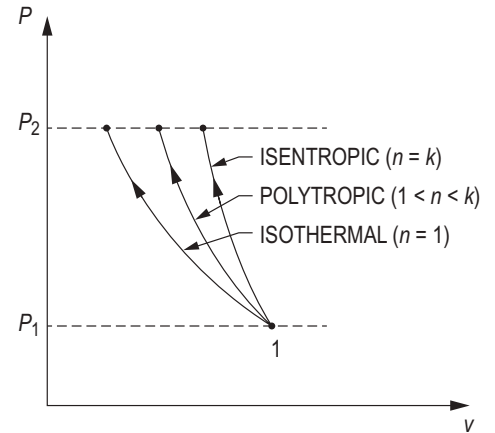
Polytropic compression:

$$Pv^n = \text{constant} \quad \text{where } n = \text{polytropic coefficient (empirical)}$$

$$w_{\text{rev}, \text{poly}} = \frac{n}{1-n} (P_2 v_2 - P_1 v_1) = \frac{n}{1-n} \frac{R(T_2 - T_1)}{MW} = \frac{n}{1-n} \frac{RT_1}{MW} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right]$$

Isotropic efficiency is the efficiency of the actual process compared to an isentropic process:

$$\eta_{\text{is}} = \frac{w_{\text{rev}, \text{is}}}{w_{\text{actual}}}$$



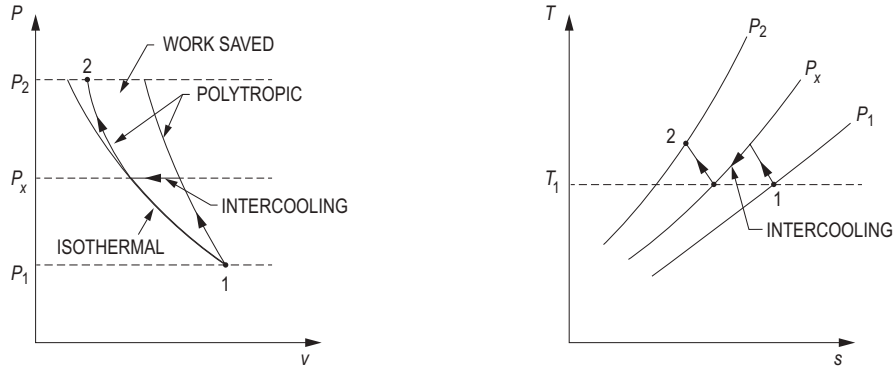
Polytropic efficiency is the efficiency of the actual process compared to a polytropic process:

$$\eta_{\text{poly}} = \frac{w_{\text{rev,poly}}}{w_{\text{actual}}}$$

For the same process, the polytropic efficiency is higher than the isentropic efficiency.

For multistage compression:

Intercooling between stages reduces the work required for compression.



Maximum reduction in work (in comparison to single-stage compression) occurs when the pressure ratio (PR) for each stage is the same:

$$PR = \left(\frac{P_2}{P_1} \right)^{\frac{1}{m}} \quad \text{where } m = \text{number of stages}$$

3.3.2.2 Efficiency of Compressors

Compressors consume power to add energy to the working fluid. This addition of energy results in an increase in fluid pressure (head).

Work of compression for an adiabatic compressor (assuming negligible changes in potential and kinetic energy and constant specific heats):

$$\dot{W}_{\text{comp}} = -\dot{m}(h_e - h_i)$$

For an ideal gas with constant specific heats:

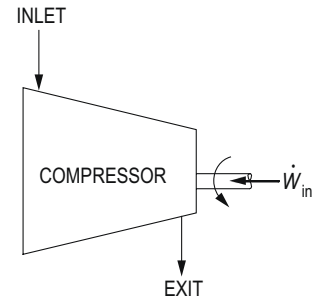
$$\dot{W}_{\text{comp}} = -\dot{m}c_p(T_e - T_i)$$

Per unit mass:

$$w_{\text{comp}} = -c_p(T_e - T_i)$$

If the change in kinetic energy is not negligible, the work of compression is:

$$\dot{W}_{\text{comp}} = -\dot{m} \left(h_e - h_i + \frac{u_e^2 - u_i^2}{2g_c} \right) = -\dot{m} \left(c_p(T_e - T_i) + \frac{u_e^2 - u_i^2}{2g_c} \right)$$



The isentropic efficiency of a compressor is the ratio between the work for the isentropic process and the work of actual process. The exit state for the actual process and the isentropic process are different, though the exit pressures are the same.

$$\eta_{c, is} = \frac{w_{actual}}{w_{isentropic}} = \frac{h_{es} - h_i}{h_e - h_i} = \frac{T_{es} - T_i}{T_e - T_i}$$

Where "i" designates the inlet conditions, "e" the exit conditions and "es" the exit conditions for the isentropic process.

Actual work of compression for an ideal gas and negligible kinetic energy:

Adiabatic compression:

$$\dot{W}_{comp} = \frac{-\dot{m} P_i k}{(k-1) \rho_i \eta_{c, is}} \left[\left(\frac{P_e}{P_i} \right)^{\frac{1}{k}} - 1 \right]$$

where

$\dot{W}_{comp}$ = fluid or gas power

P_i = inlet or suction pressure

P_e = exit or discharge pressure

ρ_i = inlet gas density

$\eta_{c, is}$ = isentropic compressor efficiency

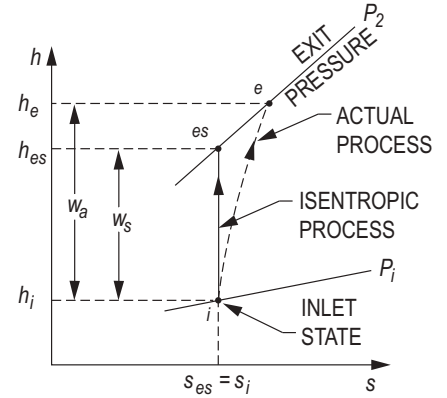
Isothermal compression:

$$\dot{W}_{comp} = \frac{\dot{m} R T_i}{MW \eta_{c, isothermal}} \ln \frac{P_e}{P_i} = \frac{\dot{m} P_i}{\rho_i \eta_C} \ln \frac{P_e}{P_i}$$

where

$\dot{W}_{comp}$, P_e , P_i , and η_C = as defined for adiabatic compression, above

T_i = inlet temperature of gas



3.3.2.3 Efficiency of Turbines

For an adiabatic turbine with $\Delta PE = 0$ and negligible ΔKE :

$$\dot{W}_{turb} = \dot{m}(h_i - h_e)$$

For an ideal gas with constant specific heats:

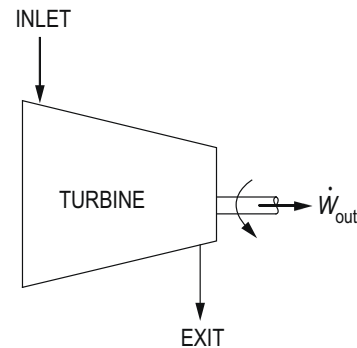
$$\dot{W}_{turb} = \dot{m} c_p (T_i - T_e)$$

Per unit mass:

$$w_{turb} = c_p (T_i - T_e)$$

For a turbine where ΔKE is included with constant specific heats:

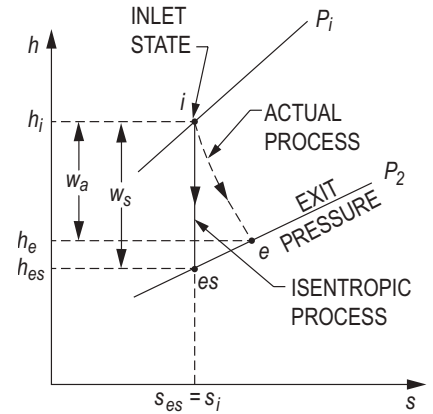
$$\dot{W}_{turb} = \dot{m} \left(h_e - h_i + \frac{u_e^2 - u_i^2}{2g_c} \right) = \dot{m} \left(c_p (T_e - T_i) + \frac{u_e^2 - u_i^2}{2g_c} \right)$$



The isentropic efficiency of a turbine is the ratio of the actual work output of the turbine to the work output that would have been achieved if the process between the inlet state and the exit pressure were isentropic. The exit state for the actual process and the isentropic process are different, though the exit pressures are the same.

$$\eta_{T,is} = \frac{w_{\text{actual}}}{w_{\text{isentropic}}} = \frac{h_i - h_e}{h_i - h_{es}} = \frac{T_i - T_e}{T_i - T_{es}}$$

Where "i" designates the inlet conditions, "e" the exit conditions and "es" the exit conditions for the isentropic process.



3.3.2.4 Efficiency of Nozzles

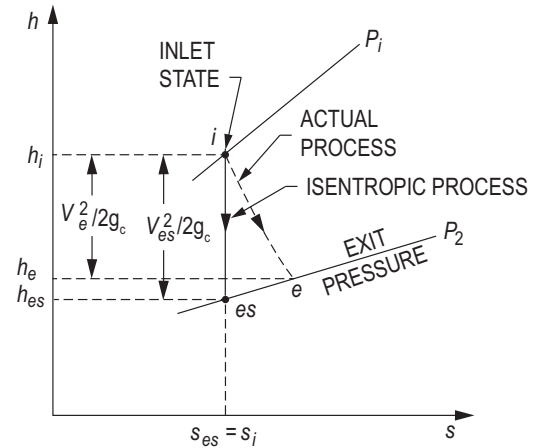
Nozzles are adiabatic devices to accelerate a fluid. A nozzle does not involve work interactions and generally, changes in potential energy are minimal, so only the change in kinetic energy is considered. Typically, inlet velocities are assumed to be negligible.

$$h_i - h_e = \frac{u_e^2}{2g_c}$$

The isentropic efficiency of a nozzle is defined as the ratio of the actual kinetic energy of the fluid at nozzle exit to the kinetic energy at the exit of an isentropic nozzle for the same inlet state and exit pressure. The exit state for the actual process and the isentropic process are different, though the exit pressures are the same.

$$\eta_{T,is} = \frac{u_e^2}{u_{es}^2} = \frac{h_i - h_e}{h_i - h_{es}}$$

Where "i" designates the inlet conditions, "e" the exit conditions and "es" the exit conditions for the isentropic process.



3.4 Power cycles

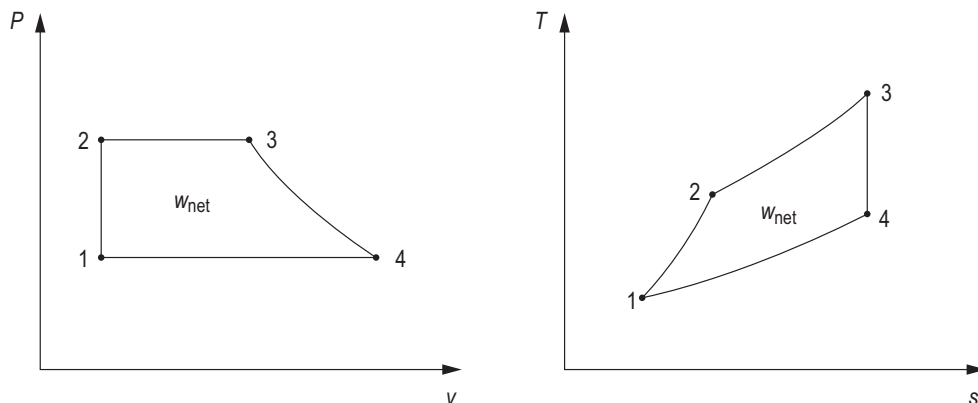
Power cycles can be characterized by

- Gas working fluid (cycle without phase change) or vapor working fluid (cycle with phase changes)
- Closed system (working fluid returned to its original state) and open system (working fluid is renewed after each cycle)
- Internal combustion (fuel burned within the system boundaries) and external combustion (energy supplied to the working fluid from an external source)

3.4.1 Efficiency of Power Cycles

Net work:

The area enclosed by the process curve on the T-s diagram and on the P-v diagram represents the net work of the cycle.



The thermal efficiency of any power cycle is defined as:

$$\eta_{th} = \frac{|w_{out}|}{|Q_{in}|}$$

Absolute-value signs are used with the heat and work quantities to make the equations independent of sign conventions.

In power production, the objective is to maximize the work obtained from the turbine.

Co-generation system:

In a co-generation system, both the work from the turbine ($W_{turbine}$) and the heat output (steam) are utilized.

Fuel Chargeable to Power (FCP) is an efficiency measure that considers both the work output from the turbine and the heat output ($Q_{steamgen}$ —i.e., the heat output used productively in the form of steam generated rather than simply the heat exhausted).

$$FCP = \frac{W_{turbine} + Q_{steamgen}}{Q_{in}}$$

Units for FCP are usually Btu/kWh produced or Btu/hp produced.

3.4.2 Gas Power Cycles

For combustion engines, the following simplifications are made (air-standard cycle):

- Working fluid is air, which behaves like an ideal gas
- All processes are reversible
- The combustion process is replaced by heat addition from an external source
- The exhaust process is replaced by a heat rejection process
- Heat capacities are constant, values at room temperature are used (cold-air-standard cycle)



Reciprocating Engines (Piston-Cylinder)

TDC = Top dead center—piston position where volume is at minimum

BDC = bottom dead center—piston position where volume is at maximum

Stroke = difference between TDC and BDC

Bore = diameter of the piston

Clearance volume = minimum volume when piston is at TDC

Displacement volume = volume between BDC and TDC

Compression ratio:

$$r = \frac{V_{\max}}{V_{\min}} = \frac{V_{BDC}}{V_{TDC}}$$

Mean Effective Pressure (MEP)

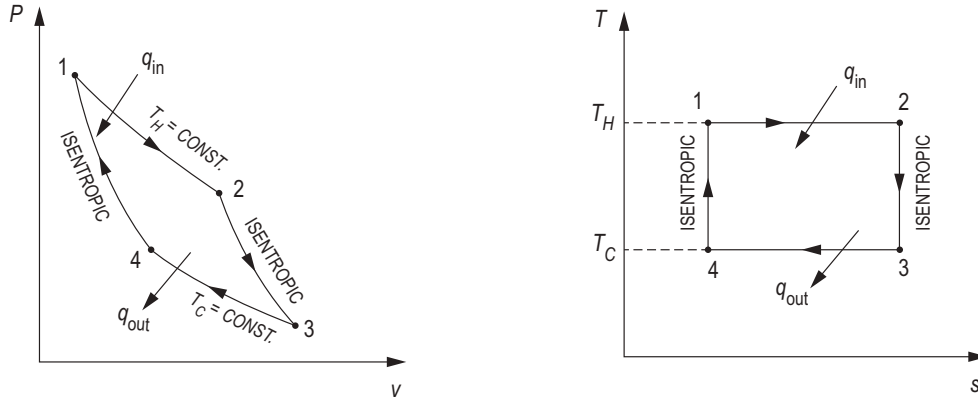
MEP is a fictitious pressure that produces the same amount of net work as the actual cycle.

$$MEP = \frac{W_{net}}{V_{\max} - V_{\min}}$$

3.4.2.1 Carnot Cycle

The Carnot cycle is used to calculate the maximum obtainable output from an ideal heat engine.

Step	Process	Work and Heat	State Functions	
1 → 2	Isothermal expansion ($\Delta T = 0$)	$w_{1 \rightarrow 2} = -RT_H \ln\left(\frac{v_2}{v_1}\right)$	$\Delta s_{1 \rightarrow 2} = -R \ln\left(\frac{v_2}{v_1}\right)$	$\Delta T_{1 \rightarrow 2} = 0$
		$q_{1 \rightarrow 2} = q_{in} = q_H$ $= T_H(s_2 - s_1) = -w_{1 \rightarrow 2}$	$\Delta u_{1 \rightarrow 2} = 0$	$P_2/P_1 = v_1/v_2$
			$\Delta h_{1 \rightarrow 2} = 0$	$v_{1 \rightarrow 2} = v_2 - v_1$
2 → 3	Isentropic expansion (turbine)	$w_{2 \rightarrow 3} = -\frac{k}{k-1}RT_H$ $\cdot \left[1 - \left(\frac{P_3}{P_2}\right)^{(k-1)/k}\right]$	$\Delta s_{2 \rightarrow 3} = 0$	$\Delta T_{2 \rightarrow 3} = T_C - T_H$
			$\Delta u_{2 \rightarrow 3} = c_v(T_C - T_H)$	$\frac{P_3}{P_2} = \left(\frac{T_H}{T_C}\right)^{\frac{k}{k-1}} = \left(\frac{v_2}{v_3}\right)^k$
		$q_{2 \rightarrow 3} = 0$	$\Delta h_{2 \rightarrow 3} = c_p(T_C - T_H)$	$\frac{v_2}{v_3} = \left(\frac{T_H}{T_C}\right)^{\frac{1}{k-1}}$
3 → 4	Isothermal compression ($\Delta T = 0$)	$w_{3 \rightarrow 4} = -RT_C \ln\left(\frac{v_4}{v_3}\right)$	$\Delta s_{3 \rightarrow 4} = -R \ln\left(\frac{v_4}{v_3}\right)$	$\Delta T_{3 \rightarrow 4} = 0$
		$q_{3 \rightarrow 4} = q_{out} = q_c$ $= T_C(s_4 - s_3) = -w_{3 \rightarrow 4}$	$\Delta u_{3 \rightarrow 4} = 0$	$\frac{P_4}{P_3} = \frac{v_3}{v_4}$
			$\Delta h_{3 \rightarrow 4} = 0$	$v_{3 \rightarrow 4} = v_4 - v_3$
4 → 1	Isentropic compression (compressor)		$\Delta s_{4 \rightarrow 1} = 0$	$\Delta T_{4 \rightarrow 1} = T_H - T_C$
		$w_{4 \rightarrow 1} = -\frac{k}{k-1}RT_C$ $\cdot \left[1 - \left(\frac{P_1}{P_4}\right)^{(k-1)/k}\right]$	$\Delta u_{4 \rightarrow 1} = c_v(T_H - T_C)$	$\left(\frac{P_1}{P_4}\right) = \left(\frac{T_C}{T_H}\right)^{\frac{k}{k-1}} = \left(\frac{v_4}{v_1}\right)^k$
		$q_{4 \rightarrow 1} = 0$	$\Delta h_{4 \rightarrow 1} = c_p(T_H - T_C)$	$\frac{v_4}{v_1} = \left(\frac{T_C}{T_H}\right)^{\frac{1}{k-1}}$



where

T_C = temperature at which the working fluid is absorbing heat, which is the same as that of the fluid entering the turbine

T_H = temperature at which heat is emitted by the working fluid, which is the same as the exit temperature from the turbine

The thermal efficiency of the Carnot Cycle is:

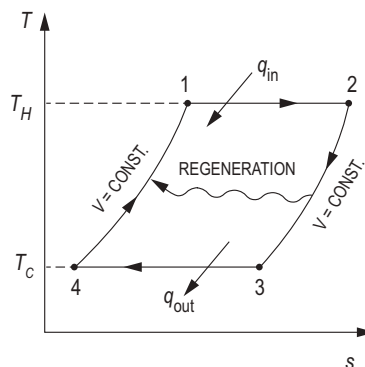
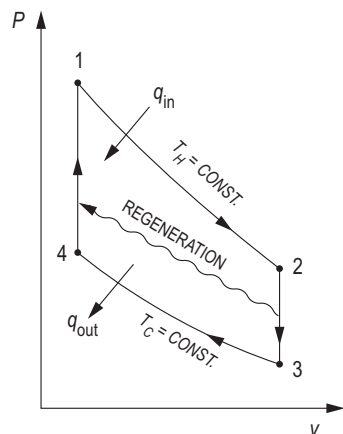
$$\eta_{th,Carnot} = \frac{W_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = \frac{T_H - T_C}{T_H}$$

The efficiency of the Carnot cycle represents an upper limit for the efficiency (maximum possible efficiency) of any power cycle operating between the two temperatures T_H and T_C .

3.4.2.2 The Stirling Cycle

The Stirling cycle is similar to the Carnot cycle, but the isentropic compression and expansion are replaced by constant volume processes with regeneration.

Step	Process	Work and Heat
1 → 2	Isothermal expansion ($\Delta T = 0$)	$q_{1 \rightarrow 2} = q_{in} = RT_H \ln\left(\frac{P_1}{P_2}\right) = -w_{1 \rightarrow 2}$
2 → 3	Isochoric regeneration (internal heat transfer from working fluid to regenerator)	$q_{2 \rightarrow 3} = q_{regen} = c_v(T_C - T_H)$ $w_{2 \rightarrow 3} = 0$
3 → 4	Isothermal compression ($\Delta T = 0$)	$q_{3 \rightarrow 4} = q_{out} = RT_C \ln\left(\frac{P_3}{P_4}\right) = -w_{3 \rightarrow 4}$
4 → 1	Isochoric regeneration (internal heat transfer back from the regenerator to the working fluid)	$q_{4 \rightarrow 1} = q_{regen} = c_v(T_H - T_C)$ $w_{4 \rightarrow 1} = 0$



The thermal efficiency of the Stirling cycle is:

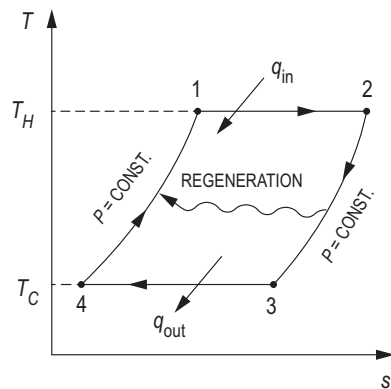
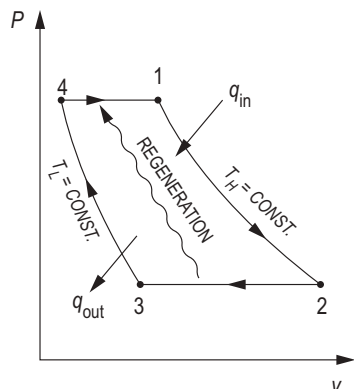
$$\eta_{\text{th, Stirling}} = \frac{W_{\text{out}}}{Q_{\text{in}}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} = \frac{T_H - T_C}{T_H}$$

The thermal efficiency of the Stirling cycle is the same as the thermal efficiency of the Carnot cycle.

3.4.2.3 The Ericsson Cycle

The Ericsson cycle is another variation of the Stirling cycle—the regeneration takes place at constant pressure rather than constant volume.

Step	Process	Work and Heat
1 → 2	Isothermal expansion ($\Delta T = 0$)	$q_{1 \rightarrow 2} = q_{\text{in}} = RT_H \ln\left(\frac{P_1}{P_2}\right) = -w_{1 \rightarrow 2}$
2 → 3	Isobaric regeneration (internal heat transfer from working fluid to regenerator)	$q_{2 \rightarrow 3} = q_{\text{regen}} = c_P(T_C - T_H)$ $w_{2 \rightarrow 3} = P_2(v_3 - v_2)$
3 → 4	Isothermal compression ($\Delta T = 0$)	$q_{3 \rightarrow 4} = q_{\text{out}} = RT_C \ln\left(\frac{P_3}{P_4}\right) = -w_{3 \rightarrow 4}$
4 → 1	Isobaric regeneration (internal heat transfer back from the regenerator to the working fluid)	$q_{4 \rightarrow 1} = q_{\text{regen}} = c_P(T_H - T_C)$ $w_{4 \rightarrow 1} = P_1(v_1 - v_4)$



The thermal efficiency of the Ericsson cycle is:

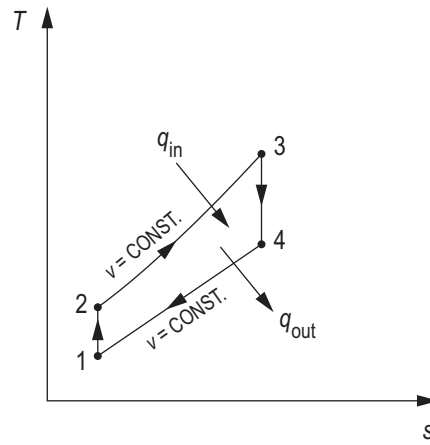
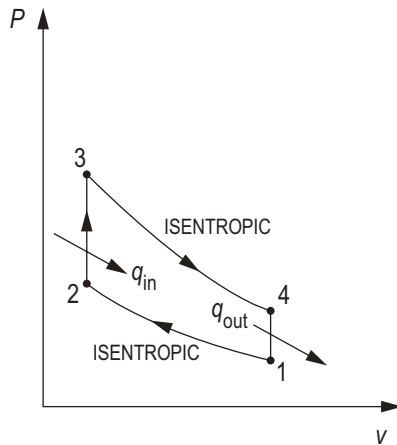
$$\eta_{th, Ericsson} = \frac{W_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = \frac{T_H - T_C}{T_H}$$

The thermal efficiency of the Ericsson cycle is the same as the thermal efficiency of the Carnot cycle.

3.4.2.4 Otto Cycle

Idealized cycle to represent spark-ignition internal combustion engines

Step	Process	Work and Heat
1 → 2	Isentropic compression	$w_{1 \rightarrow 2} = -\frac{k}{k-1}RT_1 \cdot \left[1 - \left(\frac{P_2}{P_1} \right)^{(k-1)/k} \right]$ $q_{1 \rightarrow 2} = 0$
2 → 3	Isochoric heating (represents the internal combustion at TDC)	$q_{2 \rightarrow 3} = q_{in} = c_v(T_3 - T_2)$ $w_{2 \rightarrow 3} = 0$
3 → 4	Isentropic expansion	$w_{3 \rightarrow 4} = -\frac{k}{k-1}RT_3 \cdot \left[1 - \left(\frac{P_4}{P_3} \right)^{(k-1)/k} \right]$ $q_{2 \rightarrow 3} = 0$ $v_4 = v_1 = v_{max}; v_3 = v_2 = v_{min}$
4 → 1	Isochoric cooling (represents the two strokes that exhaust the combustion gases and draw fresh air and fuel in)	$q_{4 \rightarrow 1} = q_{out} = c_v(T_1 - T_4)$ $w_{4 \rightarrow 1} = 0$



The thermal efficiency of the Otto cycle is:

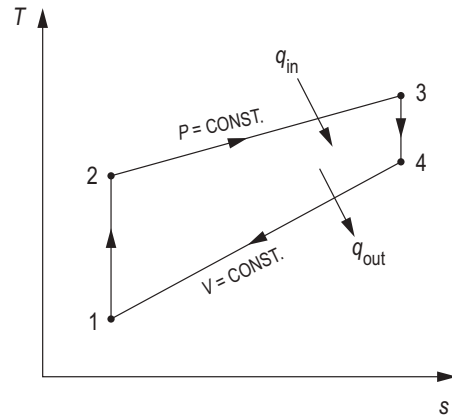
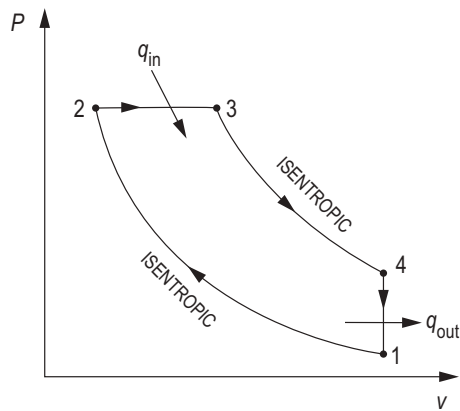
$$\eta_{th, Otto} = 1 - \frac{1}{r^{k-1}}$$

Typical compression ratios (r) are 7 – 10, higher ratios can lead to engine knock (premature auto-ignition of the fuel/air mixture)

3.4.2.5 Diesel cycle

Idealized cycle to represent compression-ignition internal combustion engines

Step	Process	Work and Heat
1 → 2	Isentropic compression	$w_{1 \rightarrow 2} = -\frac{k}{k-1} RT_1 \cdot \left[1 - \left(\frac{P_2}{P_1} \right)^{(k-1)/k} \right]$ $q_{1 \rightarrow 2} = 0$
2 → 3	Isobaric heating (represents the internal combustion, which occurs during initial part of the power stroke)	$w_{2 \rightarrow 3} = P_2(v_3 - v_2)$ $q_{2 \rightarrow 3} = q_{in} = c_p(T_3 - T_2)$
3 → 4	Isentropic expansion	$w_{3 \rightarrow 4} = -\frac{k}{k-1} RT_3 \cdot \left[1 - \left(\frac{P_4}{P_3} \right)^{(k-1)/k} \right]$ $q_{3 \rightarrow 4} = 0$ $v_4 = v_1 = v_{max}$
4 → 1	Isochoric cooling (represents the two strokes that exhaust the combustion gases and draw fresh air and fuel in)	$q_{4 \rightarrow 1} = q_{out} = c_v(T_1 - T_4)$ $w_{4 \rightarrow 1} = 0$



Cut-off ratio r_c

Ratio of the cylinder volumes after and before the combustion process

$$r_c = \frac{V_3}{V_2} = \frac{v_3}{v_2}$$

Efficiency of the Diesel Engine

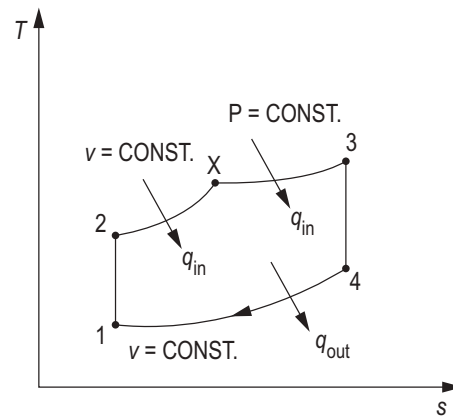
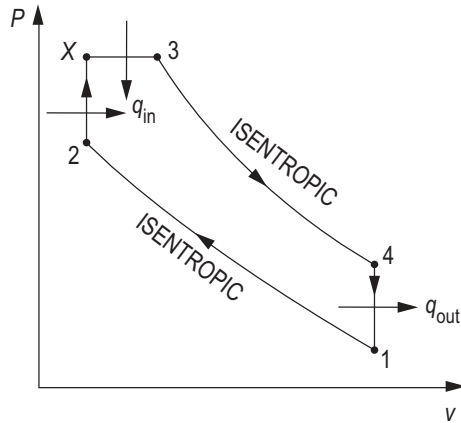
$$\eta_{th, diesel} = 1 - \frac{1}{r^{k-1}} \left[\frac{r_c^k - 1}{k(r_c - 1)} \right]$$

For the same compression ratio, $\eta_{th, diesel} < \eta_{th, Otto}$, but typical compression ratios for diesel engines are higher (about 12–24), since engine knock is not a concern for diesel engines.

3.4.2.6 Dual cycle

Combination between Otto and diesel cycle to more closely resemble the internal combustion engine

Step	Process	Work and Heat
1 → 2	Isentropic compression	$w_{1 \rightarrow 2} = -\frac{k}{k-1} R T_1 \cdot \left[1 - \left(\frac{P_2}{P_1} \right)^{(k-1)/k} \right]$ $q_{1 \rightarrow 2} = 0$
2 → X	Isochoric heating (initial combustion)	$q_{2 \rightarrow X} = q_{in,v} = c_v (T_X - T_2)$ $w_{2 \rightarrow X} = 0$
X → 3	Isobaric heating (continued combustion during power stroke)	$w_{X \rightarrow 3} = P_3 (v_3 - v_2)$ $q_{X \rightarrow 3} = q_{in,P} = c_p (T_3 - T_X)$
3 → 4	Isentropic expansion	$w_{3 \rightarrow 4} = -\frac{k}{k-1} R T_3 \cdot \left[1 - \left(\frac{P_4}{P_3} \right)^{(k-1)/k} \right]$ $q_{3 \rightarrow 4} = 0$ $v_4 = v_1 = v_{max}$
4 → 1	Isochoric cooling (represents the two strokes that exhaust the combustion gases and draw fresh air and fuel in)	$q_{4 \rightarrow 1} = q_{out} = c_v (T_1 - T_4)$ $w_{4 \rightarrow 1} = 0$



Efficiency of the Dual Cycle:

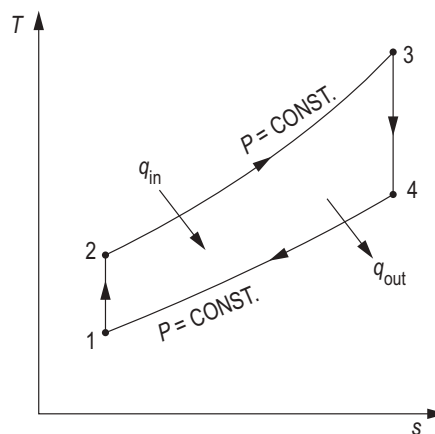
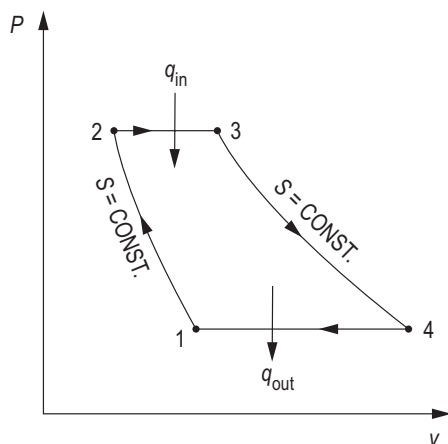
$$\eta_{th,dual} = 1 - \frac{c_v (T_4 - T_1)}{c_v (T_X - T_2) + c_p (T_3 - T_X)}$$

The Otto and the diesel cycle are special cases of the dual cycle.

3.4.2.7 The Brayton Cycle

The Brayton cycle represents compression and expansion in rotating machinery, such as gas turbines.

Step	Process	Work and Heat
1 → 2	Isentropic compression (in a compressor)	$w_{1 \rightarrow 2} = -\frac{k}{k-1}RT_1 \cdot \left[1 - \left(\frac{P_2}{P_1} \right)^{(k-1)/k} \right]$ $q_{1 \rightarrow 2} = 0$ $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}$
2 → 3	Isobaric heat addition (combustion chamber for open systems, heat exchanger for closed systems)	$w_{2 \rightarrow 3} = P_2(v_3 - v_2)$ $q_{2 \rightarrow 3} = q_{in} = c_p(T_3 - T_2)$ $T_3 = T_{max}$ $P_2 = P_3$
3 → 4	Isentropic expansion (in a turbine)	$w_{3 \rightarrow 4} = -\frac{k}{k-1}RT_3 \cdot \left[1 - \left(\frac{P_4}{P_3} \right)^{(k-1)/k} \right]$ $q_{3 \rightarrow 4} = 0$ $\frac{T_3}{T_4} = \left(\frac{P_3}{P_4} \right)^{\frac{k-1}{k}} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = \frac{T_2}{T_1}$
4 → 1	Isobaric heat rejection (exhaust and fresh air intake for open systems, heat exchanger for closed systems)	$w_{4 \rightarrow 1} = P_4(v_1 - v_4)$ $q_{4 \rightarrow 1} = q_{out} = c_p(T_1 - T_4)$ $T_1 = T_{min}$ $P_4 = P_1$



Pressure ratio for the Brayton cycle

$$r_p = \frac{P_2}{P_1} = \frac{P_3}{P_4}$$

Typical pressure ratios for gas turbines range from 11 to 16 and are limited by the maximum temperature at the inlet of the turbine.

Efficiency of the Brayton cycle

$$\eta_{th, \text{Brayton}} = 1 - \frac{1}{r_p^{(k-1)/k}} = \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

For fixed values of $T_{\min}$ and $T_{\max}$, the net work is at a maximum when:

$$r_p = \left(\frac{T_{\max}}{T_{\min}} \right)^{\frac{k}{2(k-1)}}$$

The conversion efficiency can be expressed as "heat rate" (typically in BTU/kWh, i.e., the energy required to generate 1 kWh of electricity:

$$\text{heat rate} = \eta_{th, \text{Ranking}} \cdot 3412 \frac{\text{BTU}}{\text{kWh}}$$

Back-work ratio is the ratio of the compressor work to the turbine work.

$$r_{pw} = \frac{w_{\text{compressor}}}{-w_{\text{turbine}}} = \frac{T_{\max}}{T_{\min}} \left(1 - \frac{1}{r_p^{(k-1)/k}} \right)$$

The thermal efficiency of the Brayton cycle is reduced by the efficiency of the turbine and compressor. Including the isentropic turbine and compressor efficiencies can correct for the irreversibilities in the compressor and turbine.

Regeneration

In the Brayton cycle with regeneration, the exhaust from the turbine is used to pre-heat the outlet from the compressor, prior to heating it in the combustion chamber. It reduces the required heat input and thus improves efficiency.

$$q_{\text{regen, max}} = h_{5'} - h_2 = h_4 - h_2$$

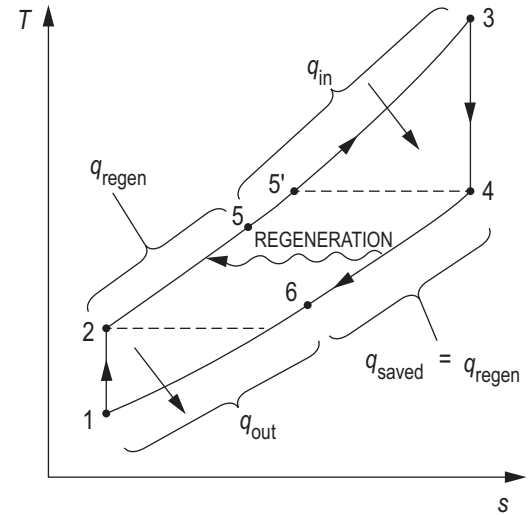
$$q_{\text{regen, act}} = h_5 - h_2 = h_4 - h_6$$

Effectiveness of the Regenerator

$$\epsilon \cong \frac{T_5 - T_2}{T_4 - T_2}$$

Efficiency of ideal Brayton cycle with regeneration

$$\eta_{th, \text{Brayton}} = 1 - \left(\frac{T_{\max}}{T_{\min}} \right) r_p^{(k-1)/k}$$



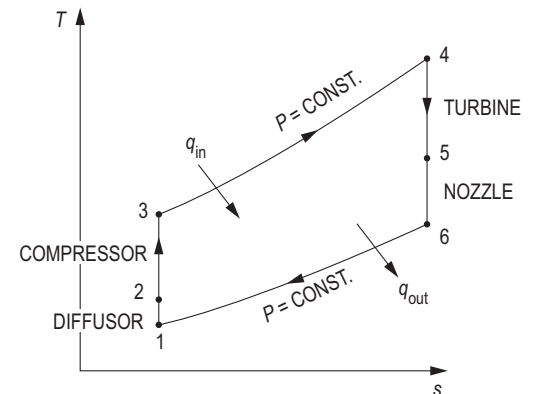
Regeneration is most effective for low pressure ratios and low temperature ratios.

Multistage compression with intercooling and multistage expansion with reheating can further increase the efficiency of the Brayton cycle.

3.4.2.8 The Jet Propulsion Cycle

The jet propulsion cycle is similar to the Brayton cycle, but the gases are expanded in the turbine only until it produces enough work to drive the compressor and then exit at high velocity to provide thrust to an aircraft. Thus, the turbine in the Brayton cycle is replaced by a combination of turbine and nozzle.

In the diffuser, the fluid changes velocity from that of the surrounding to that of the moving airplane. This is a deceleration when viewed from the airplane point of reference.



The net work for a turbojet is zero, efficiency is thus defined as propulsive efficiency based on the propulsive power ($\dot{W}_P$) and energy input rate ($\dot{Q}_{in}$):

$$\eta_P = \frac{\dot{W}_P}{\dot{Q}_{in}}$$

$$\dot{W}_P = \dot{m}(u_{exit} - u_{inlet})u_{aircraft}$$

3.4.3 Vapor Power Cycles

Vapor power cycles use a working fluid that evaporates and condenses in the course of a cycle. This allows the compression to occur in the liquid phase, where the smaller specific volume results in less work for the compression.

3.4.3.1 Rankine Cycle

The Rankine cycle is the ideal cycle for vapor power plants.

Step	Process	Work and Heat
1 → 2	Isentropic compression (of the liquid in a pump)	$w_{1 \rightarrow 2} = w_{in} = h_1 - h_2 = v(P_1 - P_2)$ $q_{1 \rightarrow 2} = 0$ $h_1 = h_{liq@P_1}$ $v \cong v_1 = v_{liq@P_1}$
2 → 3	Isobaric heat addition (in a boiler, vaporizing and superheating the liquid)	$w_{2 \rightarrow 3} = 0$ $q_{2 \rightarrow 3} = q_{in} = h_3 - h_2$
3 → 4	Isentropic expansion (of the gas in the turbine)	$w_{3 \rightarrow 4} = w_{out} = h_3 - h_4$ $q_{3 \rightarrow 4} = 0$
4 → 1	Isobaric heat rejection (in a condenser, condensing the vapor exiting the turbine)	$w_{4 \rightarrow 1} = 0$ $q_{4 \rightarrow 1} = q_{out} = h_1 - h_4$

Efficiency of the Rankine Cycle

$$\eta_{th,Ranking} = \frac{w_{net}}{q_{in}} = \frac{w_{out} - w_{in}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

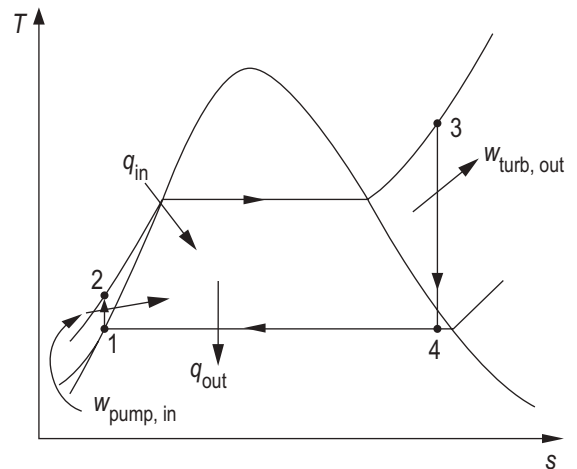
The conversion efficiency can be expressed as "heat rate" (typically in BTU/kWh, i.e., the energy required to generate 1 kWh of electricity:

$$\text{heat rate} = \eta_{th,Ranking} \cdot 3412 \frac{\text{BTU}}{\text{kWh}}$$

The backwork ratio for the Rankine cycle is much smaller than the backwork ratio for the Brayton cycle.

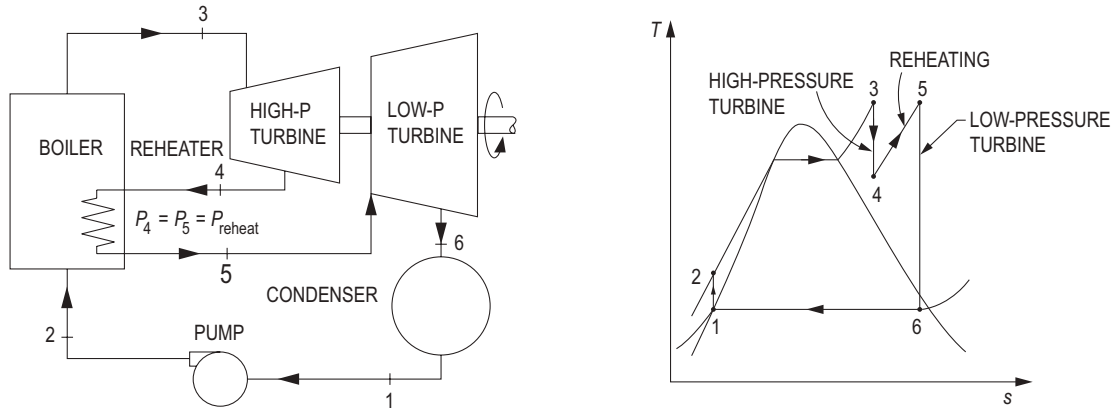
$$r_{pw} = \frac{w_{in}}{w_{out}} = \frac{w_{pump}}{w_{turbine}}$$

Efficiency calculations for an actual (not ideal) Rankine cycle include the isentropic efficiencies for the compressor and the pump.



3.4.3.2 Ideal Rankine Cycle with Reheat

To improve efficiency of the Rankine cycle, the steam is expanded in the turbine in 2 phase, with a reheat in between.



With the reheat, the heat input and work output are calculated as follows:

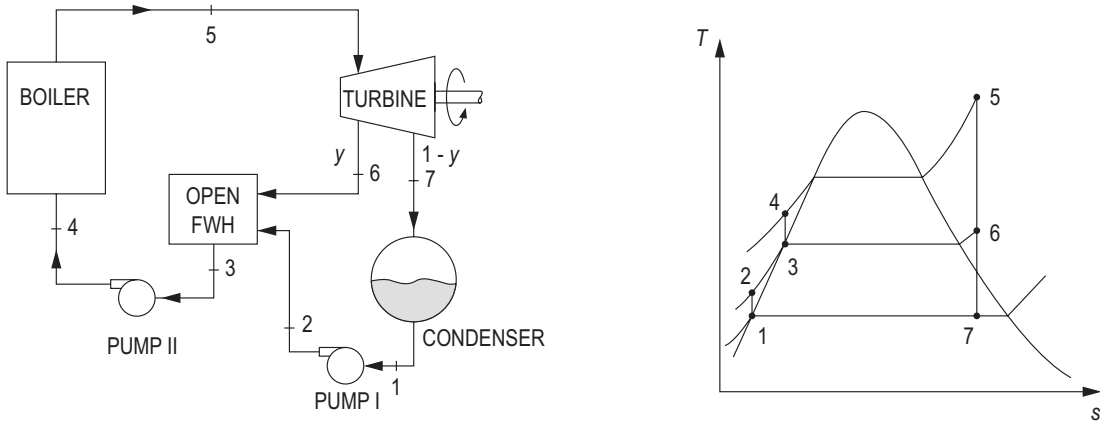
$$q_{\text{in}} = q_{\text{primary}} + q_{\text{reheat}} = (h_3 - h_2) + (h_5 - h_4)$$

$$w_{\text{out}} = w_{\text{turb, HP}} + q_{\text{turb, LP}} = (h_4 - h_3) + (h_6 - h_5)$$

3.4.3.3 Ideal Rankine Cycle with Regeneration

For regeneration, part of the steam is extracted from the turbine and used to pre-heat the feedwater. Open feedwater heaters mix the steam directly with the water; closed feedwater heater use a heat exchanger.

Regenerating increases the inlet temperature to the boiler and thus reduces the heat required in the boiler.



With the regeneration, the heat input and work output are calculated as follows (where y is the fraction of steam that is extracted from the turbine):

$$y = \frac{\dot{m}_6}{\dot{m}_5}$$

$$q_{\text{in}} = h_5 - h_4$$

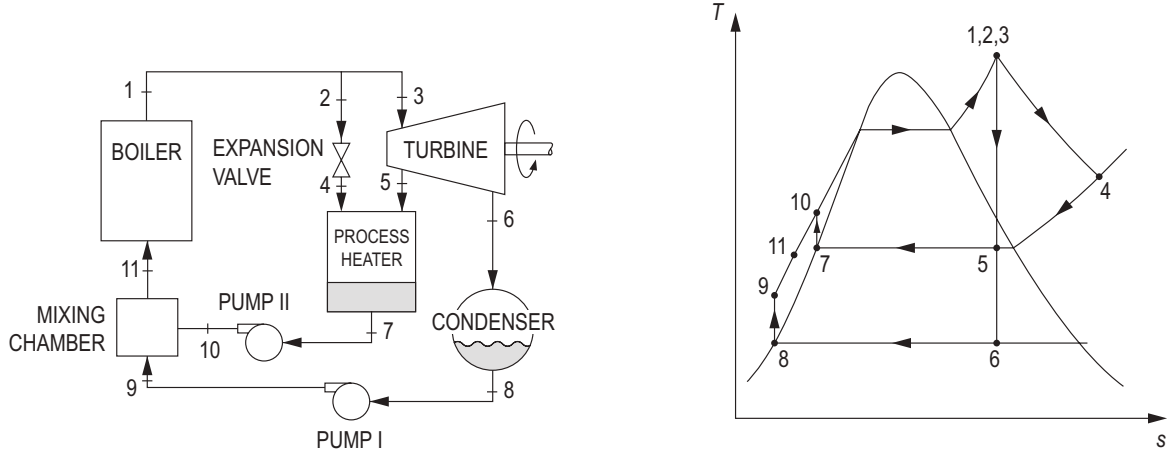
$$q_{\text{out}} = (1 - y)(h_7 - h_1)$$

$$w_{\text{out}} = (h_5 - h_6) + (1 - y)(h_6 - h_7)$$

$$w_{\text{in}} = (1 - y)w_{\text{Pump I, in}} + w_{\text{Pump II, in}} = (1 - y)v_1(P_2 - P_1) + v_3(P_4 - P_3)$$

3.4.3.4 Cogeneration Cycle

A cogeneration cycle is based on the Rankine cycle, but instead of condensing the vapor leaving the turbine, it is used for process heating. The steam is thus extracted from the turbine at higher pressures, depending on the process heating needs.



The use of the higher pressure steam reduces the net work delivered by the turbine but adds the process heat to the benefit.

In actual cogeneration cycles, some losses occur due to inefficiencies of the turbine or combustion process or heat losses from the steam piping. The utilization factor (or "fuel chargeable to power", FCP) is:

$$FCP = \frac{\dot{W}_{net} + \dot{Q}_{process}}{\dot{Q}_{in}} = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}}$$

In an ideal cogeneration cycle, the efficiency is 100%, since no heat is wasted ($\dot{Q}_{out} = 0$).

Typical cogeneration cycles include a condenser in parallel to the process heater and an expansion valve in parallel to the turbine for adjustable loads. When demand for steam is low, more power is generated by extracting less steam from the turbine. When demand for steam is high, the turbine is bypassed.

$$\begin{aligned}\dot{Q}_{in} &= \dot{m}_3(h_4 - h_3) \\ \dot{Q}_{out} &= \dot{m}_7(h_7 - h_1) \\ \dot{Q}_{process} &= \dot{m}_5 h_5 + \dot{m}_6 h_6 - \dot{m}_8 h_8 \\ \dot{W}_{turbine} &= (\dot{m}_4 - \dot{m}_5)(h_4 - h_6) + \dot{m}_7(h_6 - h_7)\end{aligned}$$

3.4.4 Refrigeration Cycles

Refrigeration cycles are the reverse of power cycles. Power cycles transfer heat from a hot reservoir to a cold reservoir and extract work. Refrigeration cycles use work to transfer heat from a cold reservoir to a hot reservoir. For a refrigerator (R), the objective is to remove heat from the low temperature reservoir. For a heat pump (HP), the objective is to add heat to the high- temperature reservoir.

3.4.4.1 Coefficient of Performance

The coefficient of performance is defined similar to the thermal efficiency, but the purpose of refrigeration cycles differs from power cycles and thus the benefit and cost are defined differently:

$$\begin{aligned}COP_R &= \frac{\text{Cooling effect}}{\text{Work input}} = \frac{Q_C}{\dot{W}_{net, in}} \\ COP_{HP} &= \frac{\text{Heating effect}}{\text{Work input}} = \frac{Q_H}{\dot{W}_{net, in}}\end{aligned}$$

Both COP_R and COP_{HP} can be > 1 . When $COP_{HP} < 1$, a simple resistance heater (which turns work into heat) would be more efficient.

For fixed values of Q_C and Q_H :

$$\text{COP}_{\text{HP}} = \text{COP}_{\text{R}} + 1$$

3.4.4.2 Reverse Carnot Cycle and Reverse Stirling Cycle

The reverse Carnot cycle is the most efficient refrigeration cycle operating between two temperatures, but it is not a realistic model for actual refrigeration cycles. It does provide a maximum achievable coefficient of performance against which other cycles can be compared.

Carnot Refrigeration cycle

$$\text{COP}_{\text{R,Carnot}} = \frac{1}{(T_H/T_C) - 1}$$

Carnot Heat Pump cycle

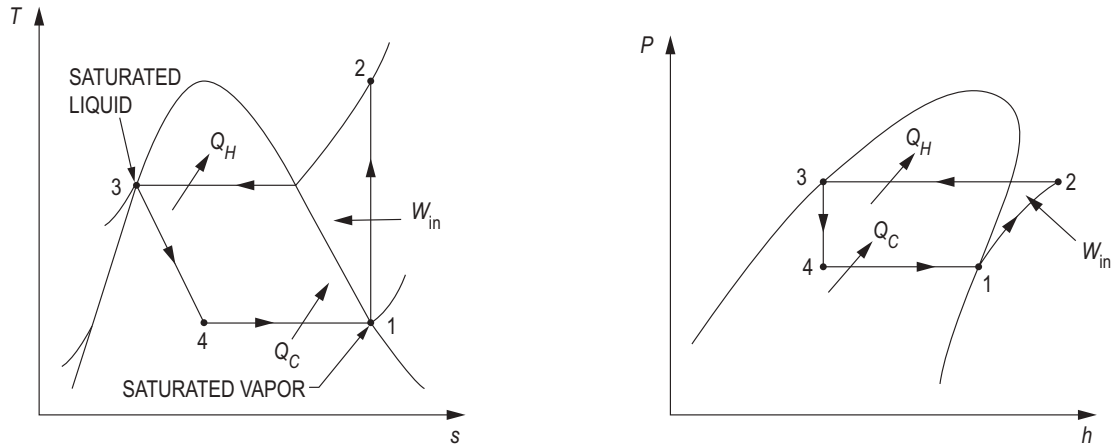
$$\text{COP}_{\text{HP,Carnot}} = \frac{1}{1 - (T_C/T_H)}$$

The reverse Stirling cycle has the same coefficients of performance and is easier to implement in practice (example: Stirling refrigerators).

3.4.4.3 The Ideal Vapor-Compression Refrigeration Cycle

The vapor-compression refrigeration cycle is the most widely used cycle for refrigerators and air conditioners. It is similar to a reverse Rankine cycle, but uses isenthalpic throttling instead of isentropic expansion.

Step	Process	Work and Heat
1 → 2	Isentropic compression (of the vapor in a compressor)	$w_{1 \rightarrow 2} = w_{\text{in}} = -\frac{k}{k-1} RT_1 \cdot \left[1 - \left(\frac{P_2}{P_1} \right)^{(k-1)/k} \right]$ $q_{1 \rightarrow 2} = 0$ $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}$ $h_1 = h_{\text{vap}}(P_1)$
2 → 3	Isobaric heat rejection (in a condenser, exit condition saturated liquid)	$w_{2 \rightarrow 3} = 0$ $q_{2 \rightarrow 3} = q_H = q_{\text{out}} = h_3 - h_2$
3 → 4	Isenthalpic throttling (of the liquid in an expansion valve or capillary tube, exit conditions: 2-phase)	$h_4 = h_3 = h_{\text{liq}}(P_3)$ $w_{3 \rightarrow 4} = 0$ $q_{3 \rightarrow 4} = 0$
4 → 1	Isobaric heat adsorption (in an evaporator, exit condition: saturated vapor)	$w_{4 \rightarrow 1} = 0$ $q_{4 \rightarrow 1} = q_C = q_{\text{in}} = h_1 - h_4$



Coefficient of Performance for the ideal Vapor-Compression Refrigeration Cycle

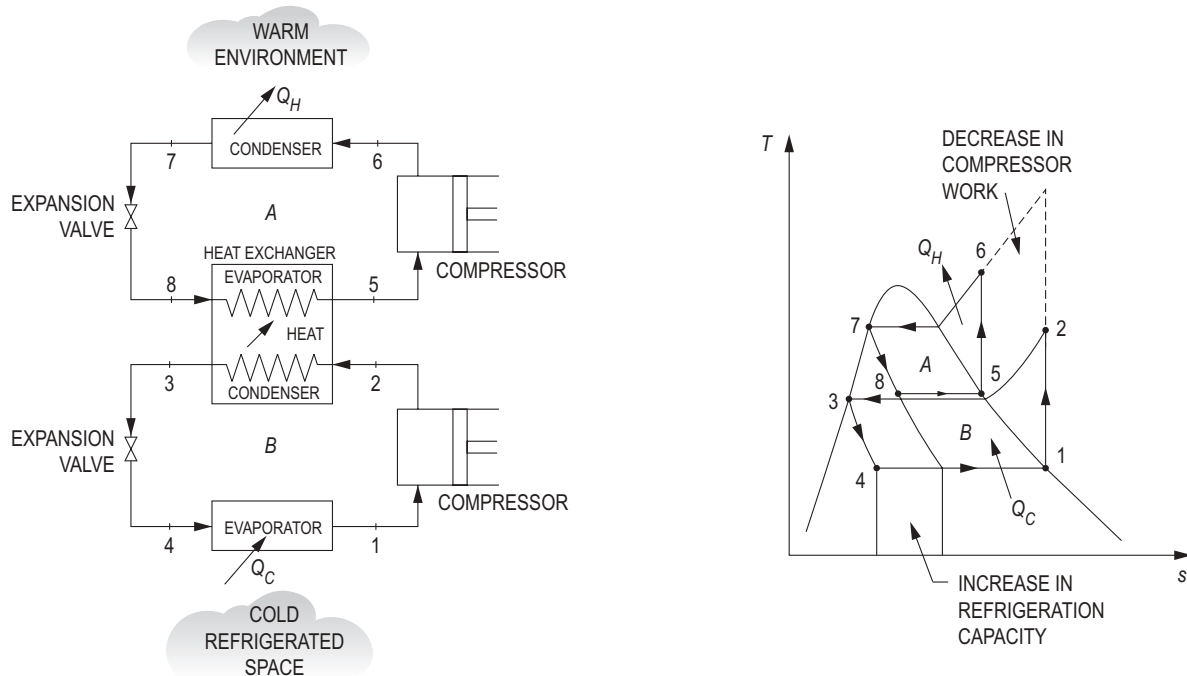
$$COP_{R,VCC} = \frac{q_c}{w_{\text{net,in}}} = \frac{h_1 - h_4}{h_2 - h_1}$$

Coefficient of Performance for the ideal Vapor-Compression Heat Pump Cycle

$$COP_{HP,VCC} = \frac{q_H}{w_{\text{net,in}}} = \frac{h_2 - h_3}{h_2 - h_1}$$

3.4.4.4 Cascade Refrigeration Systems

Cascade refrigeration system with heat exchange between the stages allows the use of different working fluids for each cycle.



The ratio of mass flow rates in each cycle is

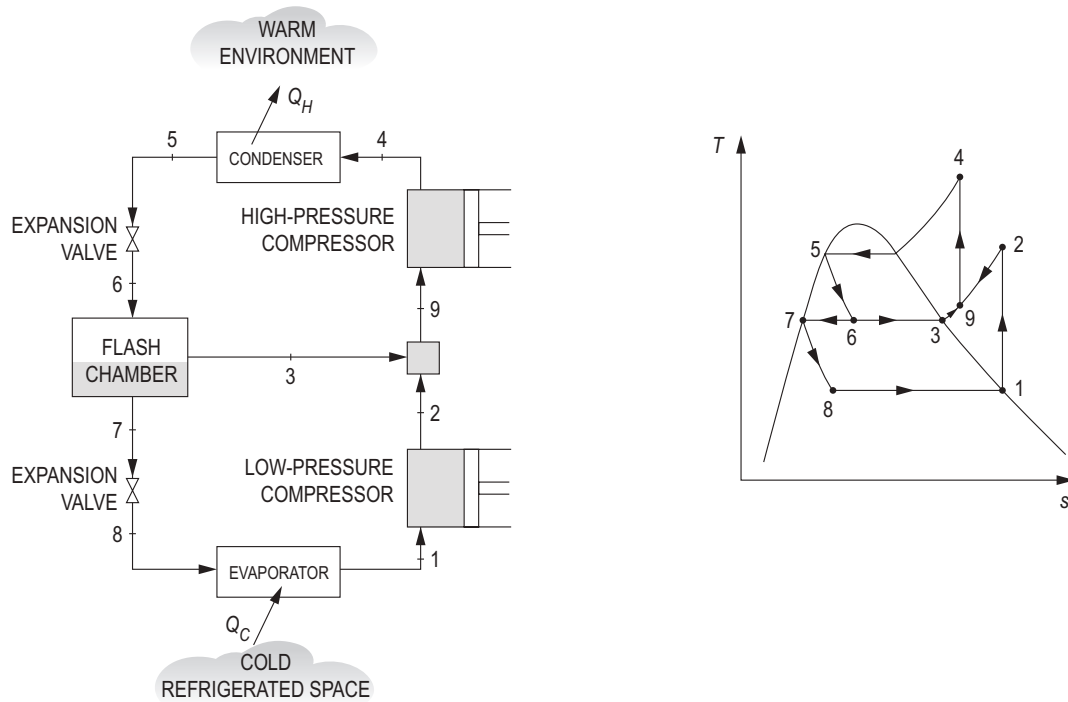
$$\frac{\dot{m}_A}{\dot{m}_B} = \frac{h_2 - h_3}{h_5 - h_8}$$

The coefficient of performance is

$$COP_{R, \text{Cascade}} = \frac{\dot{Q}_C}{\dot{W}_{\text{net, in}}} = \frac{\dot{m}_B(h_1 - h_4)}{\dot{m}_A(h_6 - h_5) + \dot{m}_B(h_2 - h_1)}$$

3.4.4.5 Multistage Compression Refrigeration Systems

Multistage compression refrigeration systems are similar to cascade system, but use the same working fluid. A mixing chamber is used in place of the heat exchanger:



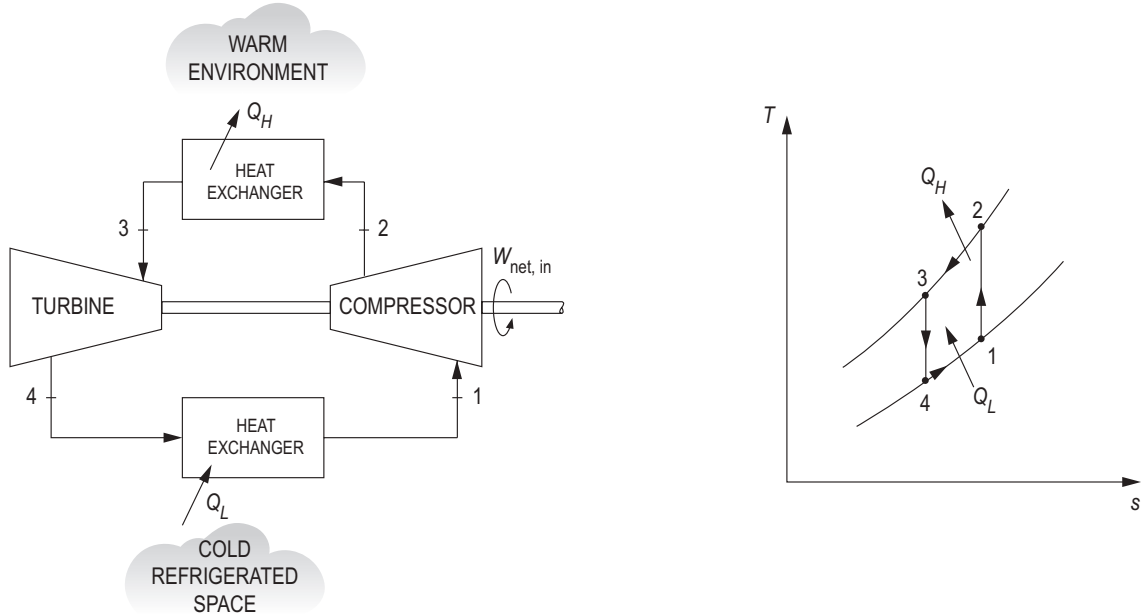
The coefficient of performance is:

$$COP_{R, \text{GasRef}} = \frac{\dot{q}_c}{\dot{w}_{\text{net, in}}} = \frac{\dot{q}_c}{\dot{w}_{\text{comp, LP}} + \dot{w}_{\text{comp, HP}}} = \frac{(1-x)(h_1 - h_8)}{(1-x)(h_2 - h_1) + (h_4 - h_9)}$$

where x is the vapor fraction after the first expansion valve (Point 6).

3.4.4.6 Gas Refrigeration Systems

The gas refrigeration cycle is the reverse Brayton cycle.

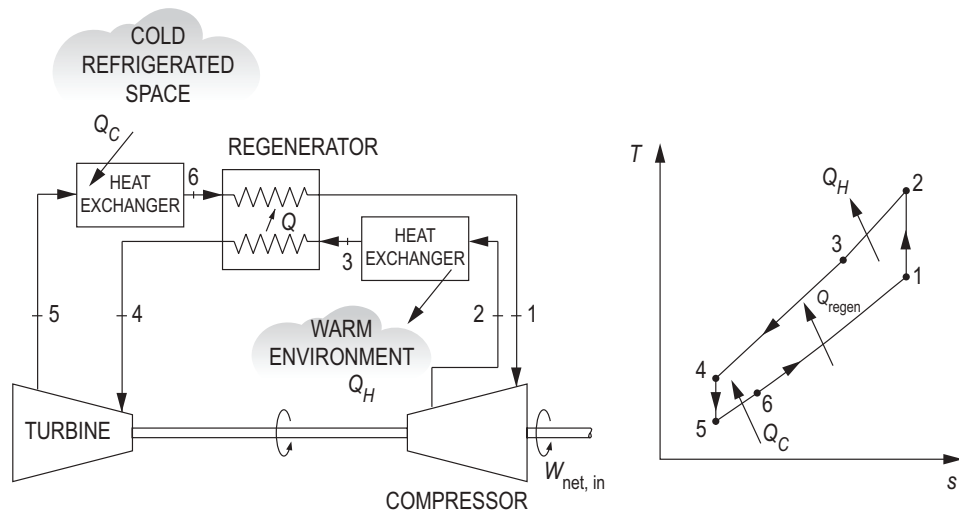


The coefficient of performance is:

$$COP_{R, GasRef} = \frac{\dot{q}_c}{\dot{w}_{net, in}} = \frac{\dot{q}_c}{\dot{w}_{comp, in} - \dot{w}_{turb, out}} = \frac{(h_1 - h_4)}{(h_2 - h_1) + (h_3 - h_4)}$$

Coefficients of performance are lower than for the Carnot cycle and the vapor-compression refrigeration cycle. They are used in airplanes for cooling and in liquefaction of gases.

Regeneration can be included if the turbine exit temperature is above the compressor inlet temperature. The heat transfer from the turbine inlet stream to the compressor inlet stream reduces the amount of heat absorbed in the cold exchanger and thus lowers the coefficient of performance. However, it also reduces the inlet temperature to the turbine and allows to achieve lower temperatures for the cold exchanger.



3.5 Chemical Reaction Equilibria

3.5.1 Gibbs Free Energy and the Equilibrium Constant

$$\Delta \hat{g}_r = -RT \ln K \quad \text{or} \quad \ln K = -\frac{\Delta \hat{g}_r}{RT}$$

where K is defined in terms of concentration or pressure in the Vapor Power Cycles and Refrigeration Cycles sections in this chapter.

3.5.2 Temperature Dependence

The change of the equilibrium constant with temperature is a function of the heat of reaction:

$$\frac{d(\ln K)}{dT} = \frac{\Delta \hat{h}_r}{RT^2}$$

The integrated equation is

$$\ln \frac{K_2}{K_1} = \frac{1}{R} \int_{T_1}^{T_2} \left(\frac{\Delta \hat{h}_r}{T^2} \right) dT$$

Over a range where $\Delta \hat{h}_r$ is nearly constant, this simplifies to:

$$\ln \frac{K_2}{K_1} = -\frac{\Delta \hat{h}_r}{R} \left(\frac{1}{T_2} - \frac{1}{T_1} \right)$$

Gibbs-Helmholtz equation:

$$-\frac{\Delta \hat{h}}{RT^2} = \left(\frac{\partial \left(\frac{\Delta \hat{g}}{RT} \right)}{\partial T} \right)_P$$

At equilibrium: $dP = 0$

$$\frac{\Delta \hat{h}^0}{RT^2} = -\frac{d \left(\frac{\Delta \hat{g}^0}{RT} \right)}{dT} = \frac{d(\ln K)}{dT}$$

3.5.3 Concentration Dependence

General reaction: $aA + bB \leftrightarrow cC + dD$

At equilibrium: $r_{\text{FWD}} = r_{\text{REV}}$

where

$$-r_{\text{FWD}} = k_1 C_A^a C_B^b$$

$$-r_{\text{REV}} = k_2 C_C^c C_D^d$$

The equilibrium constant is defined as

$$K_c = \frac{C_C^c C_D^d}{C_A^a C_B^b} = \frac{k_1}{k_2}$$

3.5.4 Pressure Dependence

For general reactions: $aA + bB \leftrightarrow cC + dD$

At equilibrium: $r_{\text{FWD}} = r_{\text{REV}}$

where

$$-r_{\text{FWD}} = k_1 P_A^a P_B^b$$

$$-r_{\text{REV}} = k_2 P_C^c P_D^d$$

The equilibrium constant is defined as

$$K_P = \frac{P_C^c P_D^d}{P_A^a P_B^b} = \frac{k_1}{k_2}$$

When $a + b = c + d$, $K_P = K_c$, and both are dimensionless.

When they are not equal:

K_P has units of pressure to the power $(c + d - a - b)$.

K_c has units of concentration to the power $(c + d - a - b)$.

Thus:

$$K_P = K_c (RT)^{(c+d-a-b)}$$

3.5.5 Le Chatelier's Principle

Le Chatelier's Principle describes the qualitative effect of pressure on equilibrium: For a gaseous reaction, increasing pressure will shift the equilibrium to the side of the reaction in the reaction equation with fewer moles.

Changes in pressure have negligible effect on liquid- or solid-phase reactions.

3.6 Phase Equilibria

3.6.1 Definitions

3.6.1.1 Phase

A phase is a homogeneous region of matter. Examples of phases include a gas, a mixture of gases, a liquid, a solution of liquids, and a solid.

3.6.1.2 Saturation Temperature

The saturation temperature is the temperature at which both liquid and vapor exist in equilibrium at a given pressure.

3.6.1.3 Triple Point

For a pure substance, the triple point is the point at which the solid, liquid, and vapor phases exist in equilibrium.

3.6.1.4 Critical Point

For a pure substance, the critical point is the temperature and pressure at which the liquid and vapor phases exhibit identical properties and are indistinguishable from each other.

3.6.1.5 Phase Rule

For nonreacting systems, the number of degrees of freedom F is the number of intensive variables (for example, temperature, pressure, and composition) that must be specified in order to fix the intensive state of a system at equilibrium.

$$F = 2 - \pi + N$$

where

π = number of phases

N = number of chemical species

For reacting systems, the number of degrees of freedom F is:

$$F = 2 - \pi + N - r$$

where

r = number of independent chemical reactions at equilibrium within the system

3.6.2 Pure Substances

3.6.2.1 Phase Transitions for Pure Substances

$\Delta S = \frac{\Delta H}{T}$ at constant pressure, and

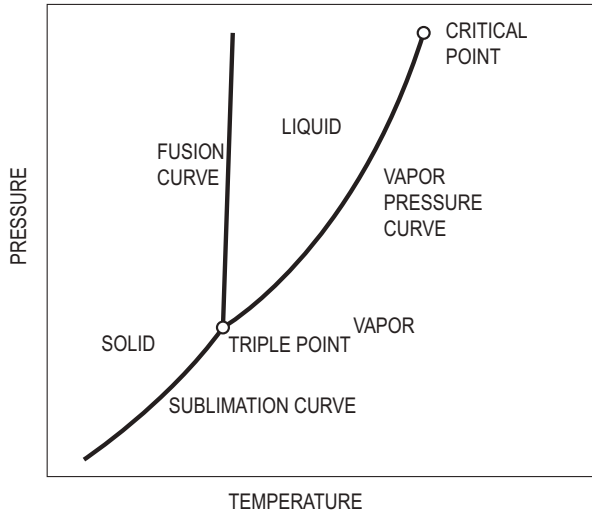
$\Delta S = \frac{\Delta U}{T}$ at constant volume

3.6.2.2 Phase Diagrams for Pure Substances

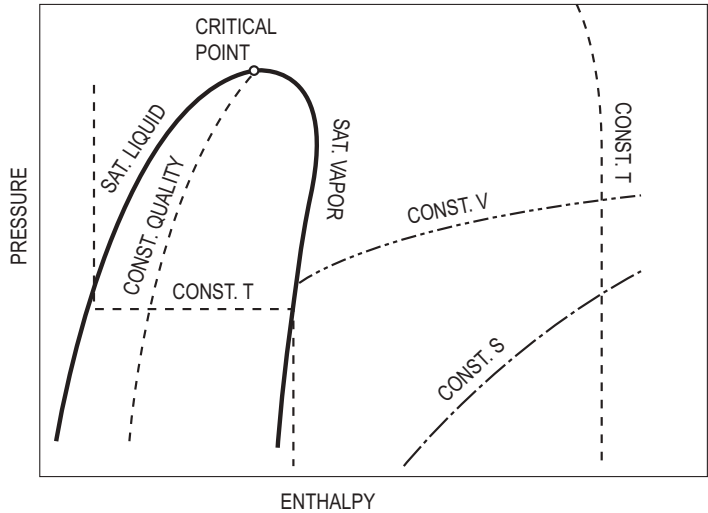
The pressure-temperature relationship for a pure fluid is often shown in a pressure-temperature plot. The intersection of the solid-liquid-vapor lines is the triple point where the three phases coexist. The critical point is where vapor and liquid properties become identical.

Four kinds of diagrams are often used for calculations involving a pure fluid. Figures below show the qualitative behavior of fluid properties.

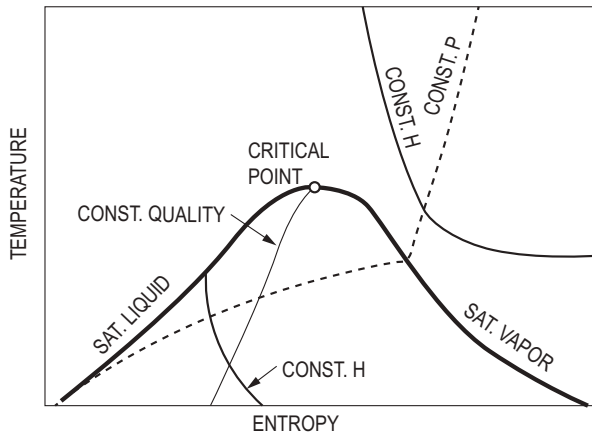
Thermodynamic Diagrams for a Pure Fluid



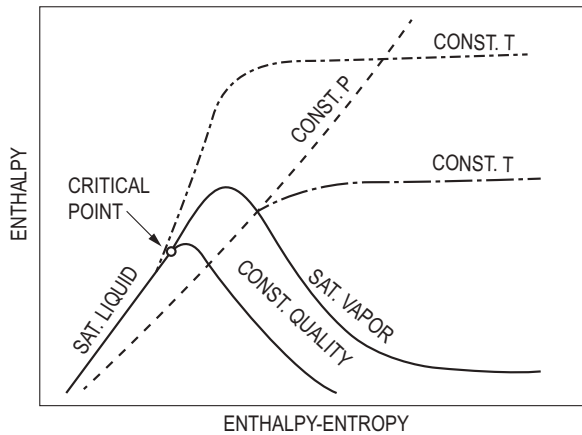
PRESSURE-TEMPERATURE DIAGRAM FOR PURE FLUID



PRESSURE-ENTHALPY DIAGRAM FOR PURE FLUID



TEMPERATURE-ENTROPY DIAGRAM FOR PURE FLUID



ENTHALPY-ENTROPY (MOLLIER) DIAGRAM FOR PURE FLUID

3.6.2.3 Vapor Pressure

Vapor pressure is the pressure in a closed system containing a pure fluid with both liquid and vapor in equilibrium at a given temperature. The equilibrium phases are saturated.

The *Antoine equation* can be used to estimate the temperature dependence of vapor pressure:

$$\log P^{\text{sat}} = A - \frac{B}{T + C}$$

where

P^{sat} = saturation pressure or vapor pressure

A , B , and C = constants for a given species

T = temperature

3.6.2.4 Clausius-Clapeyron Equation

The *Clapeyron equation* relates enthalpy change to temperature, vapor pressure, and volume in the phase change of a two-phase, single-species system.

$$\frac{dP^{\text{sat}}}{dT} = \frac{\Delta s}{\Delta v} = \frac{\Delta h}{T \Delta v}$$

where

Δh = specific latent heat for the phase change

Δs = specific entropy for the phase change

Δv = specific volume change for the phase change

For the phase transition from liquid to vapor as an ideal gas, the Clapeyron equation becomes the *Clausius-Clapeyron equation*:

$$\frac{d(\ln P^{\text{sat}})}{d\left(\frac{1}{T}\right)} = -\frac{\Delta \hat{h}_{\text{vap}}}{R}$$

Assuming a constant, or average, heat of vaporization between T_1 and T_2 , the integrated form is

$$\ln\left(\frac{P_2^{\text{sat}}}{P_1^{\text{sat}}}\right) = -\frac{\Delta \hat{h}_{\text{vap}}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

3.6.2.5 Bubbles, Cavities, and Droplets

Due to surface tension, the pressure on the inside of a curved surface is higher than on the outside (Laplace Equation):

$$P_{\text{in}} = P_{\text{out}} + 2\gamma r$$

where

r = radius of the curved surface

γ = surface tension

Droplets are spheres of the liquid phase in the vapor phase. A mist made from droplets has a higher vapor pressure than the bulk fluid (Kelvin Equation):

$$P_{\text{mist}}^{\text{sat}} = P_{\text{bulk}}^{\text{sat}} \exp\left(\frac{2\gamma \hat{v}}{rRT}\right)$$

Cavities are holes in a liquid filled with vapor. Cavities have a lower vapor pressure than the bulk fluid:

$$P_{\text{cavity}}^{\text{sat}} = P_{\text{bulk}}^{\text{sat}} \exp\left(-\frac{2\gamma \hat{v}}{rRT}\right)$$

where

$\hat{v}$ = molar volume of the liquid

Bubbles are regions of vapor or gas trapped by a thin film. Due to the double surface area, the pressure increase in bubbles is twice that of cavities.

3.6.3 Ideal Systems

3.6.3.1 Raoult's Law (for ideal solutions)

Assuming a vapor phase that is an ideal gas and a liquid phase that is an ideal solution:

$$p_i = y_i P = x_i P_i^{\text{sat}}$$

where

x_i = mole fraction of component i in liquid phase

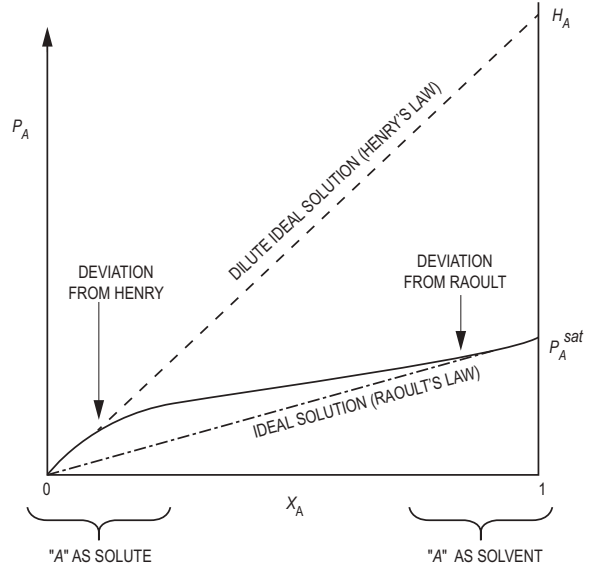
P_i^{sat} = vapor pressure of pure component i

3.6.3.2 Henry's Law (for dilute ideal solutions)

The partial pressure of a component in the gas phase is proportional to the concentration of the component in the liquid phase:

$$p_i = y_i P = x_i H_i$$

where H_i = Henry's law constant for component i



3.6.3.3 Distribution of Components Between Phases in Vapor/Liquid Equilibrium

Assume Dalton's Law and Raoult's Law apply. The distribution coefficient (K -value) is defined as:

$$K_i = \frac{y_i}{x_i} = \frac{P_i^{\text{sat}}}{P}$$

where K_i = distribution coefficient for component i

P_i^{sat} = saturation pressure of pure component i

The relative volatility is defined as:

$$\alpha_{ij} = \frac{K_i}{K_j} = \frac{y_i x_j}{y_j x_i}$$

where α_{ij} = relative volatility for components i and j

For a binary system:

$$y_1 = \frac{x_1 \alpha_{12}}{1 + x_1 (\alpha_{12} - 1)} = \frac{K_1 x_1}{K_2 + x_1 (K_1 - K_2)}$$

$$x_1 = \frac{y_1}{\alpha_{12} + y_1 (1 - \alpha_{12})} = \frac{K_2 y_1}{K_1 + y_1 (K_2 - K_1)}$$

3.6.4 Nonideal Systems

3.6.4.1 Fugacity

The criterion for the vapor-liquid equilibrium of non-ideal systems is

$$\hat{f}_i^{\text{V}} = \hat{f}_i^{\text{L}}$$

where

$\wedge$ = indicates properties of the component in the mixture

$\hat{f}_i^{\text{V}}$ = fugacity of component i in the vapor phase

$\hat{f}_i^{\text{L}}$ = fugacity of component i in the liquid phase

3.6.4.2 Fugacity of Pure Component

Vapor Fugacity of a Pure Component

$$f^V = \phi P$$

where ϕ = fugacity coefficient of pure component in vapor phase

The fugacity coefficient of a pure component is a function of temperature and pressure and may be determined from any of:

The residual Gibbs free energy (G^R) $\ln \phi = \frac{G^R}{RT}$

An equation of state

$$\ln \phi = \int_0^P (Z - 1) \frac{dP}{P}$$

A generalized correlation, e.g., $\ln \phi = (\ln \phi)^0 + \omega (\ln \phi)^1$

where ω = the acentric factor

Liquid Fugacity of a Pure Component

$$f^L = \phi^{\text{sat}} P^{\text{sat}} \phi_i \quad \phi_i = \exp \left[\frac{\hat{v}^L (P - P^{\text{sat}})}{RT} \right]$$

where

ϕ^{sat} = fugacity coefficient of pure component at saturation pressure

ϕ_i = Poynting correction factor

$\hat{v}^L$ = molar volume of pure component in the liquid phase

3.6.4.3 Fugacity of Mixtures

Vapor Fugacity of a Mixture

$$\hat{f}_i^V = \hat{\phi}_i P_i = \hat{\phi}_i y_i P$$

where $\hat{\phi}_i$ = fugacity coefficient of component i in the vapor phase

The fugacity coefficient of a component in a mixture may be determined from an equation of state and a mixing rule.

For a pure component, using the virial equation: $\ln \phi = \frac{B P}{RT}$

For a mixture, using the virial equation: $\ln \hat{\phi}_i = \left(\sum_j y_j B_{ij} - B_m \right) \frac{P}{RT}$

where

$$B_m = \sum_i \sum_j y_i y_j B_{ij}$$

B_m = second virial coefficient of the mixture

B_{ij} = virial coefficient that characterizes a bimolecular interaction between i and j

For $i = j$, $B_{ij} = B_{ji} = B_{ii}$

For $i \neq j$, B_{ij} must be obtained from measured values or mixing rules.

Liquid Fugacity of a Mixture

$$\hat{f}_i^L = \gamma_i x_i f_i^L = \gamma_i x_i \phi_i^{\text{sat}} P_i^{\text{sat}} \phi_i$$

where γ_i = activity coefficient of component i

Activity coefficients are normally based on experimental measurements and fitted to an activity coefficient model, for example the Van Laar model:

$$\ln \gamma_1 = A_{12} \left(1 + \frac{A_{12} x_1}{A_{21} x_2} \right)^{-2} \quad \text{and} \quad \ln \gamma_2 = A_{21} \left(1 + \frac{A_{21} x_2}{A_{12} x_1} \right)^{-2}$$

where A_{12} and A_{21} = Van Laar constants, typically fitted from experimental data

3.6.4.4 Vapor-Liquid Equilibrium (Gamma/Phi Approach)

$$\hat{\phi}_i y_i P = \gamma_i x_i \phi_i^{\text{sat}} P_i^{\text{sat}} \phi_i$$

The distribution coefficient for component i in a non-ideal mixture is

$$K_i = \frac{y_i}{x_i} = \frac{\gamma_i \phi_i^{\text{sat}} P_i^{\text{sat}}}{\hat{\phi}_i P}$$

Special cases:

Ideal vapor phase, ideal liquid solution, and low pressure:

Assume $\hat{\phi}_i = 1$, $\gamma_i = 1$, and $\phi_i = 1$, (Raoult's Law)

then $y_i P = x_i P_i^{\text{sat}}$ and $K_i = \frac{P_i^{\text{sat}}}{P}$

Ideal vapor phase, nonideal liquid solution, and low pressure:

Assume $\hat{\phi}_i = 1$ and $\phi_i = 1$, (Henry's Law with Henry's Law Constant $H_i = \gamma_i^\infty P_i^{\text{sat}}$)

then $y_i P = \gamma_i x_i P_i^{\text{sat}}$ and $K_i = \frac{\gamma_i P_i^{\text{sat}}}{P}$

Nonideal vapor phase, nonideal liquid solution, and low to moderate pressure:

Assume $\phi_i = 1$,

then $\hat{\phi}_i y_i P = \gamma_i x_i \phi_i^{\text{sat}} P_i^{\text{sat}}$ and $K_i = \frac{\gamma_i \phi_i^{\text{sat}} P_i^{\text{sat}}}{\hat{\phi}_i P}$

3.6.5 Phase Behavior

3.6.5.1 Lever Rule

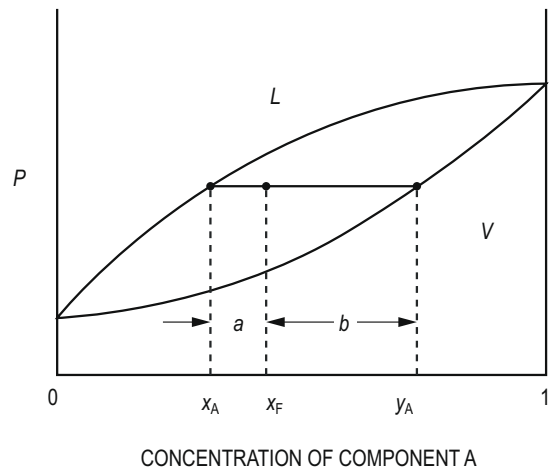
Binary Systems

For a vapor-liquid mixture of A and B, the relative amounts of the liquid and vapor phases in a mixture with an overall composition of x_F are given by the following equations:

$$\frac{m_L}{m_L + m_V} = (1 - \chi) = \frac{b}{a + b} = \frac{y_A - x_F}{y_A - x_A}$$

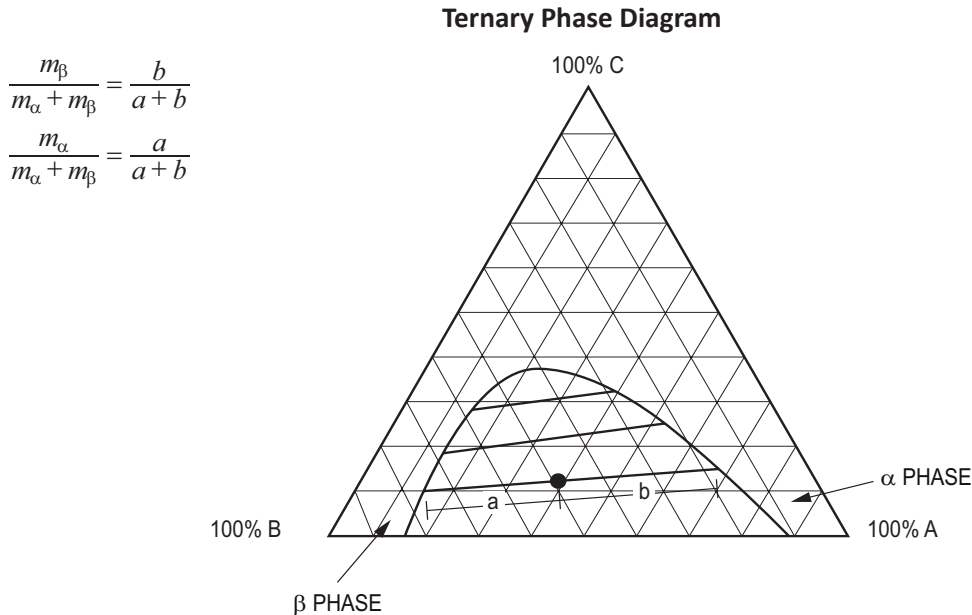
$$\frac{m_V}{m_L + m_V} = \chi = \frac{a}{a + b} = \frac{x_F - x_A}{y_A - x_A}$$

where χ is the vapor quality (i.e., the mass fraction of the vapor in the 2-phase mixture)



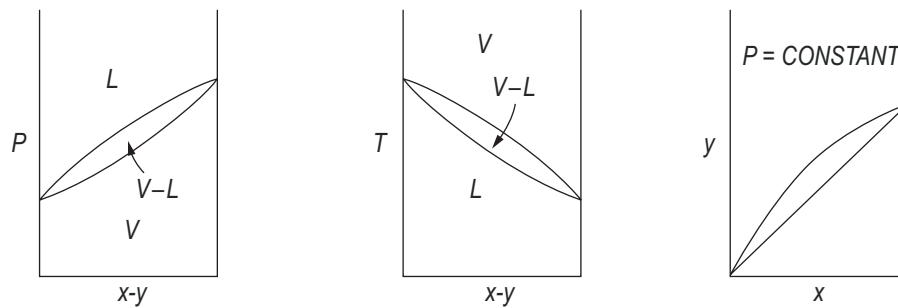
Ternary, Two-Phase System

In the following ternary phase diagram, two phases contain partially miscible components A, B, and C. One phase is rich in component B and one is lean in component B. The fraction of the B-lean phase is $\frac{a}{a+b}$, where a and b represent the length of the tie line on each side of the overall composition, denoted by the heavy black dot.



3.6.5.2 Vapor-Liquid Equilibrium in Binary, Fully Miscible System

Typical Vapor-Liquid Equilibrium Diagrams for Binary, Fully Miscible Systems



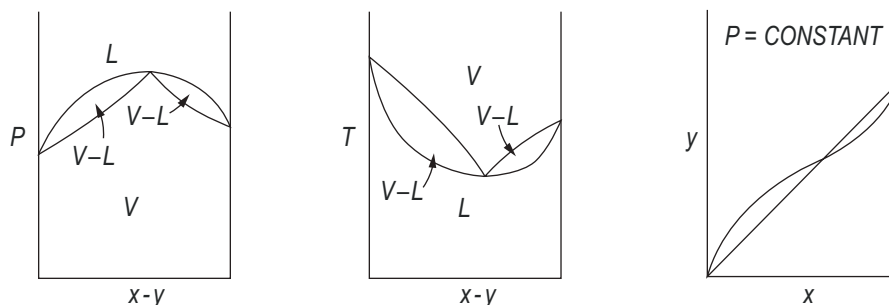
3.6.5.3 Fully Miscible, Binary System with Azeotropes

An *azeotrope* is a mixture that produces a liquid and vapor of equal composition when boiled.

For a positive azeotrope (minimum-boiling azeotrope):

- A positive deviation from Raoult's Law is exhibited on a P - xy diagram, with the P - x curve lying above that for ideal behavior. This behavior results when liquid-phase intermolecular forces between like molecules are stronger than between unlike molecules.
- The P - x curve and the P - y curve exhibit maxima at a point for which $x = y$.
- A T - xy diagram exhibits a minima at the point for which $x = y$, which represents a boiling point lower than that of any other composition.

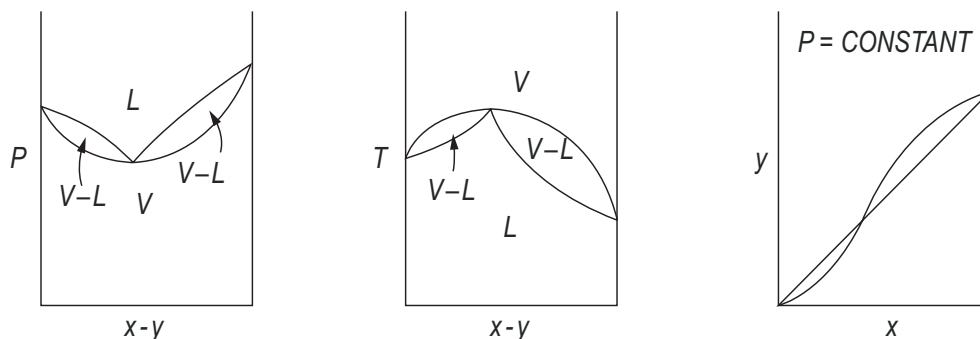
Positive Azeotrope Diagrams



For a negative azeotrope (maximum-boiling azeotrope):

- A negative deviation from Raoult's Law is exhibited on a P - xy diagram, with the P - x curve lying below that for ideal behavior. This behavior results when liquid-phase intermolecular forces between unlike molecules are stronger than between like molecules.
- The P - x curve and the P - y curve exhibit minima at a point for which $x = y$.
- The T - xy diagram exhibits a maxima at the point for which $x = y$, which represents a boiling point higher than that of any other composition.

Negative Azeotrope Diagrams



3.6.5.4 Partially Miscible Systems

Liquid-Liquid Equilibrium

Many mixtures of chemical species, when mixed in certain ranges of composition, form two liquid phases of different compositions at thermodynamic equilibrium.

The criterion for the liquid-liquid equilibrium of mixtures is

$$\hat{f}_i^\alpha = \hat{f}_i^\beta$$

where

$\hat{f}_i^\alpha$ = fugacity of component i in the liquid phase designated α

$\hat{f}_i^\beta$ = fugacity of component i in the liquid phase designated β

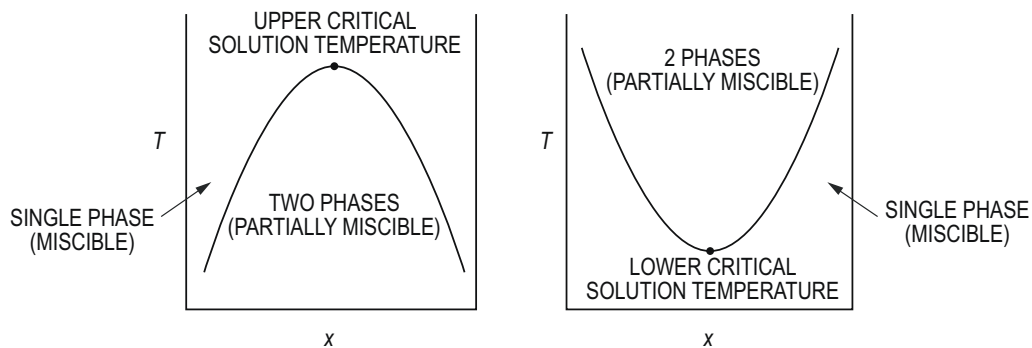
If each species exists as a liquid at the system temperature, then:

$$x_i^\alpha \gamma_i^\alpha = x_i^\beta \gamma_i^\beta$$

A solubility diagram is a T - x diagram at a constant pressure for a binary system. It depicts curves that indicate the compositions of coexisting liquid phases. Such diagrams may show:

- A lower critical solution temperature, above which two liquid phases are possible and below which a single liquid phase exists for all compositions.
- An upper critical solution temperature, below which two liquid phases are possible and above which a single liquid phase exists for all compositions.

Upper and Lower Critical Solution Temperatures



Partially Miscible Ternary Systems

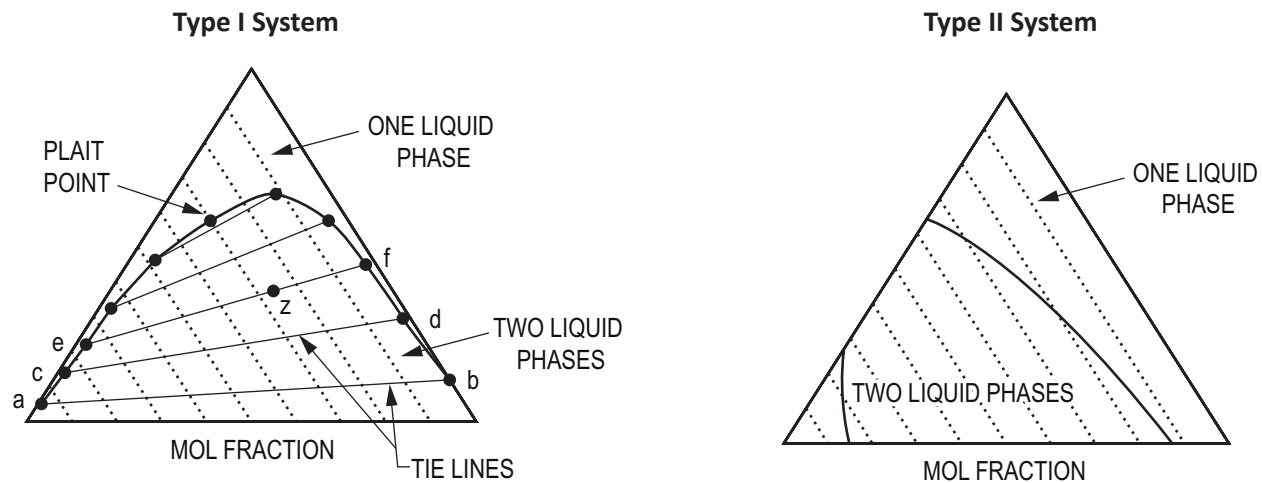
Most of the ternary or pseudoternary systems used in extraction are of two types:

Type I System: One binary pair has limited miscibility.

Type II System: Two binary pairs have limited miscibility.

The compositions of the two phases are equal at the plait point.

Examples of Type I and II systems are shown below.



Vapor-Liquid-Liquid Equilibrium

The gamma-phi approach to vapor-liquid equilibrium applies to each liquid phase. Assuming that $\phi = 1$ and $\varphi = 1$:

$$y_i^* P = \gamma_i^\alpha x_i^\alpha P_i^{\text{sat}} \quad \text{and} \quad y_i^* P = \gamma_i^\beta x_i^\beta P_i^{\text{sat}}$$

For a binary system,

$$P = y_1^* P + y_2^* P = \gamma_1^\beta x_1^\beta P_1^{\text{sat}} + \gamma_2^\alpha x_2^\alpha P_2^{\text{sat}} \quad \text{and} \quad y_i^* = \frac{\gamma_i^\beta x_i^\beta P_i^{\text{sat}}}{P}$$

where

P = total pressure

y_i^* = three-phase equilibrium concentration of component i in the vapor phase

x_1^β = concentration of Component 1 in the liquid β -phase

x_2^α = concentration of Component 2 in the liquid α -phase

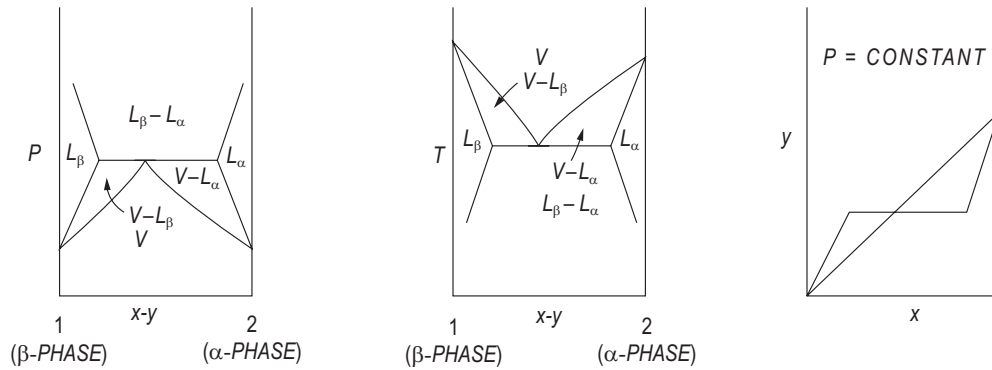
γ_1^β = activity coefficient of Component 1 in the liquid β -phase

γ_2^α = activity coefficient of Component 2 in the liquid α -phase

α = liquid phase rich in Component 2

β = liquid phase rich in Component 1

Vapor-Liquid-Liquid Equilibrium Diagrams



3.6.5.5 Immiscible Phases

In an immiscible system, x_1^β , γ_1^β , x_2^α , and γ_2^α all are equal to 1. As a result:

$$P = P_1^{\text{sat}} + P_2^{\text{sat}} \quad \text{and} \quad y_1^* = \frac{P_1^{\text{sat}}}{P_1^{\text{sat}} + P_2^{\text{sat}}}$$

For the range in which the vapor is in equilibrium with pure-liquid Component 1:

$$y_1 = \frac{P_1^{\text{sat}}}{P}$$

And similarly, for the range in which the vapor is in equilibrium with pure-liquid Component 2:

$$y_2 = \frac{P_2^{\text{sat}}}{P}$$

3.6.6 Phase Equilibrium Applications

3.6.6.1 Bubble Point

The bubble point is defined as the temperature/pressure combination in which the first bubble of vapor is formed in a liquid. It may be determined by iterative calculations from one of the following three relationships, given the liquid composition and either pressure or temperature:

$$\sum_{i=1}^n y_i = \sum_{i=1}^n K_i x_i = 1$$

For ideal mixtures: $\sum_{i=1}^n y_i = \sum_{i=1}^n \frac{x_i P_{i,\text{sat}}}{P} = 1$ or $P = \sum_{i=1}^n x_i P_{i,\text{sat}}$

If $\sum_{i=1}^n K_i x_i > 1$, decrease temperature or increase total pressure.

If $\sum_{i=1}^n K_i x_i < 1$, increase temperature or decrease total pressure.

3.6.6.2 Dew Point

The dew point is defined as the point where the vapor reaches saturation and the liquid phase begins to form. It may be determined by iterative calculations from one of the following relationships, given the vapor composition and either pressure or temperature:

$$\sum_{i=1}^n x_i = \sum_{i=1}^n \frac{y_i}{K_i} = 1$$

For ideal mixtures: $\sum_{i=1}^n x_i = \sum_{i=1}^n \frac{y_i P}{P_{i,\text{sat}}} = 1$ or $P = \frac{1}{\sum_{i=1}^n \left(\frac{y_i}{P_{i,\text{sat}}} \right)}$

If $\sum_{i=1}^n \frac{y_i}{K_i} > 1$, increase temperature or decrease total pressure.

If $\sum_{i=1}^n \frac{y_i}{K_i} < 1$, decrease temperature or increase total pressure.

3.6.6.3 Flash

A single-stage flash determines the distribution of components between the liquid and vapor phase. It may be determined by iterative calculations from one of the following relationships, given the feed composition, the relative proportions of vapor and liquid resulting from the flash, and either pressure or temperature:

$$\sum_{i=1}^n x_i = \sum_{i=1}^n \frac{z_i}{1+f(K_i-1)} = 1 \quad \text{or} \quad \sum_{i=1}^n y_i = \sum_{i=1}^n \frac{z_i K_i}{1+f(K_i-1)} = 1$$

where

z_i = mole fraction of component in the feed

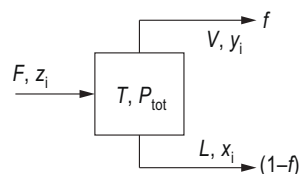
f = ratio of vapor-phase flow to the feed flow

q = ratio of liquid-phase flow to the feed flow

The lever rule may be applied to binary single-stage flash calculations as follows:

$$\frac{V}{F} = f = 1 - q = \frac{z_i - x_i}{y_i - x_i}$$

$$\frac{L}{F} = 1 - f = q = \frac{y_i - z_i}{y_i - x_i}$$



Equations for the operating line are:

$$y_i = -\left(\frac{L}{V}\right)x_i + \left(\frac{F}{V}\right)z_i = -\left(\frac{1-f}{f}\right)x_i + \frac{z_i}{f} = -\left(\frac{q}{1-q}\right)x_i + \left(\frac{1}{1-q}\right)z_i$$

where

F = total feed flow rate

V = vapor flow rate

L = liquid flow rate

3.6.6.4 Solubility of Solids

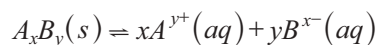
The heat of solution for a solid in water can be calculated from the following equation:

$$\Delta H_{\text{solution}}^{\circ} = \Delta H_f^{\circ}(aq) - \Delta H_f^{\circ}(s)$$

given that $\Delta H_f^{\circ}(aq)$ is known at the stated concentration.

For salt compounds, the solubility product constant describes the extent to which the salt will dissolve.

For the following general chemical reaction:



The solubility product (K_{sp}) at equilibrium is:

$$K_{sp} = [A^{y+}]^x [B^{x-}]^y$$

3.7 Tables

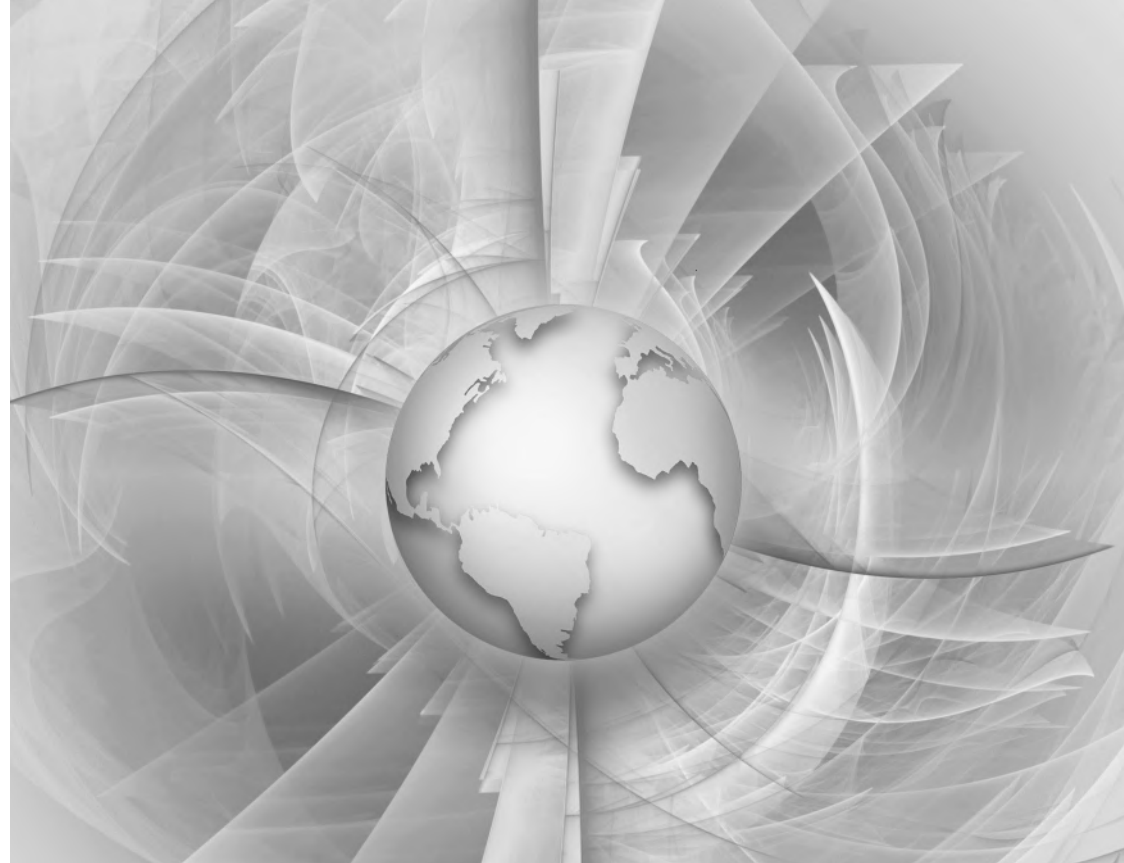
Standard Gibbs Energies of Formation at 298.15 K (25°C)

Chemical species		State (Note 2)	ΔG_f° J/gmol
Paraffins:			
Methane	CH ₄	<i>g</i>	-50,460
Ethane	C ₂ H ₆	<i>g</i>	-31,855
Propane	C ₃ H ₈	<i>g</i>	-24,290
<i>n</i> -Butane	C ₄ H ₁₀	<i>g</i>	-16,570
<i>n</i> -Pentane	C ₅ H ₁₂	<i>g</i>	-8,650
<i>n</i> -Hexane	C ₆ H ₁₄	<i>g</i>	150
<i>n</i> -Heptane	C ₇ H ₁₆	<i>g</i>	8,260
<i>n</i> -Octane	C ₈ H ₁₈	<i>g</i>	16,260
1-Alkenes:			
Ethylene	C ₂ H ₄	<i>g</i>	68,460
Propylene	C ₃ H ₆	<i>g</i>	62,205
1-Butene	C ₄ H ₈	<i>g</i>	70,340
1-Pentene	C ₅ H ₁₀	<i>g</i>	78,410
1-Hexene	C ₆ H ₁₂	<i>g</i>	86,830
Miscellaneous organics:			
Acetaldehyde	C ₂ H ₄ O	<i>g</i>	-128,860
Acetic acid	C ₂ H ₄ O ₂	<i>l</i>	-389,900
Acetylene	C ₂ H ₂	<i>g</i>	209,970
Benzene	C ₆ H ₆	<i>g</i>	129,665
Benzene	C ₆ H ₆	<i>l</i>	124,520
1,3-Butadiene	C ₄ H ₆	<i>g</i>	149,795
Cyclohexane	C ₆ H ₁₂	<i>g</i>	31,920
Cyclohexane	C ₆ H ₁₂	<i>l</i>	26,850
1,2-Ethanediol	C ₂ H ₆ O ₂		-323,080
Ethanol	C ₂ H ₆ O	<i>g</i>	-168,490
Ethanol	C ₂ H ₆ O	<i>l</i>	-174,780
Ethylbenzene	C ₈ H ₁₀	<i>g</i>	130,890
Ethylene oxide	C ₂ H ₄ O	<i>g</i>	-13,010
Formaldehyde	CH ₂ O	<i>g</i>	-102,530
Methanol	CH ₄ O	<i>g</i>	-161,960
Methanol	CH ₄ O	<i>l</i>	-166,270
Methylcyclohexane	C ₇ H ₁₄	<i>g</i>	27,480
Methylcyclohexane	C ₇ H ₁₄	<i>l</i>	20,560
Styrene	C ₈ H ₈	<i>g</i>	213,900
Toluene	C ₇ H ₈	<i>g</i>	122,050
Toluene	C ₇ H ₈	<i>l</i>	113,630

Notes:

1. The standard Gibbs energy of formation ΔG_f° is the change in the Gibbs energy when 1 mol of the listed compound is formed from its elements with each substance in its standard state at 298.15 K (25°C).
2. Standard states: (a) Gases (*g*): the pure ideal gas at 1 bar and 25°C. (c) Solutes in aqueous solution (*aq*): The hypothetical ideal 1 molal solution of the solute in water at 1 bar and 25°C.
3. Joules per mole of the substance formed

Source: TRC Thermodynamic Tables, Hydrocarbons, Thermodynamics Research Center, Texas Engineering Experiment Station, Texas A & M University System, 1985; and Gaur and Wunderlich, Tables A1-A6, "Heat Capacity and Other Thermodynamic Properties of Linear Macromolecules V. Polystyrene," *Journal of Physical and Chemical Reference Data*, vol. 11, no. 2: 1982 (all volumes available online at <https://aip.scitation.org/journal/jpr>).



4 HEAT TRANSFER

4.1 Symbols and Definitions

Symbols

Symbol	Description	Units (U.S.)	Units (SI)
A	Area	ft ²	m ²
C	Heat-capacity rate	$\frac{\text{Btu}}{\text{hr} \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{K}} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3 \cdot \text{K}}$
c_p	Heat capacity at constant pressure	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}} = \frac{\text{m}^2}{\text{s}^2 \cdot \text{K}}$
D	Diameter	ft <i>or</i> in.	m
d	Wall thickness, width	ft <i>or</i> in.	m
F	Correction factor for heat-exchanger configuration	dimensionless	
F_{ij}	Shape factor (radiation)	dimensionless	
f	Friction factor (Darcy-Weisbach)	dimensionless	
g	Acceleration of gravity	$\frac{\text{ft}}{\text{sec}^2}$	$\frac{\text{m}}{\text{s}^2}$
h	Convection heat-transfer coefficient	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{m}^2 \cdot \text{K}} = \frac{\text{kg}}{\text{s}^3 \cdot \text{K}}$
Δh_{fusion}	Latent heat of fusion	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}} = \frac{\text{m}^2}{\text{s}^2}$
Δh_{subl}	Latent heat of sublimation	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}} = \frac{\text{m}^2}{\text{s}^2}$
Δh_{vap}	Latent heat of vaporization	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}} = \frac{\text{m}^2}{\text{s}^2}$

Symbols (cont'd)

Symbol	Description	Units (U.S.)	Units (SI)
ΔH	Change in enthalpy	Btu	$J = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$
j_H	Colburn factor for heat transfer	dimensionless	
k	Thermal conductivity	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{m} \cdot \text{K}} = \frac{\text{kg} \cdot \text{m}}{\text{s}^3 \cdot \text{K}}$
L	Length	ft or in.	m
m	Mass	lbm	kg
$\dot{m}$	Mass flow rate	$\frac{\text{lbm}}{\text{hr}}$	$\frac{\text{kg}}{\text{s}}$
NTU	Number of thermal transfer units	dimensionless	
n	Number of tubes (in shell-and-tube heat exchangers)	dimensionless	
P	Pressure	$\text{psi} = \frac{\text{lbf}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
P_c	Critical Pressure	$\text{psi} = \frac{\text{lbf}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
P	Thermal efficiency	dimensionless	
Q	Heat	Btu	$J = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$
$\dot{Q}$	Rate of heat transfer	$\frac{\text{Btu}}{\text{hr}}$	$\text{W} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3}$
$\dot{q}$	Heat flux (rate of heat transfer per area)	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2}$	$\frac{\text{W}}{\text{m}^2} = \frac{\text{kg}}{\text{s}^3}$
$\dot{q}'$	Rate of heat transfer per unit length	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}}$	$\frac{\text{W}}{\text{m}} = \frac{\text{kg} \cdot \text{m}}{\text{s}^3}$
$\dot{q}_{\text{gen}}$	Rate of heat generation per volume	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^3}$	$\frac{\text{W}}{\text{m}^3} = \frac{\text{kg}}{\text{m} \cdot \text{s}^3}$
R	Heat-transfer resistance	$\frac{\text{hr} \cdot ^\circ\text{F}}{\text{Btu}}$	$\frac{\text{K}}{\text{W}} = \frac{\text{s}^3 \cdot \text{K}}{\text{kg} \cdot \text{m}^2}$
R_{NTU}	Heat-capacity rate ratio $\left(\frac{C_{\text{tube}}}{C_{\text{shell}}}\right)$ or $\left(\frac{C_{\text{min}}}{C_{\text{max}}}\right)$	dimensionless	
R_f	Fouling factor	$\frac{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}{\text{Btu}}$	$\frac{\text{m}^2 \cdot \text{K}}{\text{W}} = \frac{\text{s}^3 \cdot \text{K}}{\text{kg}}$
r	Radius	ft or in.	m
T	Temperature	$^\circ\text{F}$ or $^\circ\text{R}$	$^\circ\text{C}$ or K
ΔT_{lm}	Log-mean temperature difference	$^\circ\text{F}$ or $^\circ\text{R}$	K

Symbols (cont'd)

Symbol	Description	Units (U.S.)	Units (SI)
T_r	Reduced temperature	dimensionless	
T_n	Normal boiling point	°F <i>or</i> °R	°C <i>or</i> K
t	Time	hr	s
U_{ov}	Overall heat-transfer coefficient	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{m}^2 \cdot \text{K}} = \frac{\text{kg}}{\text{s}^3 \cdot \text{K}}$
u	Velocity	$\frac{\text{ft}}{\text{sec}}$	$\frac{\text{m}}{\text{s}}$
V	Volume	ft ³	m ³
x	Distance	ft <i>or</i> in.	m
α	Adsorptivity (radiation)	dimensionless	
α	Thermal diffusivity	$\frac{\text{ft}^2}{\text{hr}}$	$\frac{\text{m}^2}{\text{s}}$
β	Coefficient of thermal expansion	$\frac{1}{^\circ\text{R}}$	$\frac{1}{\text{K}}$
γ	Surface tension	$\frac{\text{lbf}}{\text{in.}}$	$\frac{\text{N}}{\text{m}} = \frac{\text{kg}}{\text{s}^2}$
δ	Thickness	ft <i>or</i> in.	m
ε	Emissivity of a body (radiation)	dimensionless	
ε	Heat exchanger effectiveness	dimensionless	
ϵ	Void fraction (packed bed)	dimensionless	
θ, ϕ	Angle	radians <i>or</i> degrees	
μ	Dynamic viscosity	cP <i>or</i> $\frac{\text{lbm}}{\text{ft} \cdot \text{sec}}$	$\text{Pa} \cdot \text{s} = \frac{\text{kg}}{\text{m} \cdot \text{s}}$
ν	Kinematic viscosity	$\frac{\text{ft}^2}{\text{hr}}$	$\frac{\text{m}^2}{\text{s}}$
ρ	Density	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{kg}}{\text{m}^3}$
ρ	Reflectivity (radiation)	dimensionless	
σ	Stefan-Boltzmann Constant	$\frac{\text{Btu}}{\text{ft}^2 \cdot \text{hr} \cdot ^\circ\text{R}^4}$	$\frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}$
τ	Time constant	hr	s
τ	Transmissivity (radiation)	dimensionless	

4.2 Fundamentals of Heat Transfer

4.2.1 Heat Transfer Without Phase Change

4.2.1.1 Definition of Heat

$$\dot{Q} = m c_p \frac{dT}{dt}$$

$$c_p = \frac{\Delta H}{m \Delta T}$$

Heat transferred in or out of a flowing material:

$$\dot{Q} = \dot{m} c_p \Delta T$$

4.2.1.2 Physical Properties for Heat Transfer

Thermal conductivity is a measure of the rate at which a substance transfers thermal energy:

$$k = \frac{\dot{q}}{\frac{\Delta T}{\delta}}$$

Thermal diffusivity is a measure of the rate at which a thermal disturbance is transmitted through a substance:

$$\alpha = \frac{k}{\rho c_p}$$

Kinematic viscosity (also called momentum diffusivity) is the ratio of the dynamic viscosity μ to the density of the fluid ρ :

$$\nu = \frac{\mu}{\rho}$$

4.2.1.3 Conduction

The following equations assume that the thermal conductivity is constant.

Fourier's Law of Conduction

Total heat flux (rate of heat transfer): $\dot{Q} = -k A \frac{dT}{dx}$

Heat flux per area (rate of heat transfer per area): $\dot{q} = \frac{\dot{Q}}{A} = -k \frac{dT}{dx}$

Heat flux per unit length (rate of heat transfer per unit length): $\dot{q}' = \frac{\dot{Q}}{L}$

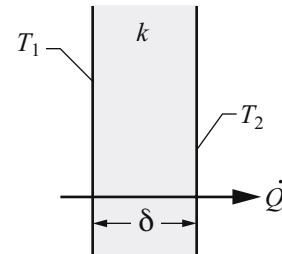
Conduction Through a Plane Wall

$$\dot{Q} = \frac{k A}{\delta} (T_1 - T_2)$$

where

T_1 = temperature of one surface of wall

T_2 = temperature of the other surface of wall



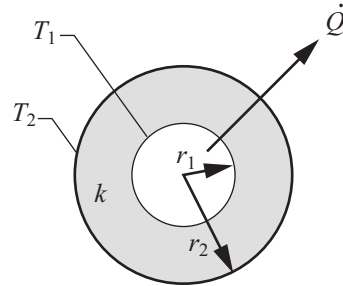
Conduction Through a Composite Wall

$$\dot{Q} = \frac{A(T_1 - T_2)}{\sum_i \frac{\delta_i}{k_i}}$$

Conduction Through a Cylindrical Wall

$$\dot{Q} = \frac{2\pi k L}{\ln\left(\frac{r_2}{r_1}\right)} (T_1 - T_2)$$

where L = cylinder length



Conduction Through a Spherical Wall

$$\dot{Q} = \frac{4\pi k r_1 r_2}{r_2 - r_1} (T_1 - T_2)$$

Conduction Through a Cube With Thick Walls (Approximation)

$$\dot{Q} \approx 0.725 \sqrt{A_{\text{outer}} A_{\text{inner}}} \left(\frac{T_{\text{inner}} - T_{\text{outer}}}{\delta} \right)$$

where $\frac{A_{\text{outer}}}{A_{\text{inner}}} \geq 2$

Steady Conduction With Internal Energy Generation

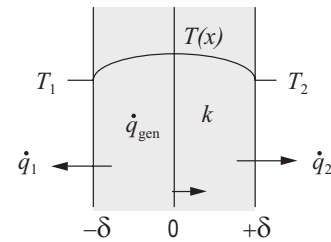
For a plane wall:

$$\frac{d^2 T}{dx^2} + \frac{\dot{q}_{\text{gen}}}{k} = 0$$

$$T(x) = \frac{\dot{q}_{\text{gen}} \delta^2}{2k} \left(1 - \frac{x^2}{\delta^2} \right) + \left(\frac{T_2 - T_1}{2} \right) \left(\frac{x}{\delta} \right) + \left(\frac{T_1 + T_2}{2} \right)$$

$$\dot{q}_1 + \dot{q}_2 = 2 \dot{q}_{\text{gen}} \delta$$

$$\dot{q}_1 = k \left(\frac{dT}{dx} \right)_{-\delta} \text{ and } \dot{q}_2 = k \left(\frac{dT}{dx} \right)_{+\delta}$$



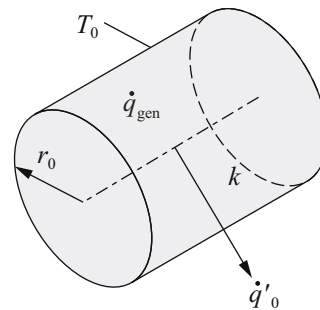
For a long circular cylinder:

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + \frac{\dot{q}_{\text{gen}}}{k} = 0$$

$$T(r) = \frac{\dot{q}_{\text{gen}} r_0^2}{4k} \left(1 - \frac{r^2}{r_0^2} \right) + T_0$$

$$\dot{q}'_0 = \pi r_0^2 \dot{q}_{\text{gen}}$$

where $\dot{q}'_0$ = heat transfer rate per unit length in $\frac{\text{Btu}}{\text{hr-ft}}$ or $\frac{\text{W}}{\text{m}}$



Transient Conduction Using the Lumped Capacitance Model

The lumped capacitance model is valid if

$$Bi = \frac{hV}{kA_s} \ll 1$$

where

Bi = Biot number

V = volume of body

A_s = surface area of body

For constant fluid temperature T_∞ and uniform body temperature T :

Heat-transfer rate at the body surface is

$$\dot{Q} = hA_s(T - T_\infty) = -\rho V c_p \frac{dT}{dt}$$

Temperature variation of the body with time is

$$T - T_\infty = (T_i - T_\infty) \exp\left(-\frac{t}{\tau}\right)$$

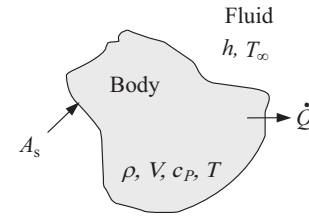
where

$$\tau = \frac{\rho V c_p}{hA_s} = \text{time constant}$$

T_i = initial temperature of the body

Total heat transferred from the body at time t is

$$Q_{\text{total}} = \rho V c_p (T_i - T) = \rho V c_p (T_i - T_\infty) \left(1 - \exp\left(-\frac{t}{\tau}\right)\right)$$



4.2.1.4 Convection

Newton's Law of Cooling

$$\dot{Q} = hA(T_w - T_\infty)$$

where

T_w = wall surface temperature

T_∞ = bulk fluid temperature

4.2.1.5 Heat Transfer Coefficients/Correlations for Forced Convection

Forced Convection: External Flow

Forced Convection—External Flow

Geometry	Correlation	Conditions		
Flat plate in parallel flow (gas or liquid) $Re_L = \frac{\rho u_\infty L}{\mu}$	$\overline{Nu}_L = \frac{\bar{h} L}{k} = 0.664 Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}}$	$Re_L < 10^5$		
	$\overline{Nu}_L = \frac{\bar{h} L}{k} = 0.0366 Re_L^{0.8} Pr^{\frac{1}{3}}$	$Re_L > 10^5$		
Long cylinder in cross flow (gas or liquid) $Re_D = \frac{\rho u_\infty D}{\mu}$	$\overline{Nu}_D = \frac{\bar{h} D}{k} = C Re_D^n Pr^{\frac{1}{3}}$	Re_D	C	n
		1–4	0.989	0.330
		4–40	0.911	0.385
		40–4000	0.683	0.466
		4000–40,000	0.193	0.618
		40,000–250,000	0.0266	0.805
Short cylinder (gas only) $Re_D = \frac{\rho u_\infty D}{\mu}$	$\overline{Nu}_D = \frac{\bar{h} D}{k} = 0.123 Re_D^{0.651} + \left(\frac{D}{L}\right)^{0.85} Re_D^{0.5}$	$70,000 < Re_D < 110,000$ $L/D < 4$		
Sphere in flow (gas or liquid) $Re_D = \frac{\rho u_\infty D}{\mu}$	$\overline{Nu}_D = \frac{\bar{h} D}{k} = 2.0 + 0.60 Re_D^{\frac{1}{2}} Pr^{\frac{1}{3}}$	$1 < Re_D < 70,000$ $0.6 < Pr < 400$		
Long, flat plate (width L), perpendicular to flow in gas $Re_L = \frac{\rho u_\infty L}{\mu}$	$\overline{Nu}_D = \frac{\bar{h} D}{k} = 0.20 Re_D^{\frac{2}{3}}$	$1 < Re_D < 400,000$		
Packed bed with gas flow –heat transfer to or from the packing $D_p = \frac{6V_p}{A_p}$ $Re_{D_p} = \frac{\rho u_s D_p}{\mu(1-\epsilon)}$ where ϵ = void fraction u_s = superficial velocity D_p = equivalent packing diameter V_p = particle volume A_p = particle surface area	$\overline{Nu}_{D_p} = \frac{\bar{h} D_p}{k} = \frac{1-\epsilon}{\epsilon} \left(0.5 Re_{D_p}^{\frac{1}{2}} + 0.2 Re_{D_p}^{\frac{2}{3}} \right) Pr^{\frac{1}{3}}$	$20 < Re_{D_p} < 10,000$ $0.34 < \epsilon < 0.78$		
	$\overline{Nu}_{D_p} = \frac{\bar{h} D_p}{k} = \frac{1.075}{\epsilon} Re_{D_p}^{0.174} Pr^{\frac{1}{3}}$	$0.01 < Re_{D_p} < 10$		

Forced Convection—External Flow (cont'd)

Geometry	Correlation		Conditions			
Packed bed with gas flow – heat transfer to or from the containment wall	$\overline{Nu}_{D_p} = \frac{\bar{h} D_p}{k} = C_1 Re_{D_p}^{\frac{1}{3}} Pr^{\frac{1}{3}} + C_2 Re_{D_p}^{0.8} Pr^{0.4}$		40 < Re _{D_p} < 2000			
			Packing Shape	C ₁	C ₂	
			Cylinder-like	2.58	0.094	
			Sphere-like	0.203	0.220	
Tube bundle in cross flow	$\overline{Nu}_D Pr^{-0.36} \left(\frac{Pr}{Pr_s} \right)^{-0.25} = C \left(\frac{S_T}{S_L} \right)^n Re_D^m$	Config.	Reynolds Range	C	m	n
		Inline	10–100	0.8	0.4	0
		Staggered	10–100	0.9	0.4	0
		Inline	1000–200,000 $\frac{S_T}{S_L} \geq 0.7$	0.27	0.63	0
		Staggered	1000–200,000 $\frac{S_T}{S_L} < 2$	0.35	0.60	0.2
		Staggered	1000–200,000 $\frac{S_T}{S_L} \geq 2$	0.40	0.60	0
		Inline	>200,000	0.021	0.84	0
		Staggered	>200,000 Pr > 1	0.022	0.84	0
	$\overline{Nu}_D = \frac{\bar{h} D}{k} = 0.019 Re_D^{0.84}$	Staggered	>200,000 Pr = 0.7	---	---	---

where

u_∞ = free stream velocity of the fluid

u_s = superficial velocity (velocity through the bed if it were empty)

$\overline{Nu}$ = average Nusselt number

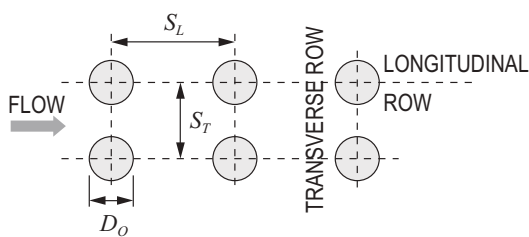
$\bar{h}$ = average heat-transfer coefficient

Pr_s = Prandtl number based on properties at tube surface

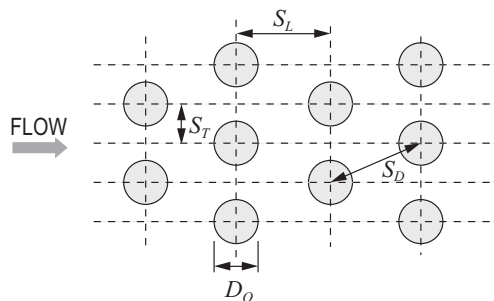
In all cases, evaluate fluid properties at average temperature between that of the body and that of the flowing fluid.

For tube bundles in cross flow, the following applies:

Inline (square pitch):



Staggered (triangular pitch):



Forced Convection: Internal Flow

Forced Convection—Internal Flow

Geometry	Correlation	Conditions
Laminar flow in circular tubes, $Re < 2300$	$Nu_D = 4.36$	Uniform heat flux, fully developed
	$Nu_D = 3.66$	Constant surface temperature, fully developed
	$\overline{Nu}_D = 1.86 \left(\frac{Re_D \cdot Pr}{L/D} \right)^{\frac{1}{3}} \left(\frac{\mu_\infty}{\mu_s} \right)^{0.14}$	Constant surface temperature, intermediate tube length with entry effects
	$\overline{Nu}_D = 3.66 + \left(\frac{0.0668 \frac{L}{D} Re_D Pr}{1 + 0.04 \frac{L}{D} Re_D Pr} \right) \left(\frac{\mu_\infty}{\mu_s} \right)^{0.14}$	Constant surface temperature, short tube length with entry effects: $100 < \left(Re_D Pr \frac{D}{L} \right) < 1500$; $Pr > 0.7$
Turbulent flow in circular tubes	$\overline{Nu}_D = 0.023 Re_D^{0.8} Pr^{\frac{1}{3}} \left(\frac{\mu_\infty}{\mu_s} \right)^{0.14}$	Uniform surface temperature or uniform heat flux: $Re > 10,000$; $Pr > 0.7$
Liquid metals	$\overline{Nu}_D = 6.3 + 0.0167 Re_D^{0.85} Pr^{0.93}$	Uniform heat flux: $0.003 < Pr < 0.05$
	$\overline{Nu}_D = 7.0 + 0.025 Re_D^{0.8} Pr^{0.8}$	Constant surface temperature: $0.003 < Pr < 0.05$

$$Re_D = \frac{\rho u_m D}{\mu} \quad \text{and} \quad \overline{Nu}_D = \frac{\bar{h} D}{k}$$

where

- u_m = mean velocity of the fluid
- μ_∞ = viscosity of the fluid at bulk fluid temperature
- μ_s = viscosity of the fluid at tube inside surface temperature

For noncircular ducts, use the equivalent hydraulic diameter: $D_H = \frac{4 \times \text{cross-sectional area}}{\text{wetted perimeter}}$

For a circular annulus, use the equivalent hydraulic diameter: $D_H = D_{\text{outer}} - D_{\text{inner}}$

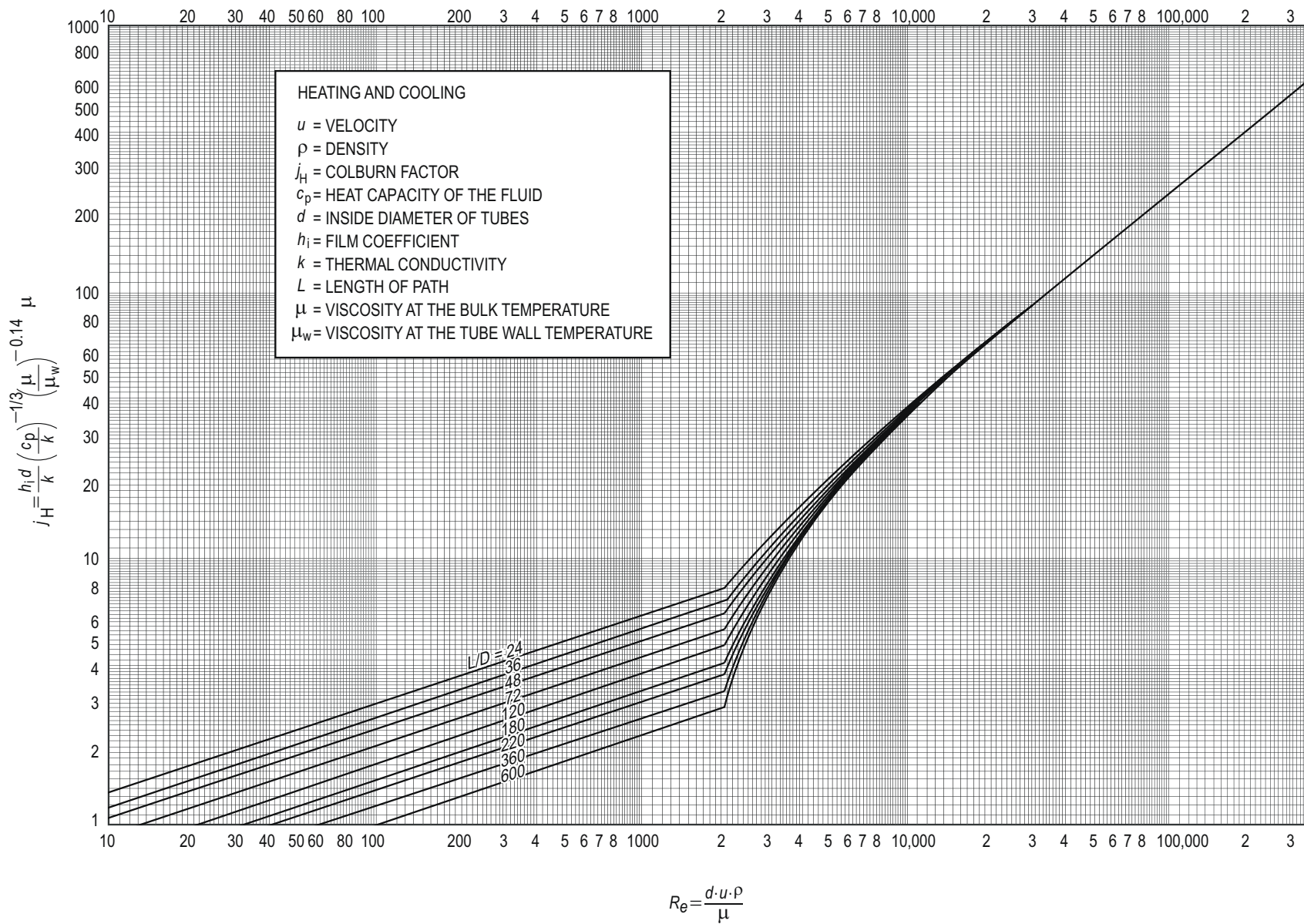
Use the friction factor f from the Moody diagram to predict heat-transfer coefficients for turbulent flow:

$$\left(\frac{Nu}{Re Pr} \right) Pr^{\frac{2}{3}} = \frac{f}{8}$$

For flow in coiled tubes with $Re > 10^4$, the forced-convection heat-transfer coefficient for the inside of the coiled pipe is:

$$h_{\text{coil}} = h_{\text{straight}} \left(1 + 3.5 \frac{D_{\text{tube}}}{D_{\text{coil}}} \right)$$

Tube-Side Heat-Transfer Curve (Adapted from Sieder and Tate)

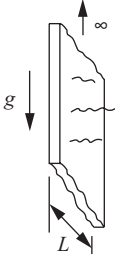
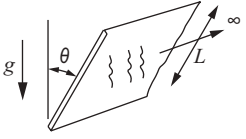
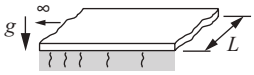
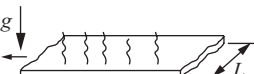
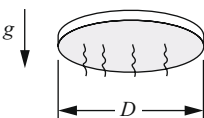
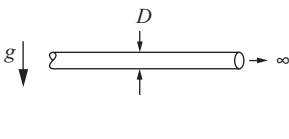
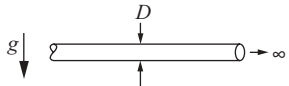


Source: Kern, Donald Q., *Process Heat Transfer*, 1990, p. 834.

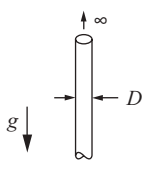
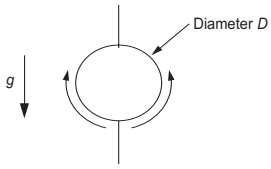
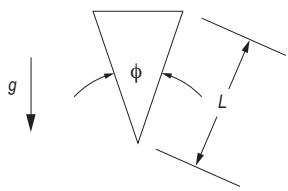
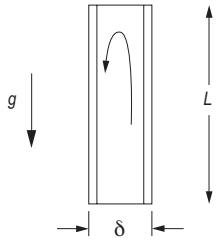
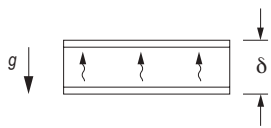
4.2.1.6 Heat Transfer Coefficients/Correlations for Free Convection

Natural (Free) Convection

Natural (Free) Convection

Geometry	Sketch	Correlation	Conditions		
Vertical plate		$\overline{Nu}_L = 1.36 (Gr_L Pr)^{\frac{1}{5}}$	$Gr_L Pr < 10^4$		
		$\overline{Nu}_L = 0.59 (Gr_L Pr)^{\frac{1}{4}}$	$10^4 < Gr_L Pr < 10^9$		
		$\overline{Nu}_L = 0.10 (Gr_L Pr)^{\frac{1}{3}}$	$10^9 < Gr_L Pr < 10^{13}$		
Long, tilted plate with heated surface facing downward		$\overline{Nu}_L = 0.56 (Gr_L Pr \cos \theta)^{\frac{1}{4}}$	$10^5 < Gr_L Pr \cos \theta < 10^{11}$ $0 \leq \theta \leq 89^\circ$		
Long, horizontal plate with heated surface facing downward		$\overline{Nu}_L = 0.58 (Gr_L Pr)^{\frac{1}{5}}$	$10^6 < Gr_L Pr < 10^{11}$		
Long, horizontal plate with heated surface facing upward		$\overline{Nu}_L = 0.16 (Gr_L Pr)^{\frac{1}{3}}$	$7 \times 10^6 < Gr_L Pr < 2 \times 10^8$		
		$\overline{Nu}_L = 0.13 (Gr_L Pr)^{\frac{1}{3}}$	$5 \times 10^8 < Gr_L Pr$		
Horizontal circular plate with heated surface facing downward		$\overline{Nu}_D = 0.82 (Gr_D Pr)^{\frac{1}{5}} Pr^{0.034}$			
Single, long horizontal cylinder		$\overline{Nu}_D = C (Gr_D Pr)^n$	Pr > 0.5		
			Gr_D • Pr	C	n
			10 ⁻³ – 10 ²	1.02	0.149
			10 ² – 10 ⁴	0.850	0.188
			10 ⁴ – 10 ⁷	0.480	0.250
		10 ⁷ – 10 ¹²	0.125	0.333	
		$\overline{Nu}_D = 0.53 (Gr_D Pr^2)^{\frac{1}{4}}$	Liquid metals, laminar flow		
Thin horizontal wire		$\overline{Nu}_D = C (Gr_D Pr)^n$	Gr_D • Pr	C	n
			< 10 ⁻⁵	0.49	0
			10 ⁻⁵ – 10 ⁻³	0.71	0.04
			10 ⁻³ – 1	1.09	0.10
			1 – 10 ⁴	1.09	0.20

Natural (Free) Convection (cont'd)

Geometry	Sketch	Correlation	Conditions
Vertical cylinder		$\overline{Nu}_D = 0.59 (Gr_D Pr)^{\frac{1}{4}}$	$10^4 < Gr_D Pr < 10^9$
		$\overline{Nu}_D = 0.10 (Gr_D Pr)^{\frac{1}{3}}$	$10^9 < Gr_D Pr < 10^{13}$
Sphere		$\overline{Nu}_D = 2 + 0.392 (Gr_D)^{\frac{1}{4}}$	$1 < Gr_D < 10^5$
Vertical cone		$\overline{Nu}_L = 0.63 (1 + 0.73 \epsilon) (Gr_L)^{\frac{1}{4}}$ $\epsilon = \frac{2}{Gr_L^{\frac{1}{4}} \tan \frac{\phi}{2}}$	$3^\circ < \phi < 12^\circ$ $7.5 < \log Gr_L < 8.7$ $0.2 < \epsilon < 0.8$
Vertical enclosed space heated from the side		$\overline{Nu}_\delta = 0.22 \left(\frac{L}{\delta} \right)^{-\frac{1}{4}} \left(\frac{Pr}{0.2 + Pr} Ra_\delta \right)^{0.28}$	$2 < \frac{L}{\delta} < 10$ $Pr < 10$ $Ra_\delta < 10^{10}$
		$\overline{Nu}_\delta = 0.18 \left(\frac{Pr}{0.2 + Pr} Ra_\delta \right)^{0.29}$	$1 < \frac{L}{\delta} < 2$ $10^{-3} Pr < 10^5$ $10^3 < \frac{Ra_\delta Pr}{0.2 + Pr}$
Horizontal enclosed space heated from below		$\overline{Nu}_\delta = 1 + 1.44 \left(1 - \frac{1708}{Ra_\delta} \right) + \left[\left(\frac{Ra_\delta}{5830} \right)^{\frac{1}{3}} - 1 \right]$	Air $1700 < Ra_\delta < 10^8$
		$\overline{Nu}_\delta = 1 + 1.44 \left(1 - \frac{1708}{Ra_\delta} \right) + \left[\left(\frac{Ra_\delta}{5830} \right)^{\frac{1}{3}} - 1 \right] + 2.0 \left(\frac{Ra_\delta^{\frac{1}{3}}}{140} \right)^{1 - \ln \left(\frac{Ra_\delta^{\frac{1}{3}}}{140} \right)}$	Water $1700 < Ra_\delta < 3.5 \cdot 10^9$

For plates and other linear geometry: $Ra_L = Gr_L Pr = \frac{g \beta (T_s - T_\infty) L^3}{\nu^2} \frac{c_p \mu}{k}$

For cylinders and spheres: $Ra_D = Gr_D Pr = \frac{g \beta (T_s - T_\infty) D^3}{\nu^2} \frac{c_p \mu}{k}$

where

T_s = surface temperature

T_∞ = bulk fluid temperature

For an ideal gas: $\frac{1}{\beta} = \frac{1}{2}(T_s + T_\infty)$

where T = absolute temperature, in K or °R

4.2.1.7 Radiation

Stefan-Boltzmann Law of Radiation

$$\dot{Q} = \epsilon \sigma A T^4$$

where T = absolute temperature, in K or °R

Types of Bodies

α = absorptivity (ratio of energy absorbed to incident energy)

ρ = reflectivity (ratio of energy reflected to incident energy)

τ = transmissivity (ratio of energy transmitted to incident energy)

$$\alpha + \rho + \tau = 1$$

Opaque body: $\tau = 0$

Gray body: $\tau = 0$ and $\alpha = \epsilon$ with $0 < \alpha < 1$ and $0 < \epsilon < 1$

Black body: $\tau = 0$ and $\alpha = \epsilon = 1$

Real bodies are frequently approximated as *gray bodies*.

A *black body* is one that absorbs all energy incident upon it. It also emits radiation at the maximum rate for a body of its size and temperature.

Shape Factor F_{ij} (Also Called View Factor or Configuration Factor)

Reciprocity relations:

$$A_i F_{ij} = A_j F_{ji}$$

where

A_i = surface area of surface i

F_{ij} = fraction of the radiation leaving surface i that is intercepted by surface j ; $0 \leq F_{ij} \leq 1$

Summation rule for N surfaces:

$$\sum_{j=1}^N F_{ij} = 1$$

Net energy exchange by radiation between two bodies:

When the body is small in comparison to its surroundings

$$\dot{Q}_{12} = \varepsilon \sigma A (T_1^4 - T_2^4)$$

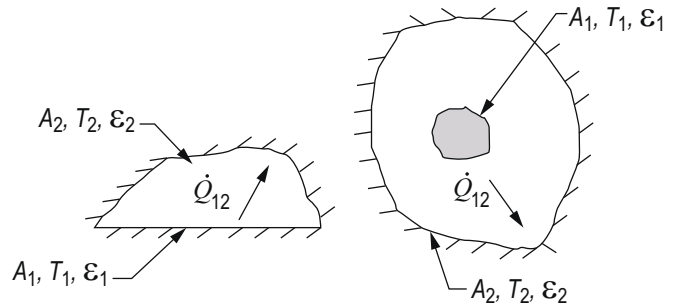
where T = absolute temperature in K or °R

When both bodies are black bodies

$$\dot{Q}_{12} = \sigma A F_{12} (T_1^4 - T_2^4)$$

Net energy exchange by radiation between two gray bodies

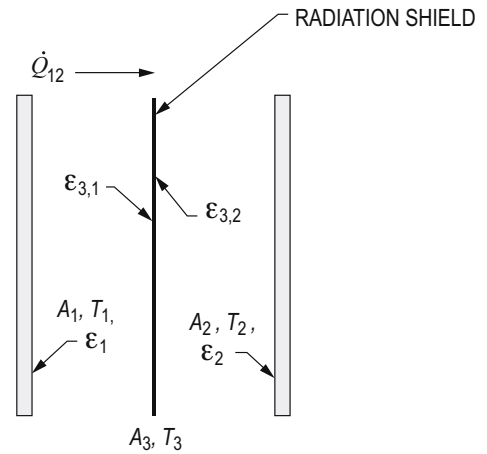
$$\dot{Q}_{12} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1 - \varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1 - \varepsilon_2}{\varepsilon_2 A_2}}$$



For radiative heat loss at night, neglect any return radiation from the clear night sky, i.e., set T_2 to 0 K or 0°R.

One-dimensional geometry with a thin, low-emissivity shield inserted between two parallel plates:

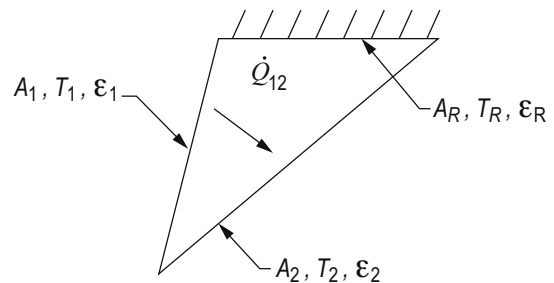
$$\dot{Q}_{12} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1 - \varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{13}} + \frac{1 - \varepsilon_{31}}{\varepsilon_{31} A_3} + \frac{1 - \varepsilon_{32}}{\varepsilon_{32} A_3} + \frac{1}{A_3 F_{32}} + \frac{1 - \varepsilon_2}{\varepsilon_2 A_2}}$$



Energy transfer by radiation from reradiating surfaces:

$$\dot{Q} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1 - \varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12} + \left(\frac{1}{A_1 F_{1R}} + \frac{1}{A_2 F_{2R}} \right)^{-1}} + \frac{1 - \varepsilon_2}{\varepsilon_2 A_2}}$$

Reradiating surfaces are considered to be insulated or adiabatic.



Radiation Heat Transfer—Special Considerations

Heat input from solar radiation:

$$\dot{Q} = \alpha A_p F_{12} \dot{q}_{\text{solar}}$$

where α = absorptivity

A_p = projected area perpendicular to the source

Simplified Equation for the Radiation Heat-Transfer Coefficient

This equation is in the same form as the equations for the conduction and convection heat-transfer coefficients and is used when there is a combination of heat-transfer coefficients. The radiation heat-transfer coefficient must be calculated at the system temperatures.

$$\dot{Q}_{\text{rad}} = h_{\text{rad}} A (T_1 - T_2)$$

where $h_{\text{rad}} = \sigma F_{12} (T_1^2 + T_2^2) (T_1 + T_2)$

4.2.1.8 Combination of Heat-Transfer Mechanisms

Overall heat-transfer coefficient U_{ov} :

$$\dot{Q} = U_{\text{ov}} A \Delta T$$

Thermal Resistance

$$\dot{Q} = \frac{\Delta T_{\text{total}}}{R_{\text{total}}}$$

$$R_{\text{total}} = \frac{1}{U_{\text{ov}} A}$$

Plane Wall	Cylindrical Wall	Spherical Wall	Convection
$R = \frac{\delta}{k A}$	$R = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k L}$	$R = \frac{r_2 - r_1}{4\pi k r_2 r_1}$	$R = \frac{1}{h A}$

Mean diameter:

Cylindrical wall $D_{\text{lm}} = \frac{D_{\text{outer}} - D_{\text{inner}}}{\ln\left(\frac{D_{\text{outer}}}{D_{\text{inner}}}\right)}$

Spherical wall $D_{\text{mean}} = \frac{D_{\text{outer}} D_{\text{inner}}}{D_{\text{outer}} - D_{\text{inner}}}$

Resistance in series:

Resistance	Heat Flux	Temperature
$R_{\text{total}} = \sum R$	$\dot{Q}_{\text{total}} = \text{constant}$	$\Delta T_{\text{total}} = \sum \Delta T$

Intermediate Temperatures: $\dot{Q} = \frac{T_{A,1} - T_{A,2}}{R_A} = \frac{T_{B,1} - T_{B,2}}{R_B} = \dots = \frac{T_{i,1} - T_{i,2}}{R_i}$

Resistance in parallel:

Resistance	Heat Flux	Temperature
$\frac{1}{R_{\text{total}}} = \sum \frac{1}{R}$	$\dot{Q}_{\text{total}} = \sum \dot{Q}$	$\Delta T_{\text{total}} = \text{constant}$

Heat flux relations: $\Delta T = \dot{Q}_1 R_1 = \dot{Q}_2 R_2 = \dots = \dot{Q}_i R_i$

Heat Transfer from Fins

For a straight fin with uniform cross-section:

For negligible heat transfer from the tip:

$$\dot{Q} = \sqrt{h P k A_c} (T_b - T_\infty) \tanh\left(L \sqrt{\frac{h P}{k A_c}}\right)$$

Including heat transfer from the tip:

$$\dot{Q} = \sqrt{h P k A_c} (T_b - T_\infty) \frac{\tanh\left(L \sqrt{\frac{h P}{k A_c}}\right) + \sqrt{\frac{h A_c}{k P}}}{1 + \sqrt{\frac{h A_c}{k P}} \tanh\left(L \sqrt{\frac{h P}{k A_c}}\right)}$$

Circular (pin) fin: $P = \pi D$ and $A_c = \frac{\pi D^2}{4}$

Rectangular fin: $P = 2(w + \delta)$ and $A_c = w\delta$

where

P = perimeter of the exposed fin cross-section

A_c = fin cross-sectional area

L = length of the fin

D = diameter of a circular fin

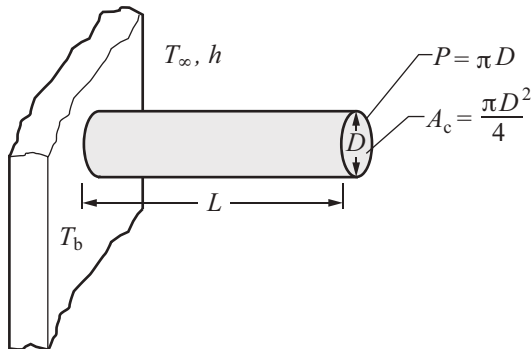
w = width of a rectangular fin

δ = height (thickness) of a rectangular fin

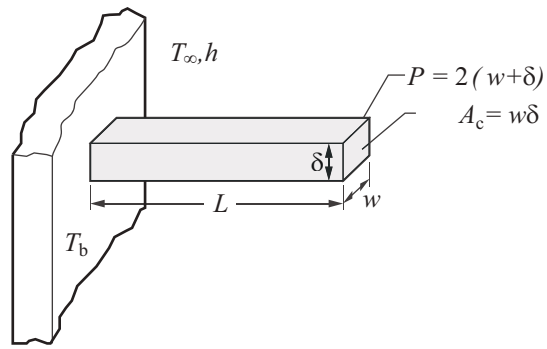
T_b = temperature at the base of the fin

T_∞ = bulk fluid temperature

Pin Fin:



Rectangular Fin:



4.2.2 Heat Transfer With Phase Change

4.2.2.1 Latent and Sensible Heat

Sensible heat: $Q_{\text{sensible}} = m c_p \Delta T$

Latent heat: $Q_{\text{latent}} = m \Delta h_{\text{phase change}}$

Heat-transfer rate during phase change: $\dot{Q}_{\text{latent}} = \dot{m} \Delta h_{\text{phase change}}$

Rate of phase change: $\frac{dm}{dt} = \frac{\dot{Q}}{\Delta h_{\text{phase change}}}$

4.2.2.2 Vaporization (Boiling)

Boiling

Evaporation is occurring at a solid-liquid interface when $T_s > T_{\text{sat}}$:

$$\dot{q} = h(T_s - T_{\text{sat}}) = h \Delta T_e$$

where

T_s = temperature of solid

T_{sat} = saturation temperature of liquid at system pressure

ΔT_e = excess temperature

Pool boiling: Liquid is quiescent; motion near the solid surface is caused by free convection and mixing induced by bubble growth and detachment.

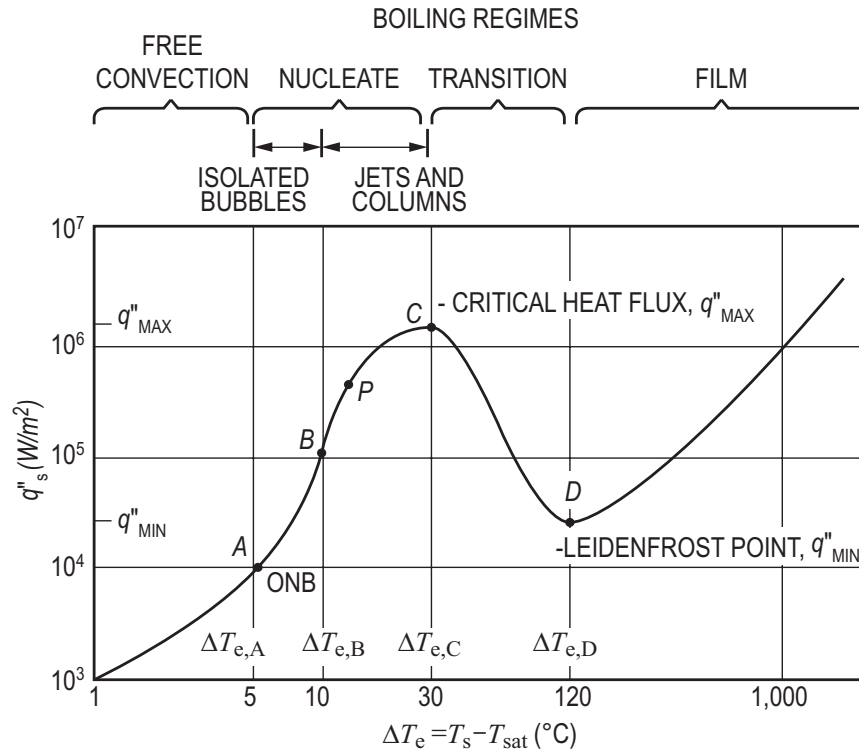
Forced convection boiling: Fluid motion is induced by external means in addition to free convection and bubble-induced mixing.

Sub-cooled boiling: Liquid temperature is below the saturation temperature; bubbles forming at the heating surface may condense in the liquid.

Saturated boiling: Liquid temperature slightly exceeds the saturation temperature; bubbles forming at the heated surface are propelled through the liquid by buoyant forces.

Typical Pool Boiling Curve for Water at One Atmosphere

Surface Heat Flux as a Function of the Excess Temperature



Free convection boiling: There is insufficient vapor in contact with the liquid phase to cause boiling at the saturation temperature.

Nucleate boiling: Isolated bubbles form at nucleation sites and separate from the surface; vapor escapes as jets or columns.

Rohsenow equation for nucleate-boiling heat flux:

$$\dot{q}_{\text{nucleate}} = \mu_{\text{liq}} \Delta h_{\text{vap}} \left[\frac{g(\rho_{\text{liq}} - \rho_{\text{vap}})}{\gamma} \right]^{1/2} \left[\frac{c_{p,\text{liq}}(T_s - T_{\text{sat}})}{C_{\text{sf}} \Delta h_{\text{vap}} \text{Pr}_{\text{liq}}^n} \right]^3$$

where

γ = surface tension of vapor-liquid interface

T_s = surface temperature of heater

T_{sat} = saturation temperature of fluid

C_{sf} = experimental constant that depends on surface-fluid combination

n = 1.0 for water and 1.7 for other liquids

Values of the Coefficient C_{sf} for Various Liquid-Surface Combinations

Fluid	Heating Surface	C_{sf}
Water	Brass	0.0060
Water	Copper	0.013
Water	Copper (emory-polished)	0.0128
Water	Copper (emory-polished, paraffin-treated)	0.0147
Water	Copper (scored)	0.0068
Water	Platinum	0.013
Water	Stainless steel (ground and polished)	0.0080
Water	Stainless steel (Teflon pitted)	0.0058
Water	Stainless steel (chemically etched)	0.0133
Water	Stainless steel (mechanically polished)	0.0132
Ethyl alcohol	Chromium	0.0027
Isopropyl alcohol	Copper	0.00225
<i>n</i> -Butyl alcohol	Copper	0.00305
<i>n</i> -Pentane	Chromium	0.015
<i>n</i> -Pentane	Copper (emory-polished)	0.0154
<i>n</i> -Pentane	Nickel (emory-polished)	0.0127
<i>n</i> -Pentane	Copper (lapped)	0.0049
<i>n</i> -Pentane	Copper (emory-rubbed)	0.0074
Benzene	Chromium	0.0100
Carbon tetrachloride	Copper	0.013
Carbon tetrachloride	Copper (emory-polished)	0.0070
35% K_2CO_3	Copper	0.0054
50% K_2CO_3	Copper	0.00275

The critical (also called "maximum" or "peak") heat flux (CHF) in nucleate pool boiling:

$$\dot{q}_{\max} = C_{cr} \Delta h_{vap} \left[\gamma g \rho_{vap}^2 (\rho_{liq} - \rho_{vap}) \right]^{1/4}$$

where C_{cr} = constant whose value depends on the heater geometry, but generally about 0.15

The critical heat flux is independent of the fluid-heating surface combination, as well as the viscosity, thermal conductivity, and heat capacity of the liquid. It increases with pressure up to about one-third of the critical pressure, and then starts to decrease and becomes zero at the critical pressure. The critical heat flux is proportional to the latent heat of vaporization; large maximum heat fluxes can be obtained using fluids with a large enthalpy of vaporization, such as water.

Values of the coefficient C_{cr} for maximum heat flux:

$$L^* = L \sqrt{\frac{g(\rho_{liq} - \rho_{vap})}{\gamma}}$$

$$K_1 = \frac{\gamma}{g(\rho_{liq} - \rho_{vap}) A_{heater}}$$

Critical Heat Flux vs. Heater Geometry

Heater Geometry	C_{cr}	Characteristic Dimension (L)	Range of L^*
Large horizontal flat heater	0.149	Width or diameter	$L^* > 27$
Small horizontal flat heater	$18.9 K_1$	Width or diameter	$9 < L^* < 20$
Large horizontal cylinder	0.12	Radius	$L^* > 1.2$
Small horizontal cylinder	$0.12 L^{*-0.25}$	Radius	$0.15 < L^* < 1.2$
Large sphere	0.11	Radius	$L^* > 4.26$
Small sphere	$0.227 L^{*-0.5}$	Radius	$0.15 < L^* < 4.26$

Minimum heat flux: This occurs at the Leidenfrost point and is of practical interest because it represents the lower limit for the heat flux in the film boiling regime.

Minimum heat flux for a large horizontal plate:

$$\dot{q}_{\min} = 0.09 \rho_{\text{vap}} \Delta h_{\text{vap}} \left[\frac{\gamma g (\rho_{\text{liq}} - \rho_{\text{vap}})}{(\rho_{\text{liq}} + \rho_{\text{vap}})^2} \right]^{\frac{1}{4}}$$

Transition boiling: Rapid bubble formation results in vapor film on surface and oscillation between film and nucleate boiling.

Film boiling: Surface is completely covered by a vapor blanket; includes significant radiation through the vapor film.

Heat flux for film boiling on a horizontal cylinder or sphere of diameter D :

$$\dot{q}_{\text{film}} = C_{\text{film}} \left\{ \frac{g k_{\text{vap}}^3 \rho_{\text{vap}} (\rho_{\text{liq}} - \rho_{\text{vap}}) [\Delta h_{\text{vap}} + 0.4 c_{p,\text{vap}} (T_s - T_{\text{sat}})]}{\mu_{\text{vap}} D (T_s - T_{\text{sat}})} \right\}^{\frac{1}{4}} (T_s - T_{\text{sat}})$$

For horizontal cylinders: $C_{\text{film}} = 0.62$

For spheres: $C_{\text{film}} = 0.67$

4.2.2.3 Evaporation

Estimate Heat of Vaporization at the Normal Boiling Point

$$\frac{\Delta H_{\text{vap},n}/T_n}{R} = \frac{1.092(\ln P_c - 1.013)}{0.930 - T_{r_n}}$$

where

T_n = normal boiling point

$\Delta H_{\text{vap},n}$ = latent heat of vaporization at T_n

P_c = critical pressure, bar

T_{r_n} = reduced temperature at T_n

Estimate heat vaporization at any temperature from a known value

$$\frac{\Delta H_{\text{vap},2}}{\Delta H_{\text{vap},1}} = \left(\frac{1 - T_{r,2}}{1 - T_{r,1}} \right)^{0.38}$$

where

ΔH_{vap} = latent heat of vaporization

T_r = reduced temperature

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4.2.2.4 Condensation

Heat-Transfer Coefficient for the Condensation of a Pure Vapor

Evaluate all liquid properties at the average temperature between the saturated temperature and the surface temperature,

where

ρ_l = density of the liquid phase of the fluid

μ_l = viscosity of the liquid phase of the fluid

k_l = thermal conductivity of the liquid phase of the fluid

$\overline{\text{Nu}}$ = average Nusselt number

$\overline{h}$ = average heat-transfer coefficient

T_{sat} = saturation temperature of the fluid

T_s = temperature of the vertical surface

P = wetted perimeter (width of a vertical plate, or πd , for a vertical tube)

$\dot{m}$ = condensate generation rate

L = length of the vertical surface

D = tube outside diameter

Condensation Film Coefficients

Geometry	Correlation	Conditions
Condensation on a vertical or angled surface, laminar flow	$\overline{Nu}_L = \frac{\bar{h} L}{k} = 0.943 \left[\frac{\rho_1^2 g \Delta h_{\text{vap}} L^3}{\mu_1 k_1 (T_{\text{sat}} - T_s)} \right]^{0.25}$	Vertical surface
	$\overline{Nu}_L = \frac{\bar{h} L}{k} = 0.943 \left[\frac{\rho_1^2 g \Delta h_{\text{vap}} L^3 \cos \theta}{\mu_1 k_1 (T_{\text{sat}} - T_s)} \right]^{0.25}$	Inclined surface, angle θ measured from the vertical
Condensation on the outside of a horizontal tube, laminar flow	$\overline{Nu}_D = \frac{\bar{h} D}{k} = 0.729 \left[\frac{\rho_1^2 g \Delta h_{\text{vap}} D^3}{\mu_1 k_1 (T_{\text{sat}} - T_s)} \right]^{0.25}$	Single tube or horizontal layer of tubes
	$\overline{Nu}_D = \frac{\bar{h} D}{k} = 0.729 \left[\frac{\rho_1^2 g \Delta h_{\text{vap}} D^3}{N \mu_1 k_1 (T_{\text{sat}} - T_s)} \right]^{0.25}$	Tube bank with N layers of horizontal tubes, arranged vertically over one another
Condensation on a tall vertical surface or on the outside of a tall vertical tube, turbulent flow	$\overline{Nu}_D = \frac{\bar{h} D}{k} = 0.0076 \text{Re}_h^{\frac{2}{3}} \left[\frac{\rho_1^2 g \Delta h_{\text{vap}} D^3}{\mu_1^2} \right]^{\frac{1}{3}}$	Condensation Reynolds number: $\text{Re}_h = \frac{4\dot{m}}{P\mu_1} > 1800$ $\dot{m} = \frac{\dot{Q}}{\Delta h_{\text{vap}}} = \frac{h A (T_{\text{sat}} - T_s)}{\Delta h_{\text{vap}}}$
Condensation on a sphere	$\overline{Nu}_D = \frac{\bar{h} D}{k} = 0.815 \left[\frac{\rho_1^2 g \Delta h_{\text{vap}} D^3}{\mu_1 k_1 (T_{\text{sat}} - T_s)} \right]^{0.25}$	

4.2.2.5 Sublimation

As shown on the thermodynamic diagram of pressure-temperature for a pure fluid, sublimation occurs when the pressure and temperature are below the triple point. As an estimate, at a constant temperature,

$$\Delta H_{\text{subl}} = \Delta H_{\text{fus}} + \Delta H_{\text{vap}}$$

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4.3 Applications of Heat Transfer

4.3.1 Heat-Exchange Equipment Design

4.3.1.1 Overall Heat-Transfer Coefficients

Energy balance around a heat exchanger:

$$\dot{Q} = \dot{m}_{\text{cold}} c_{p,\text{cold}} (T_{\text{cold,out}} - T_{\text{cold,in}}) = \dot{m}_{\text{hot}} c_{p,\text{hot}} (T_{\text{hot,in}} - T_{\text{hot,out}})$$

Rate of heat transfer in a heat exchanger:

$$\dot{Q} = U_{ov} A F \Delta T_{\text{lm}}$$

where F = LMTD correction factor based on exchanger configuration (see F-factor charts in this chapter)

Heat-transfer area in a shell-and-tube heat exchanger:

$$A_o = n \pi D_o L$$

where n = total number of tubes

Mass flow rate in a shell-and-tube heat exchanger

$$\dot{m} = n_{\text{pass}} \pi \frac{D_i^2}{4} \rho u$$

where n_{pass} = number of tubes in each pass

Overall heat-transfer coefficient for concentric tube and shell-and-tube heat exchangers:

$$\frac{1}{U_{\text{ov}} A_{\text{ref}}} = \frac{1}{h_i A_i} + \frac{R_{\text{fi}}}{A_i} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi k L} + \frac{R_{\text{fo}}}{A_o} + \frac{1}{h_o A_o}$$

$$\frac{1}{U_{\text{ov}}} = \frac{1}{h_i} \left(\frac{D_o}{D_i}\right) + R_{\text{fi}} \left(\frac{D_o}{D_i}\right) + \frac{D_o}{2k} \ln\left(\frac{D_o}{D_i}\right) + R_{\text{fo}} + \frac{1}{h_o}$$

where

A_i = inside area of the tubes

A_o = outside area of the tubes

A_{ref} = reference areas for the overall heat-transfer coefficient U_{ov} (usually the outside area)

D_i = inside diameter of the tubes

D_o = outside diameter of the tubes

h_i = convection heat-transfer coefficient for inside the tubes

h_o = convection heat-transfer coefficient for outside the tubes

R_{fi} = fouling factor for inside the tubes

R_{fo} = fouling factor for outside the tubes

4.3.1.2 Fouling Factors

Fouling factors are defined as:

$$R_f = \frac{1}{h_{\text{fouled}}} - \frac{1}{h_{\text{clean}}}$$

A table of fouling factors is shown in this chapter.

Fouling factors increase with time. Some common approximations for time dependence are as follows:

Linear: $R_f(t) = R_{f,\text{initial}} + a t$

Falling-rate: $[R_f(t)]^2 = (R_{f,\text{initial}})^2 + b t$

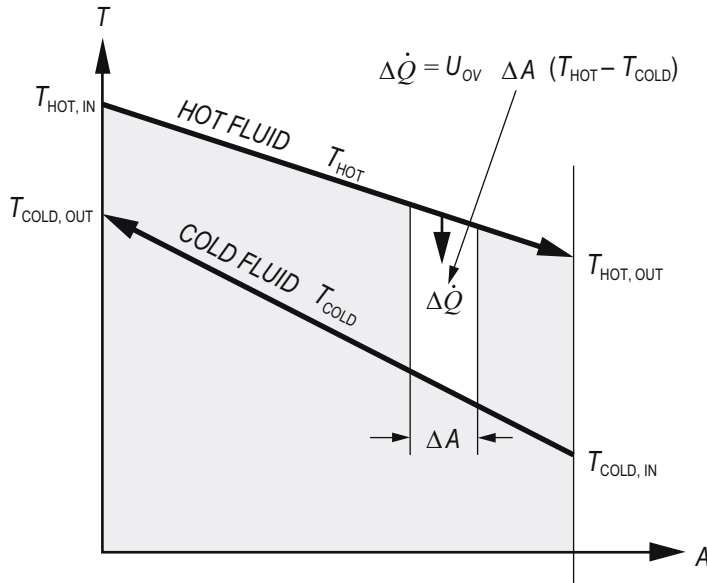
Asymptotic: $R_f(t) = R_{f,\infty} \left(1 - e^{-\frac{t}{\tau}}\right)$

where a , b , and τ = empirical constants

4.3.1.3 Log-Mean Temperature Difference

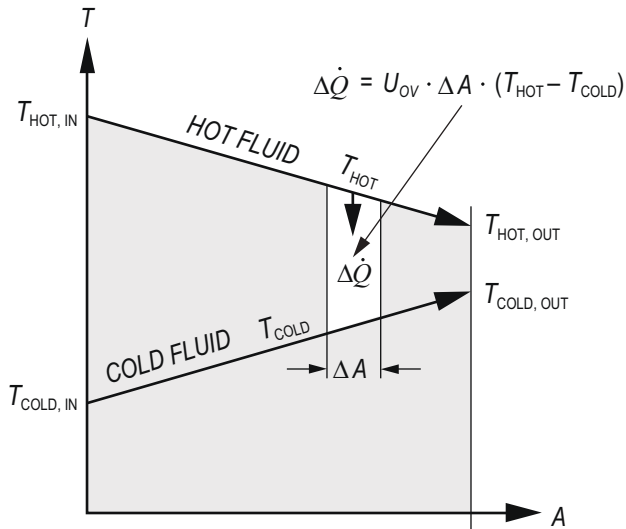
Temperature Profiles for Countercurrent and Cocurrent Heat Exchangers Without Phase Change

For countercurrent flow in heat exchangers:



$$\Delta T_{lm} = \frac{(T_{hot, out} - T_{cold, in}) - (T_{hot, in} - T_{cold, out})}{\ln \left(\frac{T_{hot, out} - T_{cold, in}}{T_{hot, in} - T_{cold, out}} \right)}$$

For cocurrent (parallel) flow in heat exchangers:

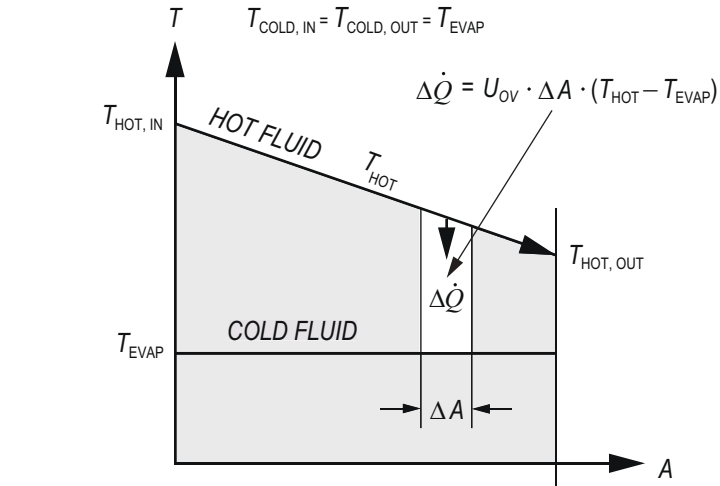


$$\Delta T_{lm} = \frac{(T_{hot, out} - T_{cold, out}) - (T_{hot, in} - T_{cold, in})}{\ln \left(\frac{T_{hot, out} - T_{cold, out}}{T_{hot, in} - T_{cold, in}} \right)}$$

Temperature Profiles for Evaporation and Condensation:

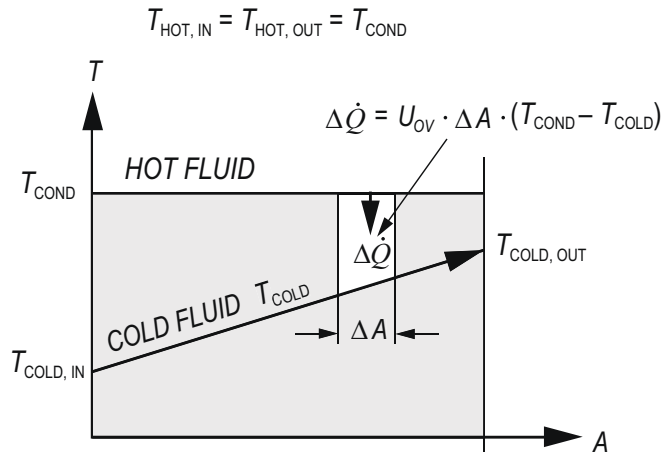
During the phase change of a pure substance, the temperature remains constant.

Evaporation:



$$\Delta T_{\text{lm}} = \frac{(T_{\text{hot, in}} - T_{\text{hot, out}})}{\ln \left(\frac{T_{\text{hot, in}} - T_{\text{evap}}}{T_{\text{hot, out}} - T_{\text{evap}}} \right)}$$

Condensation:



$$\Delta T_{\text{lm}} = \frac{(T_{\text{cold, out}} - T_{\text{cold, in}})}{\ln \left(\frac{T_{\text{cond}} - T_{\text{cold, in}}}{T_{\text{cond}} - T_{\text{cold, out}}} \right)}$$

Temperature Approach

Minimum temperature difference between a hot and a cold fluid:

$$T_{\text{approach}} = (T_{\text{hot}} - T_{\text{cold}})_{\min}$$

Cocurrent:	$T_{\text{approach}} = T_{\text{hot, out}} - T_{\text{cold, out}}$
Countercurrent, with $C_{\min} = C_{\text{hot}}$	$T_{\text{approach}} = T_{\text{hot, out}} - T_{\text{cold, in}}$
Countercurrent, with $C_{\min} = C_{\text{cold}}$	$T_{\text{approach}} = T_{\text{hot, in}} - T_{\text{cold, out}}$
Evaporation	$T_{\text{approach}} = T_{\text{hot, out}} - T_{\text{evap}}$
Condensation	$T_{\text{approach}} = T_{\text{cond}} - T_{\text{cold, out}}$

where $C = \dot{m} c_p$ = heat-capacity rate.

For $T_{\text{approach}} \rightarrow 0$

Constant heat-transfer coefficient U_{ov}	$A \rightarrow \infty$
Constant heat-transfer rate $\dot{Q}$	$\dot{m} \rightarrow \dot{m}_{\min}$
Constant flow rate $\dot{m}$	$\dot{Q} \rightarrow \dot{Q}_{\max}$

4.3.1.4 F-Factor

$$\Delta T_{\text{mean}} = F \Delta T_{\log \text{ mean}}$$

Temperature efficiency:

$$P = \frac{T_{\text{tube, out}} - T_{\text{tube, in}}}{T_{\text{shell, in}} - T_{\text{tube, in}}}$$

Ratio of heat-capacity rates:

$$R_{\text{TS}} = \frac{T_{\text{shell, in}} - T_{\text{shell, out}}}{T_{\text{tube, out}} - T_{\text{tube, in}}} = \frac{C_{\text{tube}}}{C_{\text{shell}}}$$

where $C = \dot{m} c_p$ = heat-capacity rate

Charts of the F-factors for various configurations are shown in this chapter.

4.3.1.5 Equipment Selection

Types of Heat Exchangers

Flow Types

Cocurrent or parallel flow: both fluids flow in same direction

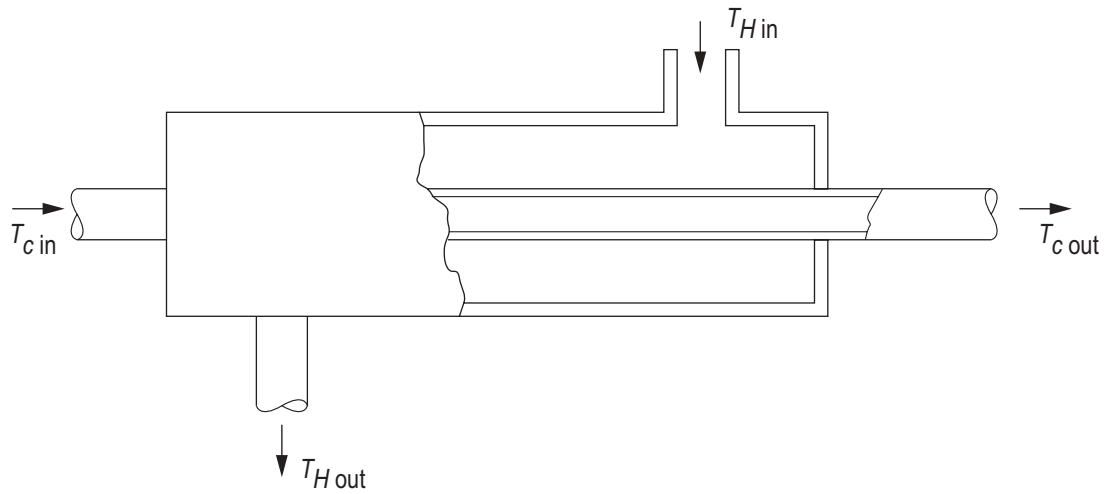
Countercurrent flow: both fluids flow in opposite direction

Crossflow: both fluids flow at right angles to each other

Mixed flow: one or both fluids are mixed through means of baffles or other geometry

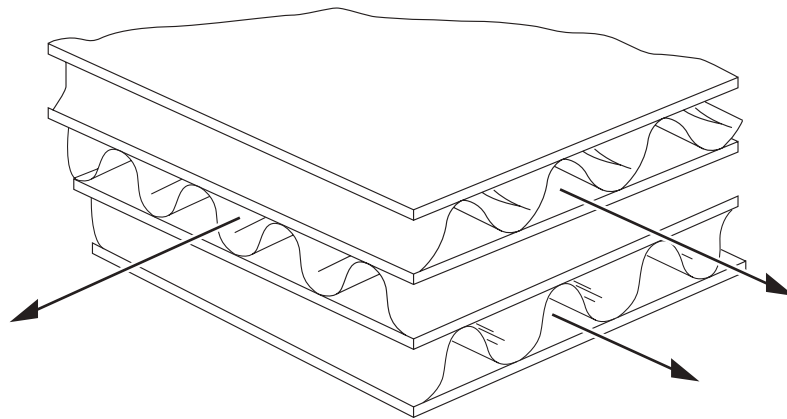
Types of Construction

Double pipe: One pipe flows inside of a second larger pipe.



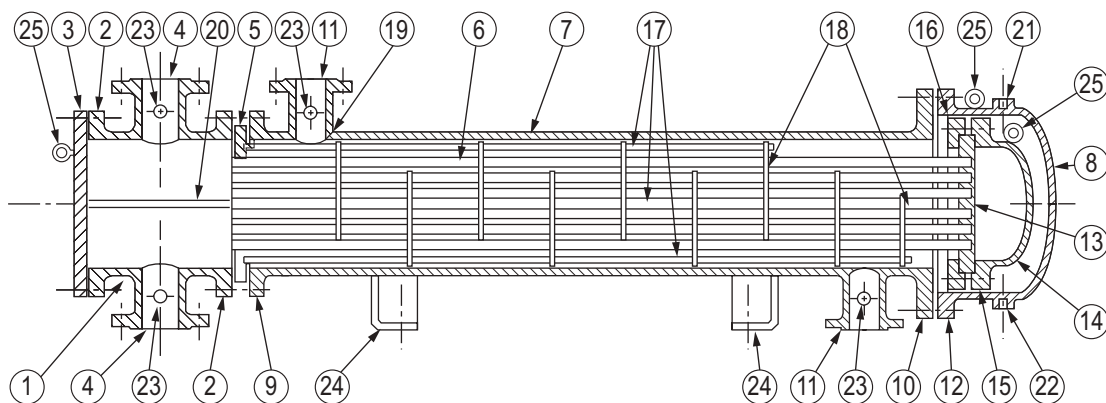
DOUBLE-PIPE HEAT EXCHANGER

Cross-flow: Stacked layers of fluid flow at right angels to each other.



CROSSFLOW HEAT EXCHANGER

Shell-and-tube: Smaller tubes are passed through a larger housing, or shell. This flexible configuration is one of the most common in industry. It allows for varied numbers of tubes or passes. Mixing can be accomplished with baffles. The chart at the end of the chapter shows the standard configurations established by the Tubular Exchanger Manufacturers Association (TEMA).



1. STATIONARY HEAD – CHANNEL	14. FLOATING HEAD COVER
2. STATIONARY HEAD FLANGE– CHANNEL OR BONNET	15. FLOATING HEAD FLANGE
3. CHANNEL COVER	16. FLOATING HEAD BACKING DEVICE
4. STATIONARY HEAD NOZZLE	17. TIE RODS AND SPACERS
5. STATIONARY TUBESHEET	18. TRANSVERSE BAFFLES OR SUPPORT PLATES
6. TUBES	19. IMPINGEMENT PLATE
7. SHELL	20. PASS PARTITION
8. SHELL COVER	21. VENT CONNECTION
9. SHELL FLANGE– STATIONARY HEAD END	22. DRAIN CONNECTION
10. SHELL FLANGE– REAR HEAD END	23. INSTRUMENT CONNECTION
11. SHELL NOZZLE	24. SUPPORT SADDLE
12. SHELL COVER FLANGE	25. LIFTING LUG
13. FLOATING TUBESHEET	

Example of shell-and-tube heat exchanger (TEMA-type AES), and heat exchanger component nomenclature.

Source: Republished with permission of McGraw-Hill, Inc., from *Perry's Chemical Engineers' Handbook*, 8th ed., Don W. Green and Robert H. Perry, New York, 2008; permission conveyed through Copyright Clearance Center, Inc.

Compact Heat Exchangers

Plate-and-Frame: A series of corrugated plates are compressed between two pressure-retaining frame plates, and sealed with elastomeric gaskets. They are useful for this compact size and easily expandable capacity.

Spiral-Plate: Two rolled strips of plate with spacer studs are welded onto each other in clock-spring shape. The high shear rates as compared to tubular designs prevent many forms of fouling, and the pure countercurrent flow leads to a LMTD correction factor that is essentially equal to 1.0.

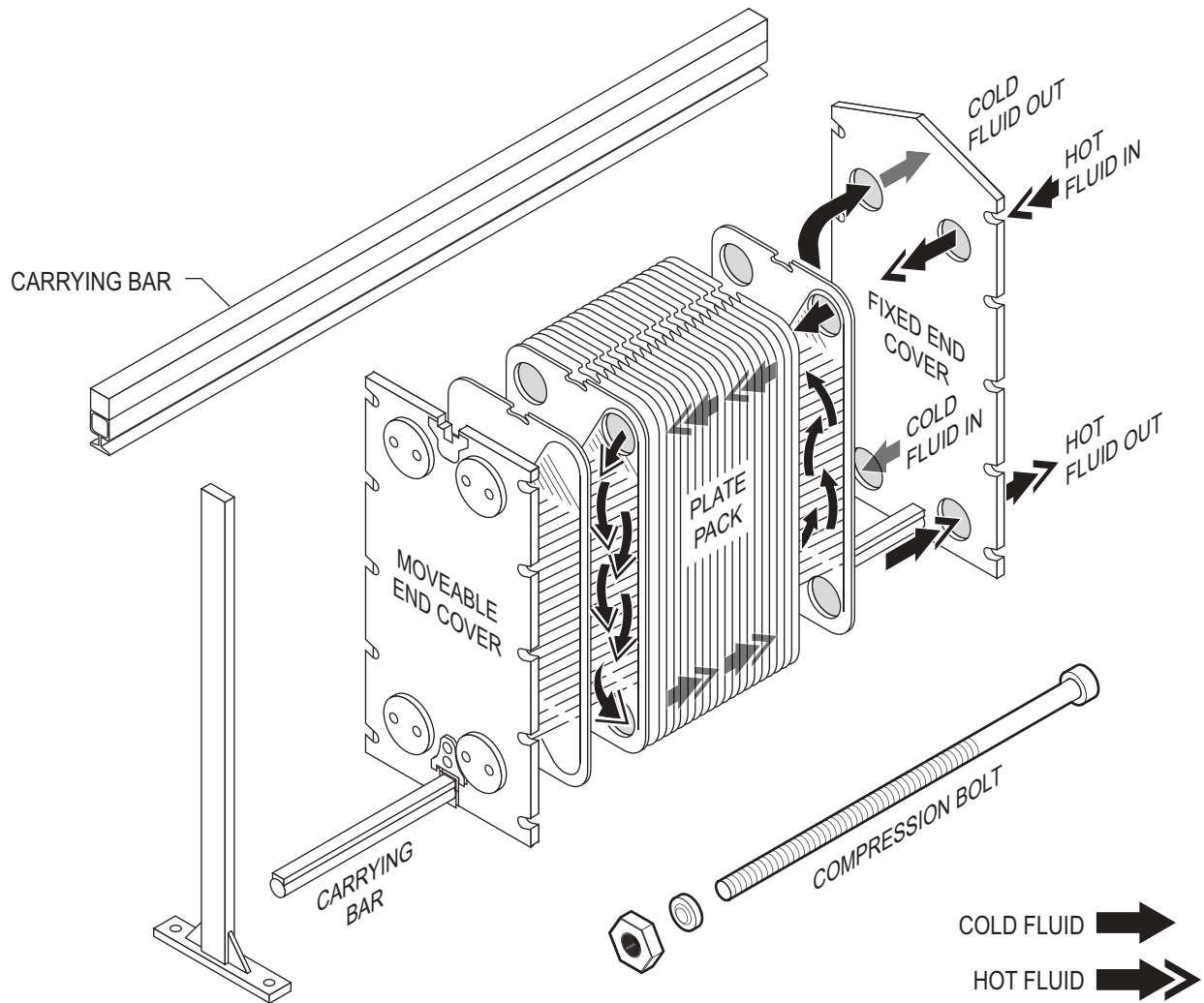
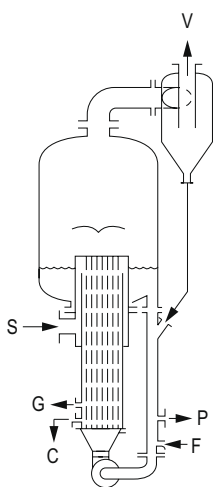
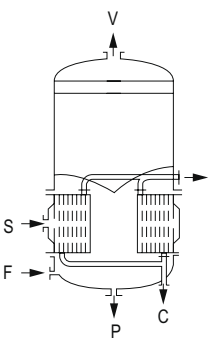


PLATE-AND-FRAME HEAT EXCHANGER

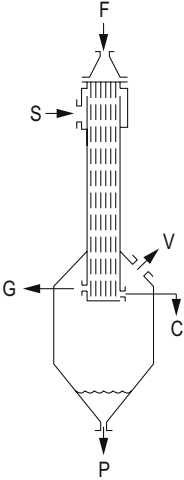
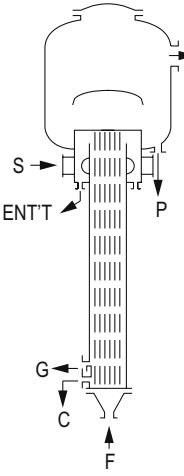
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Types of Evaporators

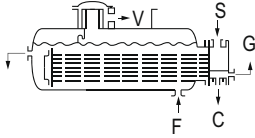
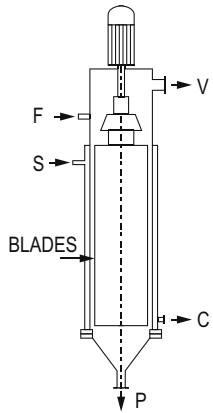
Evaporators

Type and Schematic	Description and Applications	Advantages and Disadvantages
Forced-Circulation Evaporator 	Description: Circulating pump withdraws liquor from the flash chamber and forces it past the heating surfaces. Typically, heating tubes are submerged and hydrostatic heads prevents boiling; evaporation occurs in the flash chamber. Higher heat-transfer rates can be achieved if boiling is allowed in the tubes but then scaling and salt formation may occur. The forced circulation keeps solids in suspension. Tube velocities are limited by erosion and typically are 4–10 ft/s. Applications: • Crystalline products • Corrosive solutions • Viscous solutions	Advantages: • High heat-transfer coefficients • Positive circulation • Relative freedom from salting, scaling, and fouling Disadvantages: • High cost • Power required for circulating pump • High hold-up and residence time Difficulties: • Plugging of tube inlets by detached salt deposits • Corrosion/erosion • Salting due to boiling in the tubes • Poor circulation due to high head losses
Short-Tube Vertical Evaporator 	Description: Circulation past the heating surface is generated by boiling in the tubes. The liquid then returns to the chamber through a central well. For crystallizing solutions, a propeller placed in the lower end of the central well will keep solids in suspension. Best heat transfer is achieved when liquid level is halfway up the tubes. Scaling occurs in the tubes where evaporation takes place but can be mechanically cleaned, because the tubes are relatively wide (2–3") and short (4–6'). Applications: • Clear liquids • Crystalline products (if using propeller) • Noncorrosive liquids • Mild scaling solutions	Advantages: • High heat-transfer coefficients • Low head room • Easy mechanical descaling • Relatively inexpensive Disadvantages: • Poor heat transfer at low ΔT • High floor space and weight • High hold-up • Poor heat transfer for viscous liquids Difficulties: • Large body makes use of corrosion-resistant higher alloys cost-prohibitive • Corrosion/erosion • Salting due to boiling in the tubes • Poor circulation due to high head losses

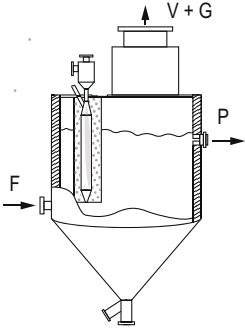
Evaporators

Type and Schematic	Description and Applications	Advantages and Disadvantages
Long-Tube Vertical Evaporator (Falling Film) 	Description: Liquid is fed to the top of vertical tubes. Tubes are narrow (1–2") and long (20–35'). The liquid flows down the walls as a film. Pressure drop in the tubes is low and the temperature of the liquid is essentially the same as that of the vapor head. Vapor-liquid separation typically occurs at the bottom. To ensure proper wetting of the tubes, external recirculation is usually required unless feed-to-evaporation rates are high. Applications: • Heat-sensitive materials • Foaming liquids • Low temperature operation • Large evaporation loads	Advantages: • Low hold-up • Cheapest per unit of capacity • Small floor space • Good heat-transfer coefficients at all temperatures Disadvantages: • High head room • Not suitable for scaling or salting liquids • External recirculation usually required Difficulties: • Poor feed distribution • Plugging of the feed distributor if solids are present in the liquid
Long-Tube Vertical Evaporator (Rising Film) 	Description: Liquid enters the long, vertical heating tubes from the bottom and rises up, propelled by the vapors generated by the evaporation. Boiling occurs in the tubes. On top of the tubes is a small vapor head with almost no liquid hold-up, where the liquid and vapor separate. The product line can be connected to the feed line to create recirculation. Applications: • Black liquid (pulp and paper) • High temperature differences • High evaporation loads	Advantages: • Good heat-transfer coefficients at reasonable temperatures • Simple construction and compactness enables use of corrosion-resistant alloys • Low cost • Low hold-up • Small floor space Disadvantages: • High head room • Not suitable for scaling or salting liquids • Poor heat-transfer coefficients at lower temperatures Difficulties: • Sensitivity to changes in operating conditions

Evaporators

Type and Schematic	Description and Applications	Advantages and Disadvantages
Horizontal Tube Evaporator 	Description: The evaporating liquid is on the shell side and the heating medium on the tube side. This evaporator is mainly used for boiler feedwater. It has low entrainment and can be designed for high steam and vapor temperatures and pressures. Tubes can be designed so that they deform when shocked (sprayed with cold water while still heated with steam), which causes the scale to crack off, making this evaporator suitable for severe scaling applications, such as hard water. Applications: - Boiler feedwater - Severely scaling liquids (bent-tube type)	Advantages: - Large vapor-liquid disengaging area - Good heat-transfer coefficients - Semiautomatic descaling (bent-tube type) - Low cost (straight-tube type) - Minimal head room required Disadvantages: - Not suitable for salting liquids - Not suitable for scaling liquids (straight-tube type) - High cost (bent-tube type) - Typically small capacity
Wiped Film (Agitated Film) Evaporator 	Description: The liquid is spread on the tube wall by a rotating assembly of blades that maintain close clearance from the wall or ride on the film. The heating surface is one large-diameter tube that may be straight or tapered, horizontal or vertical. The expensive construction limits application to the most difficult materials. Applications: - Extremely viscous materials - Heat-sensitive materials in which exposure to high temperature must be minimized	Advantages: - Very short residence time - Ability to handle extremely viscous materials - High feed-to-product ratios without need for recirculation Disadvantages: - Low heat-transfer coefficients - High installation costs - High operating costs

Evaporators

Type and Schematic	Description and Applications	Advantages and Disadvantages
Submerged Combustion Evaporator 	Description: Heat transfer is provided by bubbling combustion gases through the liquid; thus no heat-transfer surfaces are used. The evaporator consists of a tank holding the liquid, a burner, and a gas distributor. The vapor from the evaporation is mixed with the combustion gases, making it impossible to recover the heat from the vapor. Applications: • Highly corrosive solutions • Severely scaling liquids	Advantages: • No surface on which scale can form • Use of special alloys or nonmetallic materials is possible Disadvantages: • High entrainment losses • No heat recovery from the vapor, resulting in high fuel costs • Cannot control crystal size in crystallization applications

where

C = counterscale

F = feed

G = vent

P = product

S = steam

V = vapor

EWT'T = separated entrainment outlet

Source of first 5 schematics: Perry, Robert H. and Cecil H. Chilton, *Chemical Engineer's Handbook*, 5th ed., New York: McGraw-Hill, 1973.

Source of the sixth schematic: *Chemical Engineering Research and Design* (www.ichemejournals.com), © Institute of Chemical Engineers, published by Elsevier.

Source of the seventh schematic: China Manufacturers and Suppliers of Oil, Gas and Petrochemical Equipment, <http://www.china-ogpe.com/buyingguide/>, January 2016.

Heat-Transfer Calculations for Evaporators

While the general heat-transfer equations apply, evaporators have some special considerations:

Heat-transfer coefficient: Depends strongly on the temperature difference.

Heat-transfer area: Surface area through which the heat transfer takes place, measured on the liquid side.

Apparent temperature difference: The temperature difference can be difficult to determine because it varies along the length of the evaporator tubes. The apparent temperature difference is calculated as the difference between the heating-medium and boiling-liquid temperatures. Heating-medium temperature is the saturation temperature of the steam at steam pressure. (Superheat or subcooling are not considered.) Boiling-liquid temperature is the saturation temperature of the liquid at vapor head pressure—thus assuming a negligible boiling-point rise.

Temperature corrected for boiling-point rise: Boiling-point rise is the difference between the boiling point of the solution and the boiling point of the pure solvent at the same pressure. The temperature corrected for the boiling-point rise is the apparent temperature difference minus the boiling-point rise. This is typically used as the basis for the calculation of heat-transfer coefficients and also as a basis for comparing efficiencies of different evaporator types.

Multi-Effect Evaporators

Multi-effect evaporators reduce the energy needed for evaporation by using the steam generated in one stage as the heating medium for another stage.

The temperature difference for heat transfer in each effect is:

$$\Delta T = T_{\text{cond, steam}} - T_{\text{evap, liquid}}$$

where the condensation temperature of the steam is determined by the pressure in the effect where the steam was generated:

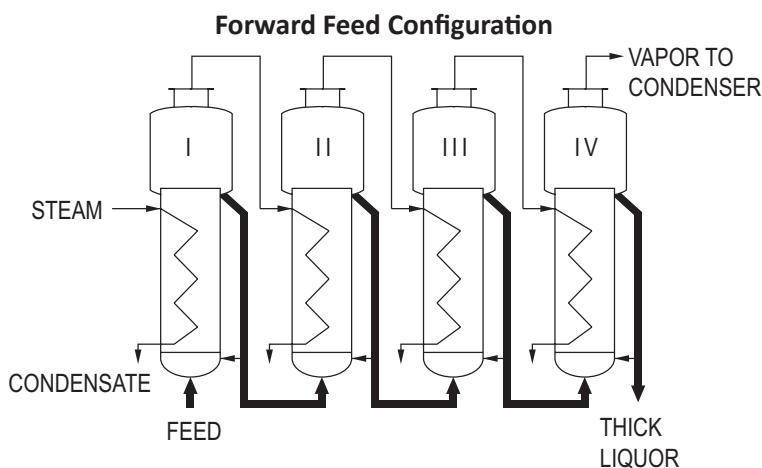
$$T_{\text{cond, steam}} = T_{\text{sat}} \text{ at } P_{n-1}$$

The evaporation temperature of the liquid is determined by the pressure in the current effect:

$$T_{\text{evap, liquid}} = T_{\text{sat}} \text{ at } P_n$$

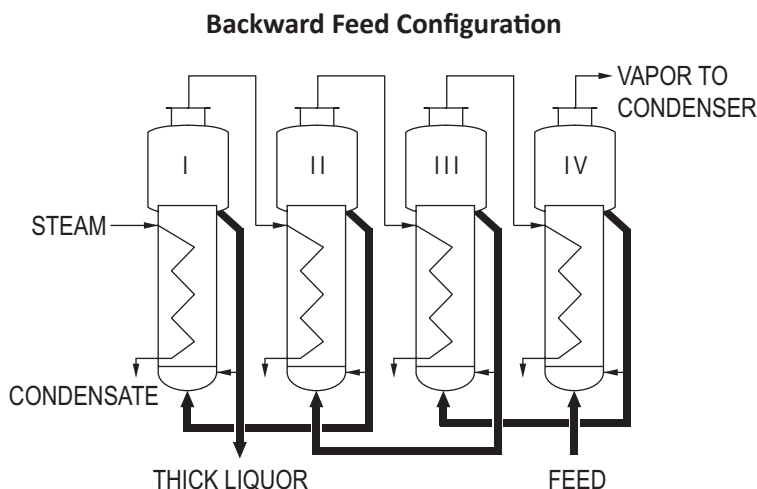
Different feed arrangements are common:

1. In the *forward feed configuration*, the product and vapor flow are parallel. This configuration is used when the feed is near the boiling point or when the product is heat-sensitive or prone to scaling and requires low temperature differences. One additional advantage is that flow of the product from one effect can be achieved by pressure difference alone, so that no intermediate liquor pumps are needed.



Source: McCabe, Warren L., Julian Smith, and Peter Harriott, *Unit Operations of Chemical Engineering*, 5th ed., New York: McGraw-Hill, 1993.

2. In the *backward feed configuration*, the product and vapor flow are countercurrent. It is used when the feed is cold, because most of the feed preheating is done by the vapor generated in the previous effect. It is preferred for highly viscous liquor, because the temperature in the effect will be higher as the liquor becomes more concentrated.



Source: McCabe, Warren L; Julian Smith; and Peter Harriott. Unit Operations of Chemical Engineering, 5th ed., New York: McGraw-Hill, 1993.

4.3.1.6 Insulation

Heat loss from cylindrical, insulated pipe:

$$\dot{Q} = \frac{2\pi k_{\text{ins}} L (T_1 - T_{\infty})}{\ln\left(\frac{r_2}{r_1}\right) + \frac{k_{\text{ins}}}{h_{\infty} r_2}}$$

Surface temperature of the insulation:

$$T_2 = \frac{T_1 + T_{\infty} \frac{h_{\infty} r_2}{k} \ln\left(\frac{r_2}{r_1}\right)}{1 + \frac{h_{\infty} r_2}{k} \ln\left(\frac{r_2}{r_1}\right)}$$

Critical insulation radius (where heat loss is at a minimum): $\frac{d\dot{Q}}{dr_2} = 0$
(for $k_{\text{ins}} \ll k_{\text{pipe}}$)

$$r_{2, \text{crit}} = \frac{k_{\text{ins}}}{h_{\infty}}$$

$$\dot{Q}_{\text{min}} = \frac{2\pi k_{\text{ins}} L (T_1 - T_{\infty})}{1 + \ln\left(\frac{k_{\text{ins}}}{h_{\infty} r_1}\right)}$$

$$T_{2, \text{crit}} = \frac{T_1 + T_{\infty} \ln\left(\frac{k_{\text{ins}}}{h_{\infty} r_1}\right)}{1 + \ln\left(\frac{k_{\text{ins}}}{h_{\infty} r_1}\right)}$$

where

T_1 = surface temperature of the pipe

T_2 = surface temperature of the insulation

T_∞ = temperature of surroundings

r_1 = outer radius of the pipe

r_2 = outer radius of the insulation

k_{ins} = thermal conductivity of the insulation

h_∞ = convective heat-transfer coefficient for the surroundings

4.3.2 Heat-Exchange Equipment Analysis

4.3.2.1 Pressure Drop and Surface Temperatures

Pressure Drop for Single-Phase Heat Transfer

Tube-side pressure drop for incompressible, single-phase flow in a shell-and-tube exchanger (including pressure drop in the tubes, in the heads for a multipass exchanger, and at the inlet and outlet nozzles):

$$\Delta P_{\text{tubeside}} = \left\{ 1.5 + n \left[2 f \left(\frac{L}{D} \right) \left(\frac{\mu}{\mu_w} \right)^m + 2.5 \right] \right\} \frac{\rho u_t^2}{2}$$

where

u_t = velocity in the tubes

f = friction factor (Darcy-Weisbach, see Moody diagram)

$m = 0.25$ for laminar flow ($\text{Re} < 2,100$)

$m = 0.14$ for turbulent flow ($\text{Re} > 2,100$)

Pressure Drop in Rising Film Evaporators

In rising film evaporators, the pressure drop in the tubes is comprised of frictional pressure drop and acceleration pressure drop from the increased velocity of the flow due to volume change during evaporation. If the inlet flow is liquid, the acceleration pressure drop is calculated from:

$$\Delta P_a = y \left(\frac{\dot{m}}{A_{\text{cross}}} \right)^2 \left(\frac{1}{\rho_{\text{vap}}} - \frac{1}{\rho_{\text{liq}}} \right) \frac{1}{g_c}$$

where

ΔP_a = acceleration pressure drop

y = vapor fraction (by weight)

A_{cross} = cross-sectional area of the tube

Surface Temperature for Condensation

Surface temperature for the condensation of a superheated vapor:

$$T_{\text{surface}} = T_{\text{coolant}} + T_{\text{vapor}} \left(1 - \frac{U_{\text{ov}}}{h} \right)$$

where

h = sensible heat-transfer coefficient for the vapor

U_{ov} = overall heat-transfer coefficient, based on h

Condensation occurs only if $T_{\text{surface}} \leq T_{\text{sat}}$.

4.3.2.2 Performance Evaluations (Number of Thermal Transfer Units)

Heat-Exchanger Effectiveness (ϵ):

$$\epsilon = \frac{\dot{Q}}{\dot{Q}_{\text{max}}} = \frac{\text{actual heat-transfer rate}}{\text{maximum possible heat-transfer rate}}$$

$$\epsilon = \frac{C_{\text{hot}}(T_{\text{hot, in}} - T_{\text{hot, out}})}{C_{\text{min}}(T_{\text{hot, in}} - T_{\text{cold, in}})} = \frac{C_{\text{cold}}(T_{\text{cold, out}} - T_{\text{cold, in}})}{C_{\text{min}}(T_{\text{hot, in}} - T_{\text{cold, in}})}$$

$C_{\text{hot}} = C_{\text{min}}$	$C_{\text{cold}} = C_{\text{min}}$
$\epsilon_{\text{hot}} = \frac{T_{\text{hot, in}} - T_{\text{hot, out}}}{T_{\text{hot, in}} - T_{\text{cold, in}}}$	$\epsilon_{\text{cold}} = \frac{T_{\text{cold, out}} - T_{\text{cold, in}}}{T_{\text{hot, in}} - T_{\text{cold, in}}}$

Heat-capacity rate is C :

$$C = \dot{m} c_p$$

$$C_{\text{min}} = \text{smaller of } C_{\text{hot}} \text{ and } C_{\text{cold}}$$

$$C_{\text{max}} = \text{larger of } C_{\text{hot}} \text{ and } C_{\text{cold}}$$

Ratio of heat-capacity rates is R_{NTU} :

$$R_{NTU} = \frac{C_{\text{min}}}{C_{\text{max}}}$$

where

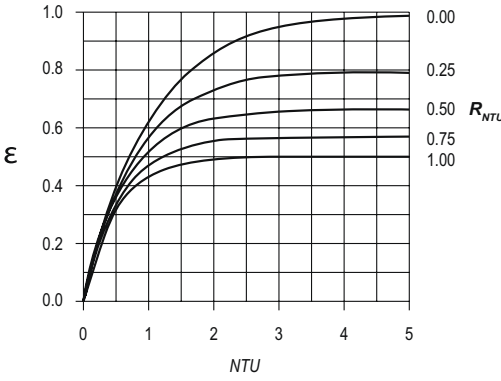
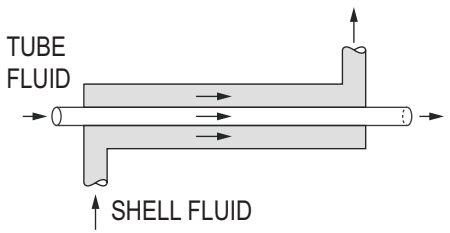
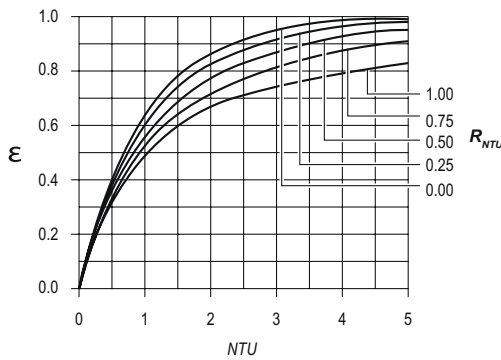
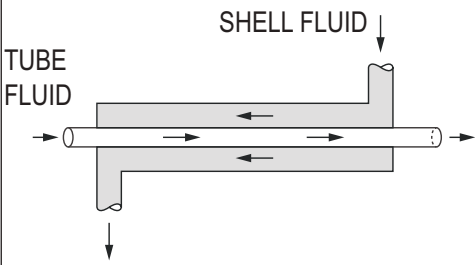
$$0 \leq R_{NTU} \leq 1$$

$$R_{NTU} = 0 \text{ for exchangers with phase change (condensation or evaporation)}$$

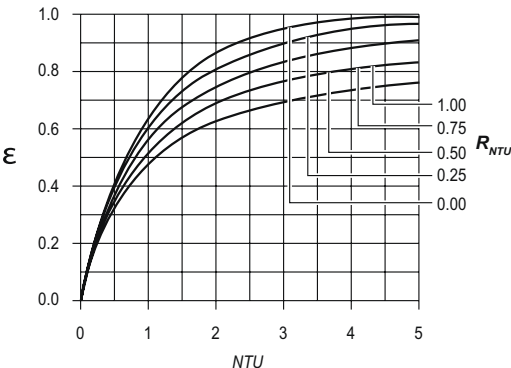
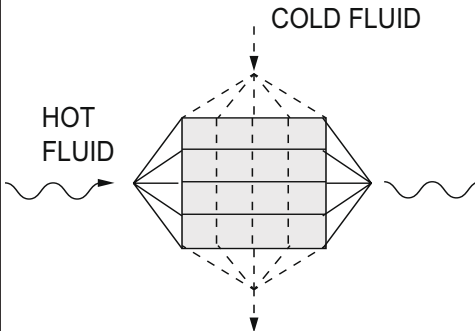
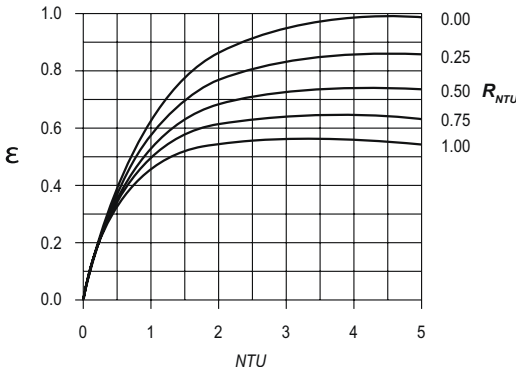
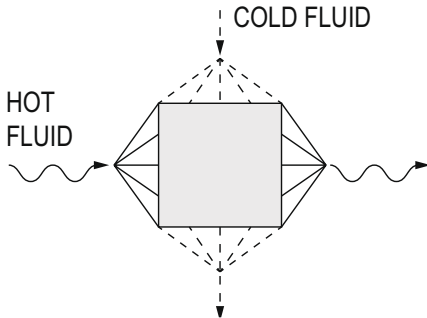
Number of Transfer Units (NTU)

$$NTU = \frac{U_{\text{ov}} A}{C_{\text{min}}}$$

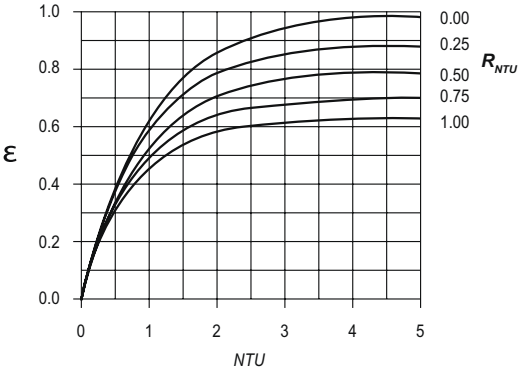
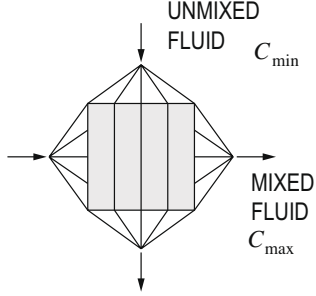
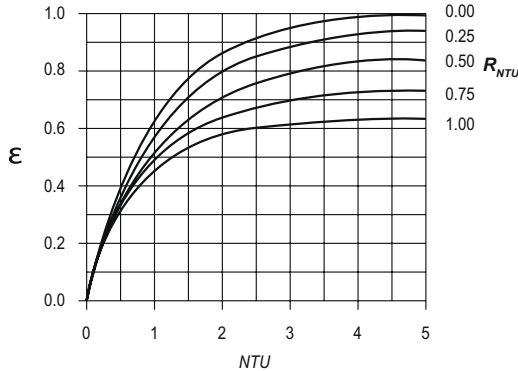
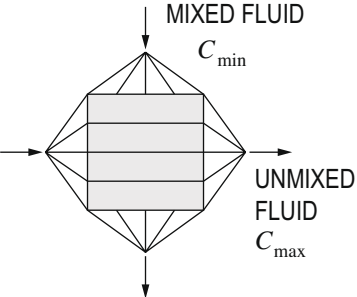
Heat-Exchanger Effectiveness and NTU Relations

Flow Geometry	Effectiveness and Transfer Unit Equations, Schematic, and Graphical Solution		
Double-Pipe			
Cocurrent	ε	$\varepsilon = \frac{1 - \exp[-NTU(1 + R_{NTU})]}{1 + R_{NTU}}$	
	NTU	$NTU = \frac{-\ln[1 - \varepsilon(1 + R_{NTU})]}{1 + R_{NTU}}$	
			
Countercurrent	ε	$\varepsilon = \frac{1 - \exp[-NTU(1 - R_{NTU})]}{1 - R_{NTU} \exp[-NTU(1 - R_{NTU})]}$	$R_{NTU} = 1: \varepsilon = \frac{NTU}{NTU + 1}$
	NTU	$NTU = \frac{1}{R_{NTU} - 1} \ln\left(\frac{\varepsilon - 1}{R_{NTU} \varepsilon - 1}\right)$	$R_{NTU} = 1: NTU = \frac{\varepsilon}{1 - \varepsilon}$
			

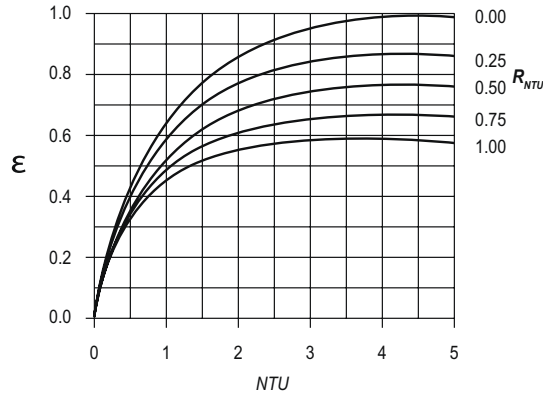
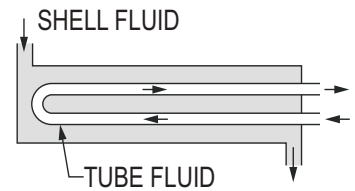
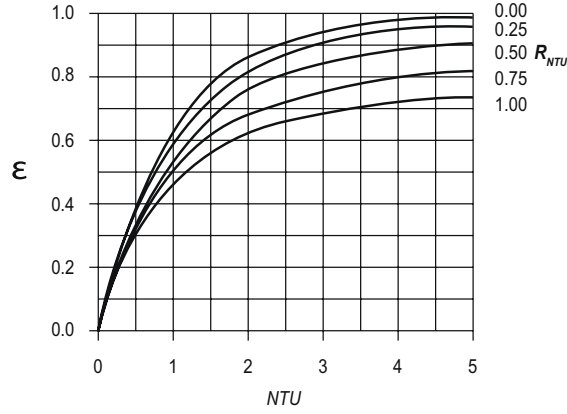
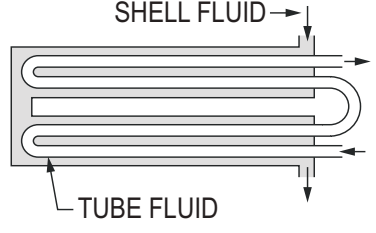
Heat-Exchanger Effectiveness and NTU Relations (cont'd)

Flow Geometry	Effectiveness and Transfer Unit Equations, Schematic, and Graphical Solution		
Cross-Flow			
Both Fluids, Unmixed	ε	$\varepsilon = 1 - \exp \left[\frac{\exp(-NTU^{0.78} R_{NTU}) - 1}{NTU^{-0.22} R_{NTU}} \right]$	
			
Both Fluids, Mixed	ε	$\frac{1}{\varepsilon} = \frac{1}{1 - \exp(-NTU)} + \frac{R_{NTU}}{1 - \exp(-NTU R_{NTU})} - \frac{1}{NTU}$	
			

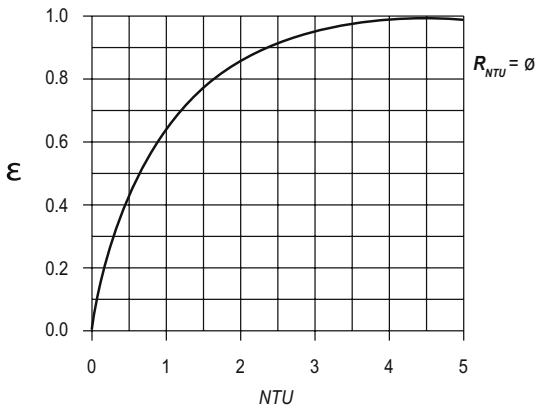
Heat-Exchanger Effectiveness and NTU Relations (cont'd)

Flow Geometry	Effectiveness and Transfer Unit Equations, Schematic, and Graphical Solution	
$C_{\max}$ Mixed $C_{\min}$ Unmixed	ε	$\varepsilon = \frac{1}{R_{NTU}} \left\{ 1 - \exp \left[-R_{NTU} (1 - \exp(-NTU)) \right] \right\}$
	NTU	$NTU = -\ln \left[1 + \frac{1}{R_{NTU}} \ln(1 - R_{NTU} \varepsilon) \right]$
	 	
$C_{\max}$ Unmixed $C_{\min}$ Mixed	ε	$\varepsilon = 1 - \exp \left\{ -\frac{1}{R_{NTU}} [1 - \exp(-NTU R_{NTU})] \right\}$
	NTU	$NTU = -\frac{1}{R_{NTU}} [1 + R_{NTU} \ln(1 - \varepsilon)]$
	 	

Heat-Exchanger Effectiveness and NTU Relations (cont'd)

Flow Geometry	Effectiveness and Transfer Unit Equations, Schematic, and Graphical Solution	
Shell-and-Tube		
One shell pass; 2, 4, 6 tube passes	ϵ	$\frac{2}{\epsilon} = 1 + R_{NTU} + \sqrt{1 + R_{NTU}} \frac{1 + \exp(-NTU \sqrt{1 + R_{NTU}^2})}{1 - \exp(-NTU \sqrt{1 + R_{NTU}^2})}$
	NTU	$NTU = -\frac{1}{\sqrt{1 + R_{NTU}}} \ln \left[\frac{\frac{2}{\epsilon} - 1 - R_{NTU} - \sqrt{1 + R_{NTU}^2}}{\frac{2}{\epsilon} - 1 - R_{NTU} + \sqrt{1 + R_{NTU}^2}} \right]$
	One shell pass; 2, 4, 6 tube passes  	
Two shell passes; 2, 4, 6 tube passes	Two shell passes; 2, 4, 6 tube passes  	

Heat-Exchanger Effectiveness and NTU Relations (cont'd)

Flow Geometry	Effectiveness and Transfer Unit Equations, Schematic, and Graphical Solution	
All Exchangers With Evaporation and Condensation		
$R_{NTU} = 0$	ε	$\varepsilon = 1 - \exp(-NTU)$
	NTU	$NTU = -\ln(1 - \varepsilon)$
		

4.4 Tables and Graphs

4.4.1 Tables of Heat-Transfer Data

4.4.1.1 Heat Capacity

Typical Ranges of Heat Capacity at Ambient Temperatures

Material	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$
Gases at 1 atm	0.15–1	0.60–4
Nonorganic liquids	0.50–1.20	2–5
Organic liquids	0.25–0.75	1–3
Solid nonmetals	0.20–0.50	0.80–2
Metals	0.03–0.20	0.12–0.80

4.4.1.2 Thermal Conductivity

Typical Ranges of Thermal Conductivity

Material	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{m} \cdot \text{K}}$
Gases at 1 atm	0.004–0.10	0.007–0.17
Insulators	0.02–0.12	0.03–0.21
Nonmetallic liquids	0.05–0.40	0.09–0.70
Nonmetallic solids	0.02–1.50	0.03–2.60
Liquid metals	5–45	8.7–78
Metallic alloys	8–70	14–120
Pure metals	30–240	52–420

Heat-Transfer Properties of Building and Insulating Materials (U.S. Units)

Materials	Density $\frac{\text{lbm}}{\text{ft}^3}$	Heat Capacity $\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	Temperature $^\circ\text{F}$	Thermal Conductivity $\frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$
Asbestos	36	0.183	–300	0.055
			–100	0.082
			32	0.088
			200	0.111
			800	0.130
Brick (building)	94	0.199	—	0.416
Brick (fireclay)	165	0.229	—	0.578
Calcium silicate	—	—	100	0.033
			400	0.046
			600	0.060
Cardboard (corrugated)	—	—	68	0.037
Cellular glass	—	—	100	0.030
			600	0.073
Clay	91	—	100	0.751
Concrete	144	0.270	68	0.739
Cork	8	—	68	0.025
Cotton	6	0.311	68	0.028
Diatomaceous earth	10.6	—	100	0.026
Fiberglass	—	—	100	0.026
			300	0.034
Glass (window)	156	0.160	68	0.430
Gypsum	30	—	68	0.045
Kaolin firebrick	19	—	400	0.050
			1400	0.110
			2100	0.260
Leather	62	—	86	0.092
Magnesia (85%)	17	—	100	0.034
			400	0.044
Mineral wool	10	—	100	0.030
			600	0.057
Plywood	34	—	68	0.069
Rubber	72	0.332	68	0.116
Rubber, foam	4.4	—	68	0.017
Sand	95	0.420	68	0.191
Sawdust	12	—	68	0.034
Urethane foam	4.4	0.251	100	0.016
Wood (oak—with the grain)	48	0.568	68	0.210
Wood (oak—against the grain)			68	0.120
Wood (pine—with the grain)	33	0.657	68	0.148
Wood (pine—across the grain)			68	0.062
Wool	8.5	—	100	0.027

Heat-Transfer Properties of Building and Insulating Materials (SI Units)

Materials	Density $\frac{\text{kg}}{\text{m}^3}$	Heat Capacity $\frac{\text{W}}{\text{kg} \cdot \text{K}}$	Temperature $^{\circ}\text{C}$	Thermal Conductivity $\frac{\text{W}}{\text{m} \cdot \text{K}}$
Asbestos	577	765	−200	0.094
			−75	0.142
			0	0.152
			100	0.192
			420	0.225
Brick (building)	1500	835	—	0.720
Brick (fireclay)	2640	960	—	1.000
Calcium silicate	—	—	40	0.057
			200	0.080
			320	0.104
Cardboard (corrugated)	—	—	20	0.064
Cellular glass	—	—	40	0.052
			320	0.126
Clay	1460	—	40	1.300
Concrete	2300	1130	20	1.279
Cork	128	—	20	0.043
Cotton	96	1300	20	0.048
Diatomaceous earth	170	—	40	0.045
Fiberglass	—	—	40	0.045
			150	0.059
Glass (window)	2500	670	20	0.744
Gypsum	481	—	20	0.078
Kaolin firebrick	304	—	200	0.087
			750	0.190
			1150	0.450
Leather	1000	—	30	0.159
Magnesia (85%)	272	—	40	0.059
			200	0.076
Mineral wool	160	—	40	0.052
			320	0.099
Plywood	540	—	20	0.120
Rubber	1150	1392	20	0.200
Rubber, foam	70	—	20	0.030
Sand	1522	1759	20	0.330
Saw dust	192	—	20	0.059
Urethane foam	70	1050	40	0.028
Wood (oak—with the grain)	770	2380	20	0.363
Wood (oak—against the grain)		—	20	0.207
Wood (pine—with the grain)	525	2750	20	0.256
Wood (pine—across the grain)		—	20	0.107
Wool	136	—	40	0.047

4.4.1.3 Heat-Transfer Coefficients (Film Coefficients, h)

Typical Values of Heat-Transfer Coefficients Without Phase Change

System Description	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{m}^2 \cdot \text{K}}$
Air and gas (free convection)	0.2–4	1–20
Air and gas (flowing—low pressure)	2–20	10–100
Air and gas (flowing—high pressure)	20–60	100–360
Liquid (free convection)	10–175	50–1000
Oils and heavy organics (flowing)	35–200	200–1200
Molten salts and brines	100–200	500–1000
Heat-transfer fluids and refrigerants	175–450	1000–2700
Water (flowing)	150–450	900–2700

Typical Values of Heat-Transfer Coefficients With Phase Change

System Description	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{m}^2 \cdot \text{K}}$
Condensation		
Condensing organic vapors	150–250	850–1500
Condensing ammonia	500–800	2800–4500
Condensing steam	700–900	4000–5000
Boiling		
Boiling organics	125–250	700–1500
Boiling ammonia	200–350	1100–2000
Boiling water	280–500	1600–2800

4.4.1.4 Overall Heat-Transfer Coefficients (U_{ov})

Typical Overall Heat-Transfer Coefficients for Building Applications

Building Component	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{m}^2 \cdot \text{K}}$
Brick wall, uninsulated	0.45	2.55
Frame wall, uninsulated	0.25	1.42
Frame wall, with Rockwool	0.07	0.4
Single-pane glass window	1.1	6.2
Double-pane glass window	0.4	2.3

Typical Overall Heat-Transfer Coefficients for Air Coolers

System	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{m}^2 \cdot \text{K}}$
Finned air cooler/condensing steam	5–50	30–300
Finned air cooler/water	4–10	25–60
Air cooler (fin-fan)/water	50–80	300–450
Air cooler (fin-fan)/light organics	50–125	300–700
Air cooler (fin-fan)/heavy organics	12–25	70–150
Air cooler (fin-fan)/condensing hydrocarbons	50–100	300–600
Air cooler (fin-fan)/condensing ammonia	110	650
Air cooler (fin-fan)/condensing Freon	70	400
Air cooler (fin-fan)/gas <5–10 bar/60–130 psig	10–20	60–120
Air cooler (fin-fan)/gas >10–30 bar/130–420 psig	20–50	100–300

Typical Overall Heat-Transfer Coefficients in Exchangers Without Phase Change (Shell-and-Tube Exchangers)

System	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{m}^2 \cdot \text{K}}$
Gas/gas	2–10	10–50
Water or brine/compressed gas	10–30	60–200
Water/hydrogen with natural gas	80–125	450–700
Water/brine	100–200	600–1200
Water/water	150–300	850–1700
Water/alcohol, organic solvents	50–150	280–850
Water/gasoline	60–90	340–510
Water/gas oil, distillate	35–60	200–340
Water/heavy oil	10–50	60–300
Freon or ammonia/water	40–90	220–510
Light organics/light organics	40–75	220–425
Medium organics/medium organics	20–60	110–340
Heavy organics/heavy organics	10–40	57–220
Heavy organics/light organics	10–60	57–340
Crude oil/gas oil	30–55	170–310

**Typical Overall Heat-Transfer Coefficients in Water-Cooled Condensers
(Shell-and-Tube Exchangers)**

Condensing Fluid	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{m}^2 \cdot \text{K}}$
Alcohol vapors	45–125	250–700
Ammonia vapors	150–250	850–1400
Freon vapors	45–150	250–850
Aqueous vapors	200–1000	1100–5600
Condensing oil	40–100	220–570
Organic vapors	125–175	700–1000
Organic vapors with noncondensables	90–125	500–700
Vacuum condensers	35–90	200–500

**Typical Overall Heat-Transfer Coefficients in Heaters With
Condensing Steam (Shell-and-Tube Exchangers)**

Heated Fluid	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{m}^2 \cdot \text{K}}$
Gas	5–50	30–300
Heavy oil	10–50	60–300
Light oil	35–100	200–600
Kerosene/gasoline	50–200	300–1100
Organic Solvents	90–175	500–1000
Water	250–700	1500–4000

Typical Overall Heat-Transfer Coefficients for Immersed Heating Coils

Immersed Coils		$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$		$\frac{\text{W}}{\text{m}^2 \cdot \text{K}}$	
Pool Liquid	Heating Medium	Natural convection	Agitated	Natural convection	Agitated
Dilute aq. solution	Steam	100–200	130–275	500–1000	700–1600
Light oil	Steam	35–50	50–100	200–300	300–600
Heavy oil	Steam	15–30	50–70	90–170	300–400
Molten sulfur	Steam	20–35	35–45	100–200	200–250
Molasses/corn syrup	Steam	15–30	60–80	70–170	350–450
Aqueous solution	Water	70–100	110–160	400–600	400–650
Light oil	Water	20–25	35–50	100–150	200–300

Typical Overall Heat-Transfer Coefficients for Plate Exchangers

Plate Exchangers		$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{m}^2 \cdot \text{K}}$
Hot Fluid	Cold Fluid		
Light organic	Light organic	450–900	2500–5000
Light organic	Viscous organic	45–90	250–500
Viscous organic	Viscous organic	20–35	100–200
Light organic	Process water	450–600	2500–3500
Viscous organic	Process water	45–90	250–500
Light organic	Cooling water	350–800	2000–4500
Viscous organic	Cooling water	45–80	250–450
Condensing steam	Light organic	450–600	2500–3500
Condensing steam	Viscous organic	45–90	250–500
Process water	Process water	900–1300	5000–7500
Process water	Cooling water	90–1200	500–7000
Dilute aqueous solutions	Cooling water	900–1200	5000–7000
Condensing steam	Process water	600–800	3500–4500

Typical Overall Heat-Transfer Coefficients in Evaporators

System	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{m}^2 \cdot \text{K}}$
Agitated film		
Newtonian liquid, $\mu = 1$ cP	400	2000
Newtonian liquid, $\mu = 100$ cP	300	1500
Newtonian liquid, $\mu = 10,000$ cP	120	700
Vertical long tube		
Natural circulation	200–600	1000–3500
Forced circulation	400–1000	2000–6000

4.4.1.5 Representative Values for Fouling Factors

Representative Values for Fouling Factors
Values for $\leq 125^{\circ}\text{F}/50^{\circ}\text{C}$, unless specified otherwise

Material	$\frac{\text{hr} \cdot \text{ft}^2 \cdot ^{\circ}\text{F}}{\text{Btu}}$	$\frac{\text{m}^2 \cdot \text{K}}{\text{W}}$
Water		
Seawater, brine, salt water	0.0005	0.00009
Seawater, brine, salt water ($> 125^{\circ}\text{F}/50^{\circ}\text{C}$)	0.0010	0.00018
River water (brackish)	0.0020	0.00035
River water (muddy, silty)	0.0030	0.00053
Hard water	0.0033	0.00059
City/well water	0.0010	0.00018
Untreated boiler feedwater ($> 125^{\circ}\text{F}/50^{\circ}\text{C}$)	0.0010	0.00018
Treated boiler feedwater	0.0010	0.00018
Untreated cooling tower water	0.0020	0.00035
Treated cooling tower water	0.0010	0.00018
Distilled water	0.0005	0.00009
Hydrocarbons		
Fuel oil	0.0050	0.00088
Asphalt and residue	0.0100	0.00176
Vegetable oil and heavy gas oil	0.0030	0.00054
Light hydrocarbons	0.0010	0.00018
Heavy hydrocarbons	0.0040	0.00072
Other		
Quenching liquids	0.0040	0.00070
Refrigerating liquids, brines	0.0010	0.00018
Heat-transfer media	0.0010	0.00018
Polymer forming liquids	0.0050	0.00090
Vaporizing liquids (organic and inorganic)	0.0020	0.00035
Condensing organic liquids	0.0010	0.00018
Organic vapors and liquids (including condensing)	0.0010	0.00018
Gases and Vapors		
Steam (clean)	0.0005	0.00009
Steam (oil-bearing)	0.0010	0.00018
Alcohol vapors	0.0005	0.00009
Industrial air or other dirty (oil-bearing) gases	0.0020	0.00035
Diesel exhaust ($> 125^{\circ}\text{F}/50^{\circ}\text{C}$)	0.0100	0.00176

4.4.1.6 Nucleate Boiling Heat-Transfer Data

Relative Magnitude of Nucleate Boiling Heat-Transfer Coefficients at 1 atm, Referenced to Value for Water

Fluid	$\frac{h}{h_{\text{water}}}$
Water	1.0
Water with 20% sugar	0.87
Water with 10% Na ₂ SO ₄	0.94
Water with 26% glycerin	0.83
Water with 55% glycerin	0.75
Water with 24% NaCl	0.61
Isopropanol	0.70
Methanol	0.53
Toluene	0.36
Carbon-tetrachloride	0.35
<i>n</i> -Butanol	0.32

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Maximum Heat Flux in Nucleate Boiling (Burnout Heat Flux)

Fluid	Surface	Heat Flux $\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2} \times 10^{-3}$	ΔT °F	Heat Flux $\frac{\text{kW}}{\text{m}^2}$	ΔT °C
Water	Copper	200–270		620–850	
	Chrome-plated copper	300–400	42–50	940–1260	23–28
	Steel	410	54	1290	30
Benzene	Copper	43.5	—	130	—
	Aluminum	50.5	—	160	—
Propanol	Nickel-plated copper	67–110	76–90	210–340	42–50
Butanol	Nickel-plated copper	79–105	60–70	250–330	33–39
Ethanol	Aluminum	55	—	170	—
	Copper	80.5	—	250	—
Methanol	Copper	125	—	390	—
	Chrome-plated copper	111	—	350	—
	Steel	125	—	390	—
Liquid H ₂	Any metal surface	9.53	4	30	2
Liquid N ₂	Any metal surface	31.7	20	100	11
Liquid O ₂	Any metal surface	47.5	20	150	11

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4.4.1.7 Solar Radiation Data

Maximum Expected Solar Radiation at Various North Latitudes

Month	$\frac{\text{Btu}}{\text{hr-ft}^2}$						$\frac{\text{W}}{\text{m}^2}$					
	30° North		40° North		45° North		30° North		40° North		45° North	
	24-hr avg.	noon	24-hr avg.	noon	24-hr avg.	noon	24-hr avg.	noon	24-hr avg.	noon	24-hr avg.	noon
January	65	240	40	170	30	135	205	757	126	536	95	426
February	75	270	55	210	45	175	237	852	174	662	142	552
March	90	305	75	255	65	230	284	962	237	804	205	726
April	110	340	95	300	90	280	347	1073	300	946	284	883
May	120	360	120	335	115	320	379	1136	379	1057	363	1009
June	130	365	130	345	130	335	410	1151	410	1088	410	1057
July	130	365	130	350	130	340	410	1151	410	1104	410	1073
August	125	360	125	340	120	325	394	1136	394	1073	379	1025
September	115	350	105	315	100	300	363	1104	331	994	315	946
October	100	315	80	270	75	245	315	994	252	852	237	773
November	80	270	60	215	50	185	252	852	189	678	158	584
December	65	240	45	175	35	140	205	757	142	552	110	442

Source: Langhaar, J.W., "Cooling Pond May Answer Your Water Cooling Problem," Chem.Eng. 60(8), 1953, pages 194–198.

4.4.1.8 Emissivity (ϵ)

Emissivity of Building Materials at Ambient Temperature
(Unless Specified Otherwise)

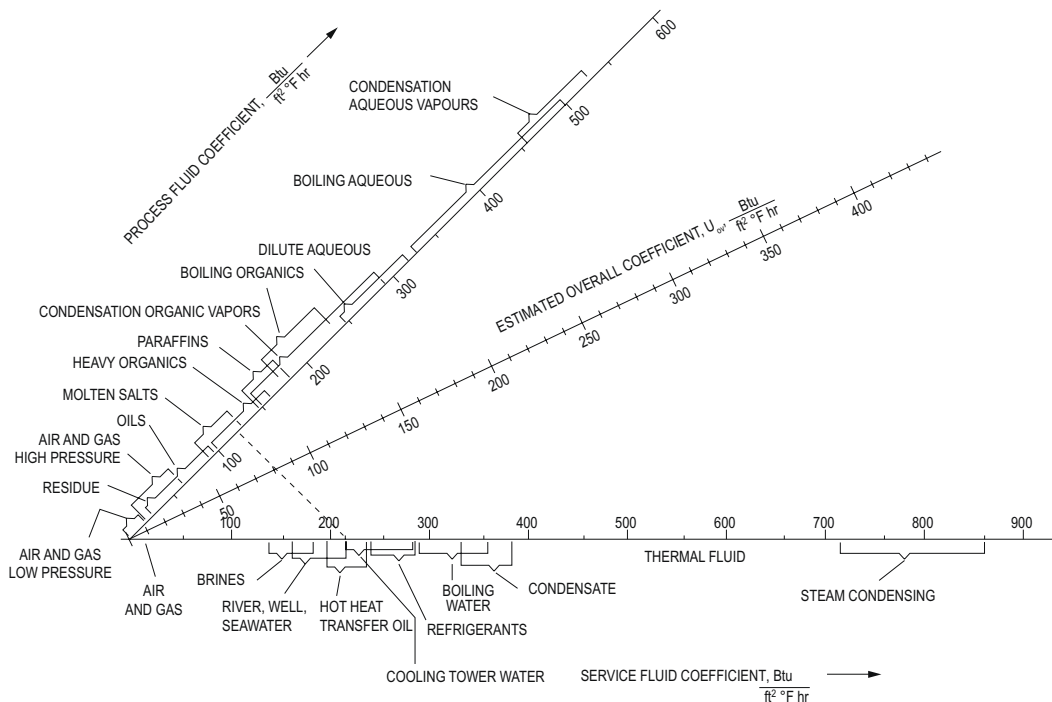
Material	Emissivity
Asbestos	0.96
Brick (building)	0.93
Brick (fireclay) at 2000°F/1100°C	0.75
Enamel (white)	0.90
Glass (smooth)	0.94
Gypsum	0.90
Marble	0.93
Oak	0.90
Oil	0.82
Plaster	0.91
Refractory (good radiator) at 1500°F/800°C	0.85
Refractory (poor radiator) at 1500°F/800°C	0.70
Roofing paper	0.91
Rubber (grey, soft)	0.86
Rubber (hard)	0.95
Water	0.96

Emissivity of Metals at Ambient and Elevated Temperatures

Material	Emissivity at Ambient Temperatures	Emissivity at ~1000°F/540°C
Aluminum, polished	0.04	0.08
Aluminum, anodized	0.94	0.60
Aluminum, surface roofing	0.22	—
Brass, polished	0.10	—
Brass, oxidized	0.61	—
Chromium, polished	0.08	0.26
Copper, polished	0.02	0.18
Copper, oxidized	0.78	0.77
Gold, polished	0.02	0.04
Iron, polished	0.06	0.13
Iron, cast, oxidized	0.63	0.76
Iron, galvanized	0.25	0.6
Iron, oxide	0.90	0.85
Magnesium	0.07	0.18
Stainless steel, polished	0.15	0.22
Stainless steel, weathered	0.85	0.85
Tungsten	0.03	0.10
Zinc, polished	0.05	0.04
Zinc, galvanized	0.25	—

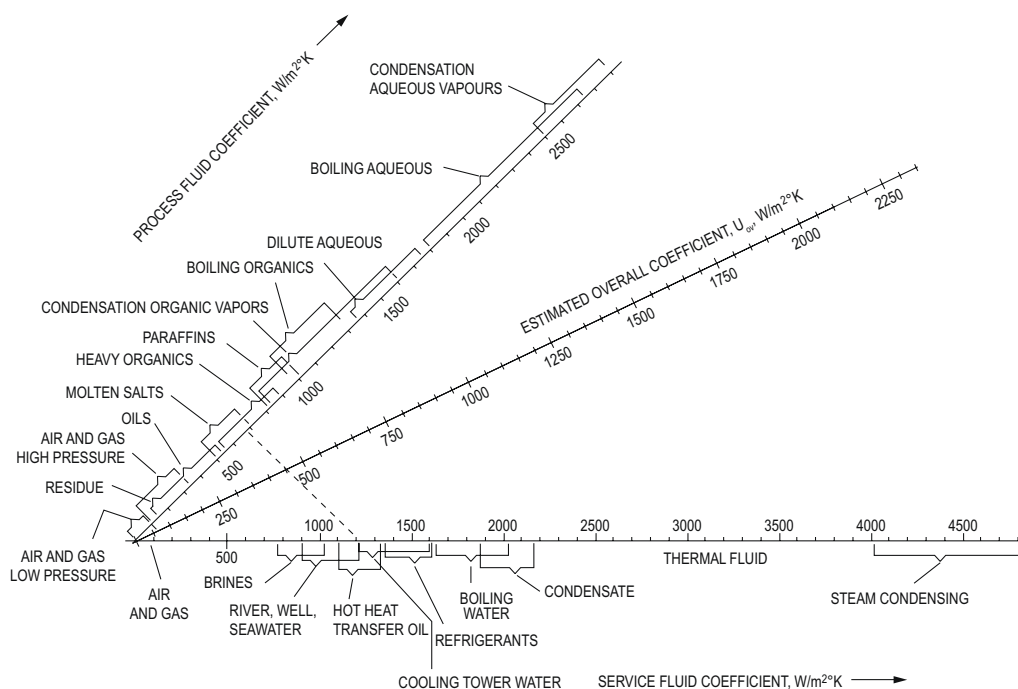
4.4.2 Charts with Heat-Transfer Data

Overall Heat-Transfer Coefficients for Various Applications (U.S. Units): $\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$



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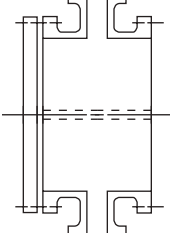
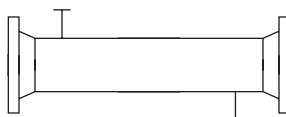
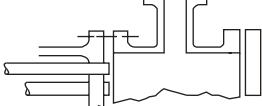
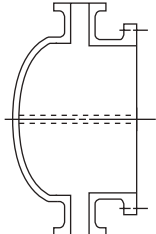
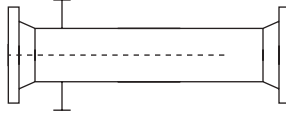
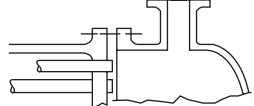
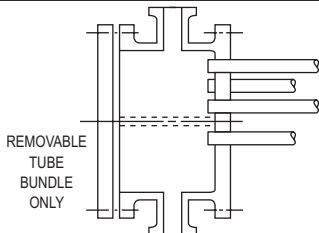
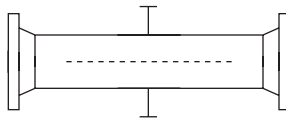
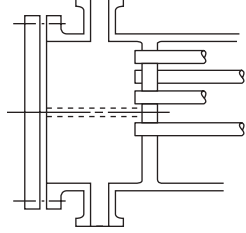
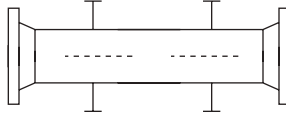
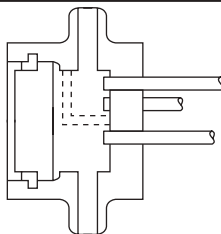
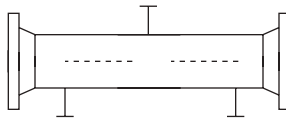
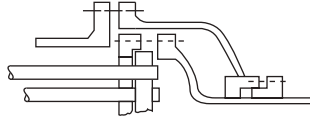
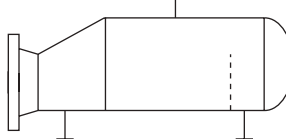
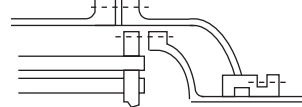
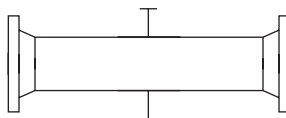
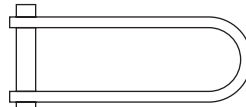
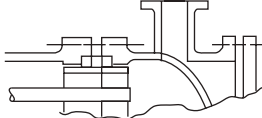
Overall Heat-Transfer Coefficients for Various Applications (SI Units): $\frac{\text{W}}{\text{m}^2 \cdot \text{K}}$



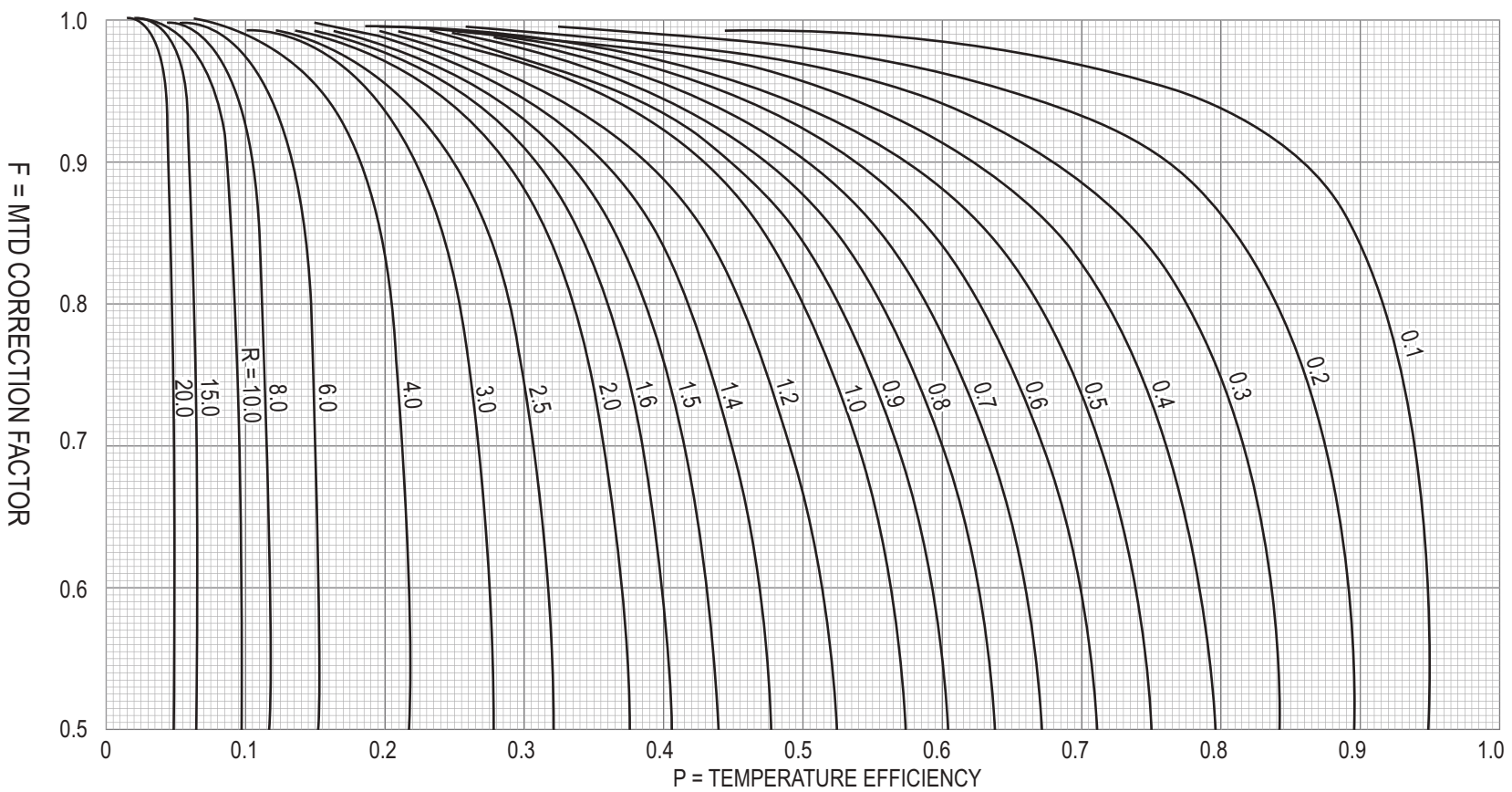
Source: Reprinted from *Chemical Engineering Design*, 2nd ed., Gavin Towler and Ray Sinnott, Chapter 19: Heat Transfer Equipment, © 2013, with permission from Elsevier.

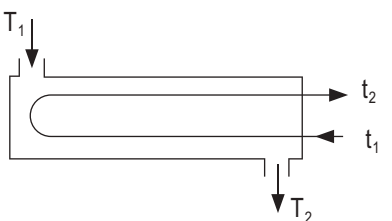
4.4.3 Heat-Exchanger Design Information

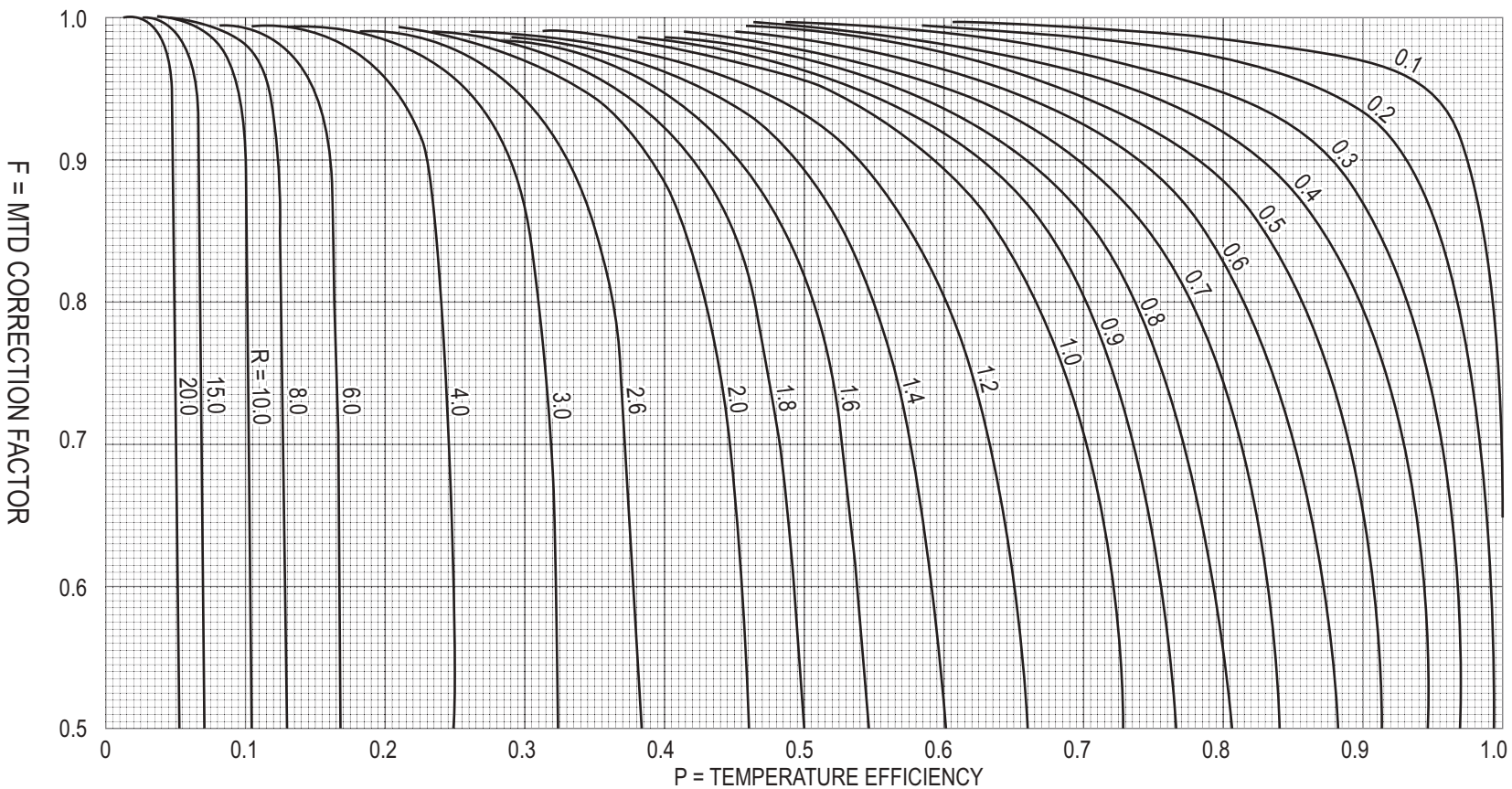
TEMA Heat Exchanger Types

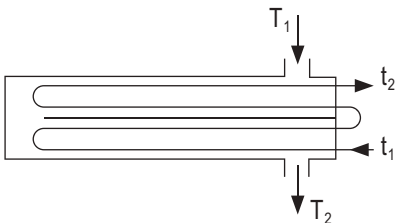
FRONT-END STATIONARY HEAD TYPES		SHELL TYPES		REAR-END HEAD TYPES	
A	 CHANNEL AND REMOVABLE COVER	E	 ONE-PASS SHELL	L	 FIXED TUBE SHEET LIKE "A" STATIONARY HEAD
B	 BONNET (INTEGRAL COVER)	F	 PASS SHELL WITH LONGITUDINAL BAFFLE	M	 FIXED TUBE SHEET LIKE "B" STATIONARY HEAD
C	 REMOVABLE TUBE BUNDLE ONLY CHANNEL INTEGRAL WITH TUBE SHEET AND REMOVABLE COVER	G	 SPLIT FLOW	N	 FIXED TUBE SHEET LIKE "N" STATIONARY HEAD
N	 CHANNEL INTEGRAL WITH TUBE SHEET AND REMOVABLE COVER	H	 DOUBLE SPLIT FLOW	P	 OUTSIDE PACKED FLOATING HEAD
D	 SPECIAL HIGH-PRESSURE CLOSURE	J	 DIVIDED FLOW	S	 FLOATING HEAD WITH BACKING DEVICE
		K	 KETTLE-TYPE REBOILER	T	 PULL-THROUGH FLOATING HEAD
		X	 CROSS FLOW	U	 U-TUBE BUNDLE
				W	 EXTERNALLY SEALED FLOATING TUBE SHEET

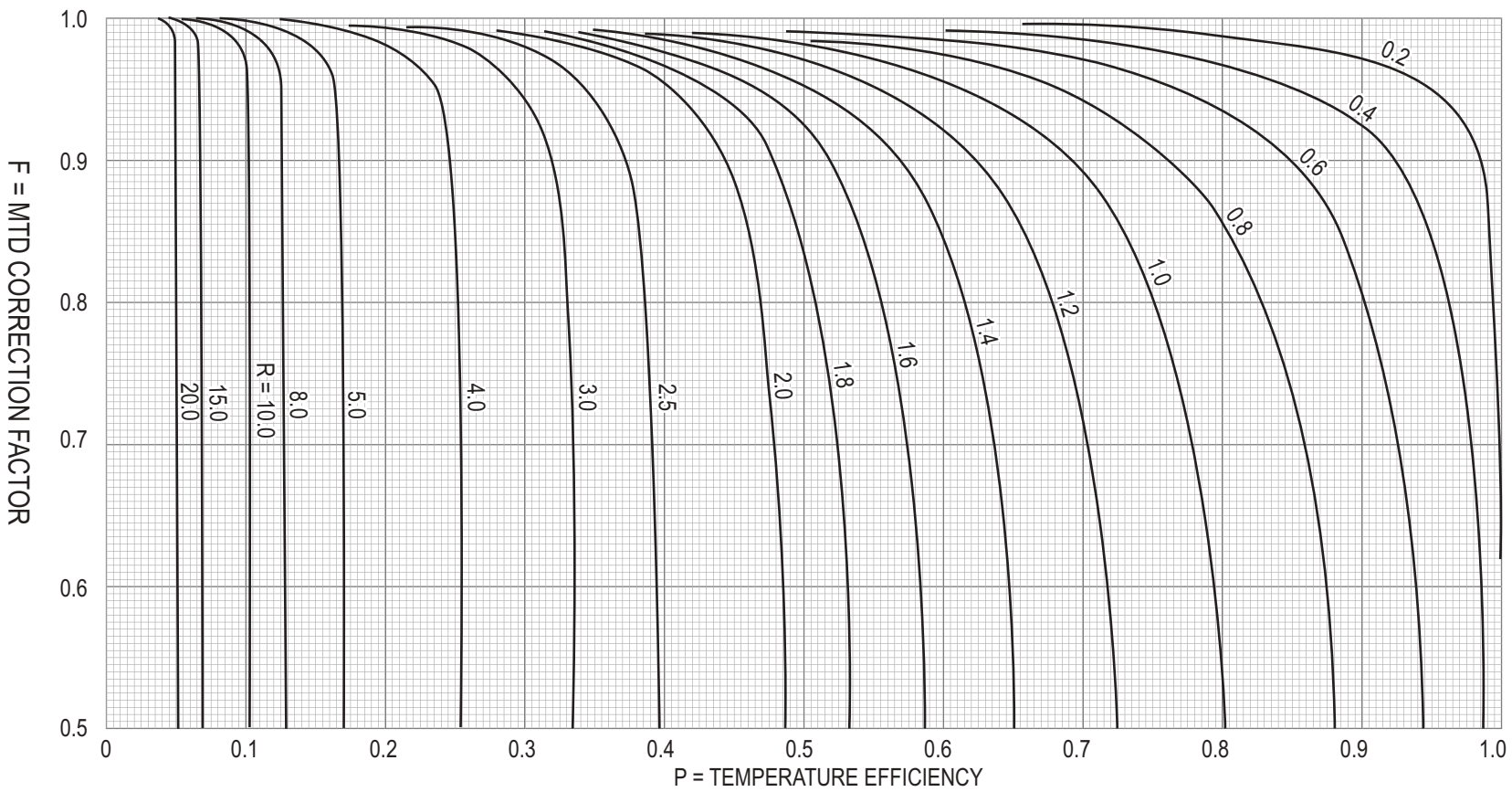
4.4.4 F-Factor Charts

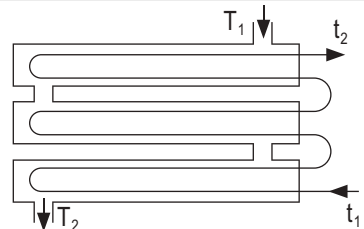


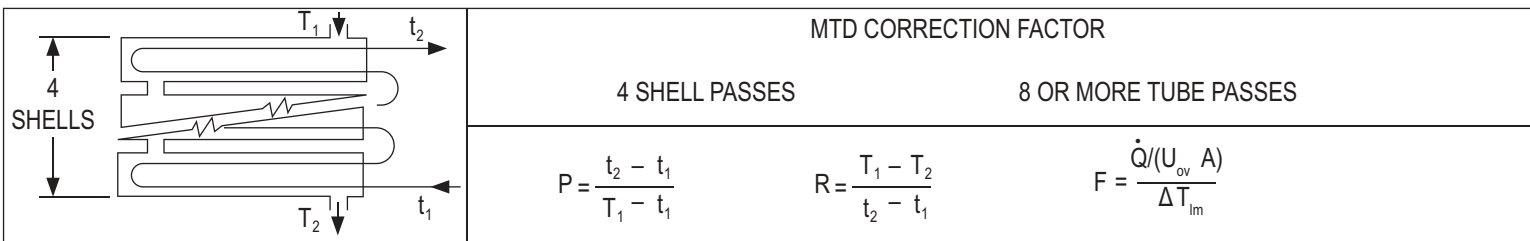
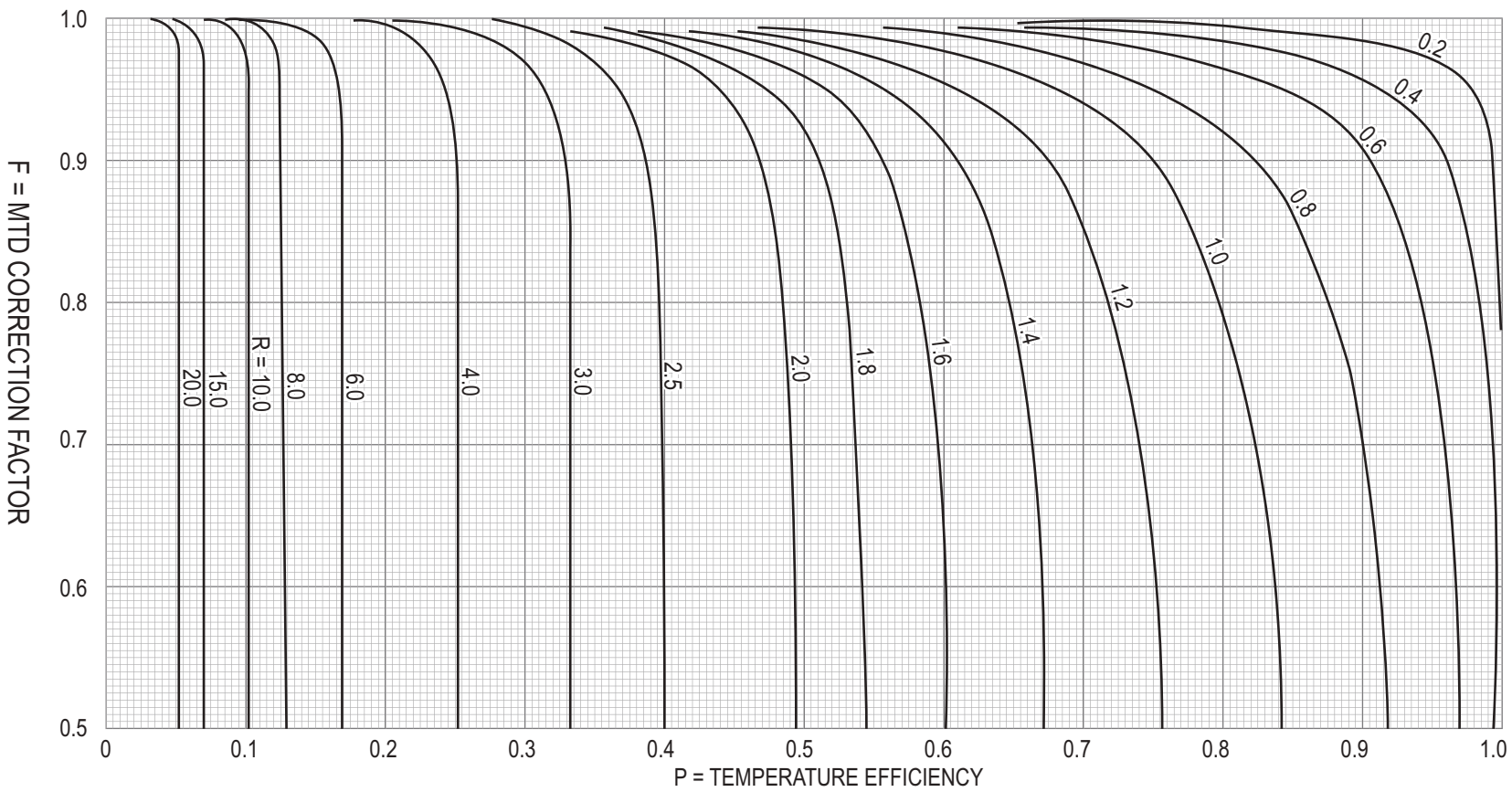
	MTD CORRECTION FACTOR	
	1 SHELL PASS	2 OR MORE TUBE PASSES
	$P = \frac{t_2 - t_1}{T_1 - t_1}$	$R = \frac{T_1 - T_2}{t_2 - t_1}$ $F = \frac{\dot{Q}/(U_{ov} A)}{\Delta T_{lm}}$

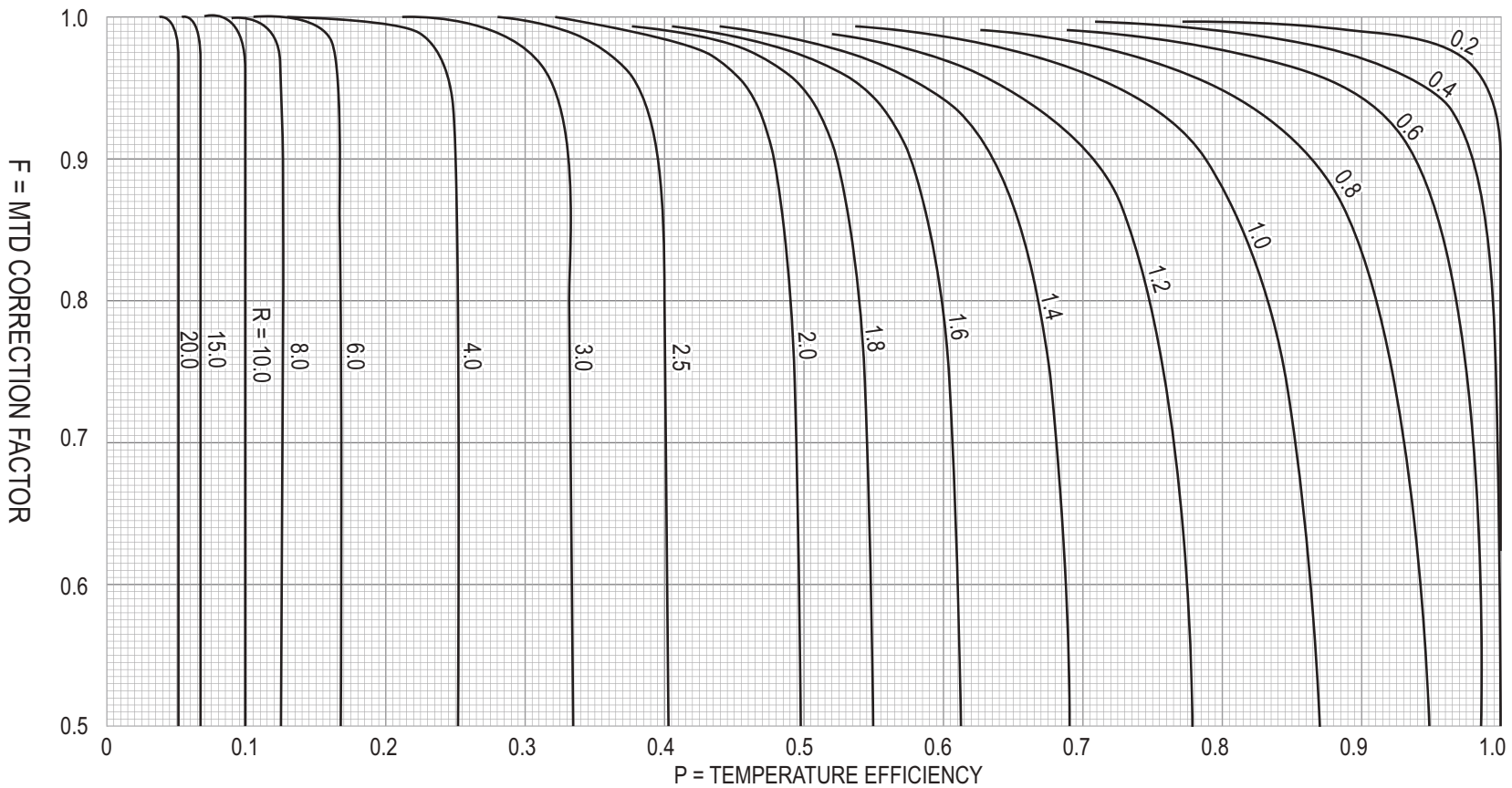


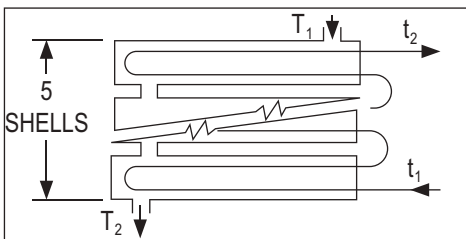
	MTD CORRECTION FACTOR		
	2 SHELL PASS	4 OR MORE TUBE PASSES	
	$P = \frac{t_2 - t_1}{T_1 - t_1}$	$R = \frac{T_1 - T_2}{t_2 - t_1}$	$F = \frac{\dot{Q}/(U_{ov} A)}{\Delta T_{lm}}$

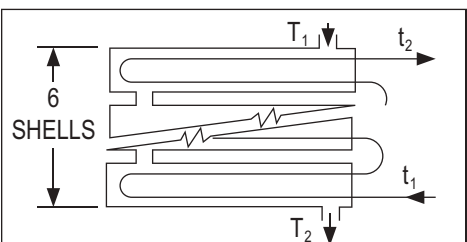
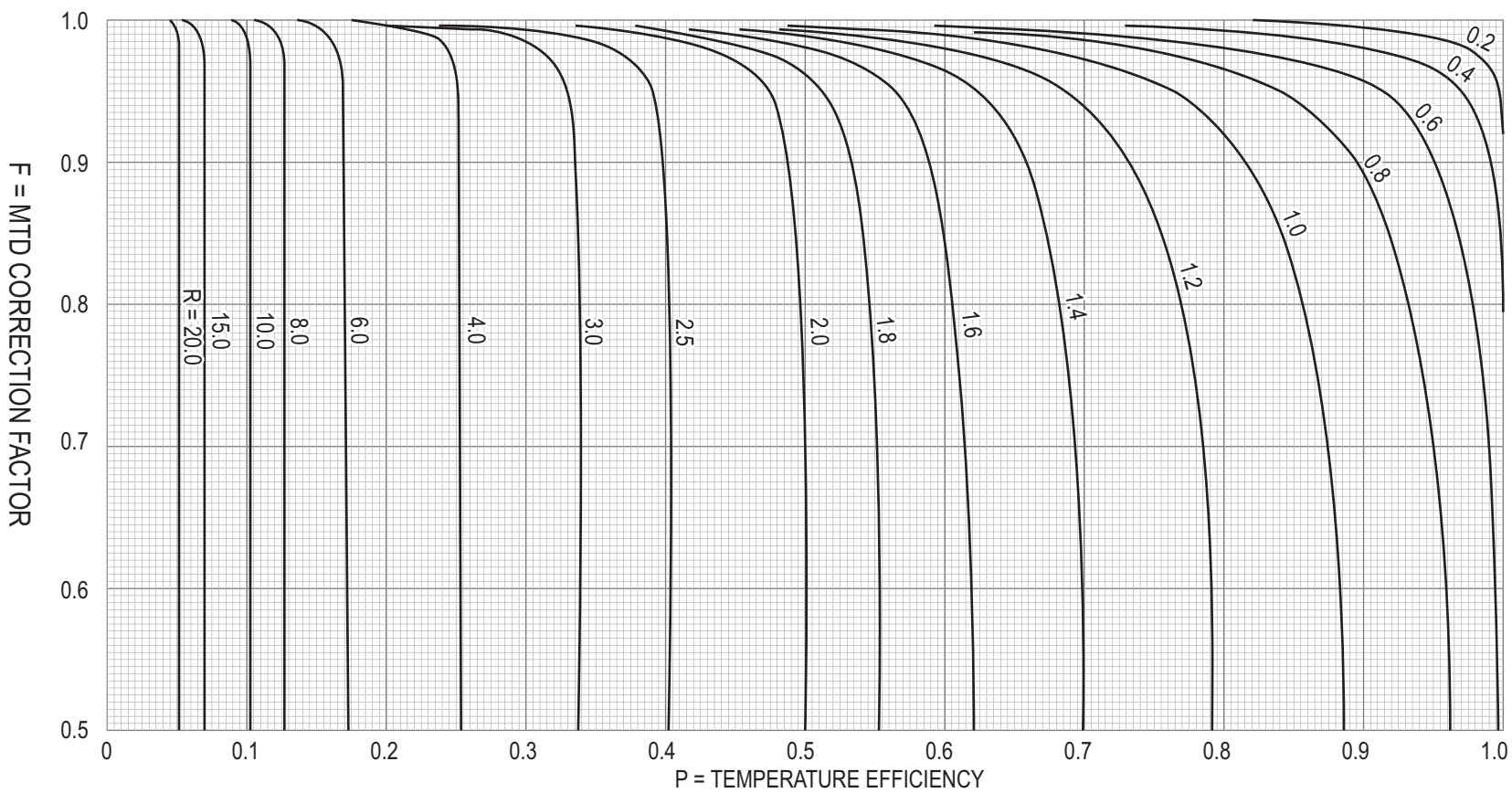


	MTD CORRECTION FACTOR		
	3 SHELL PASSES	6 OR MORE TUBE PASSES	
	$P = \frac{t_2 - t_1}{T_1 - t_1}$	$R = \frac{T_1 - T_2}{t_2 - t_1}$	$F = \frac{\dot{Q}/(U_{ov} A)}{\Delta T_{lm}}$

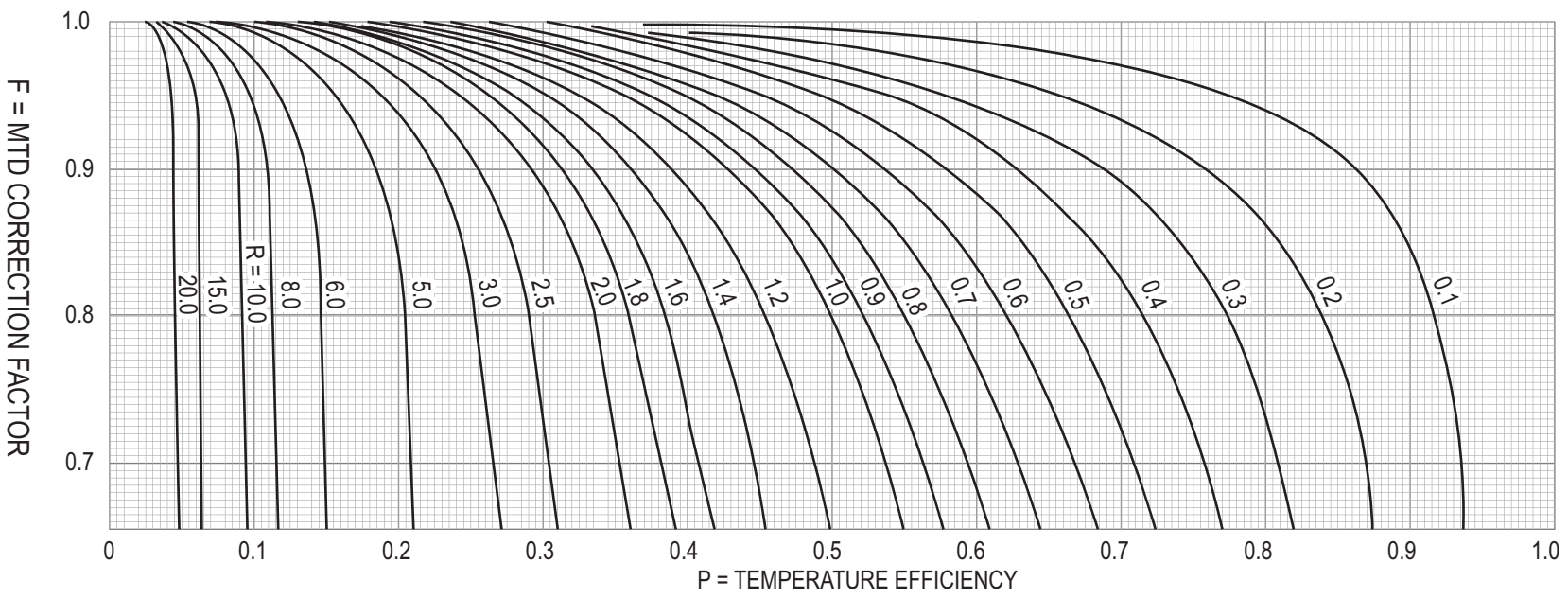


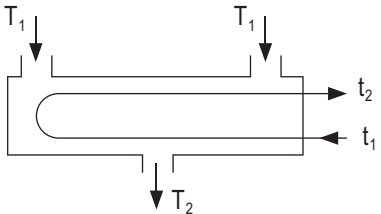


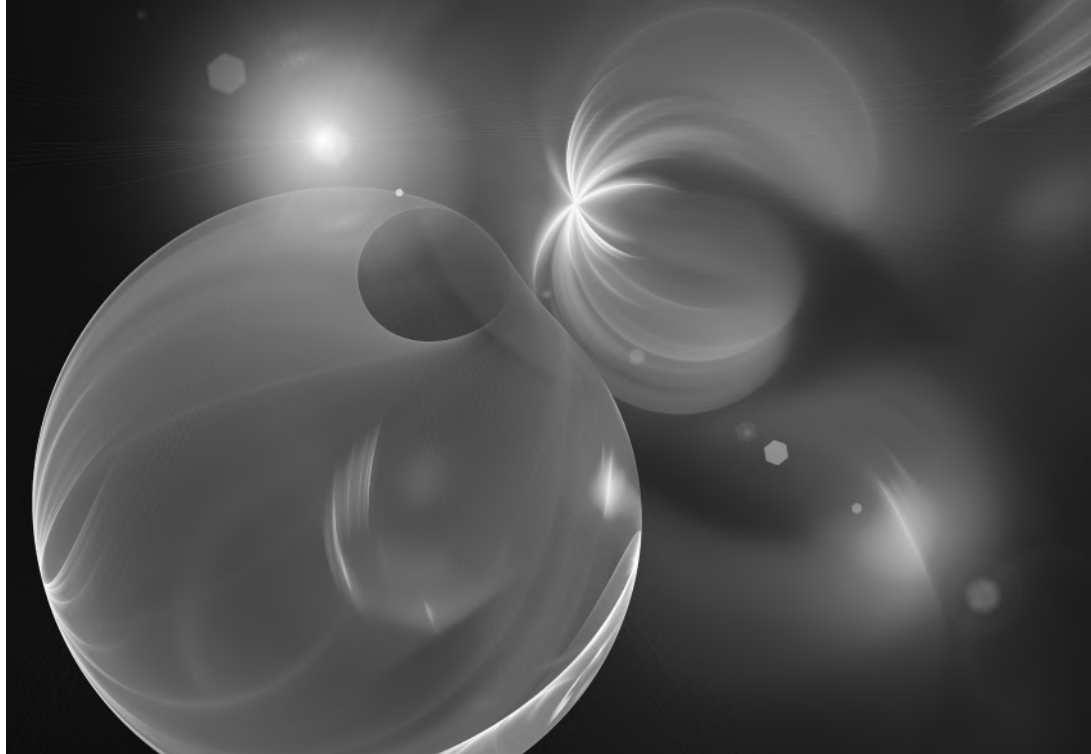
	MTD CORRECTION FACTOR	
	5 SHELL PASSES	10 OR MORE TUBE PASSES
	$P = \frac{t_2 - t_1}{T_1 - t_1}$	$R = \frac{T_1 - T_2}{t_2 - t_1}$
	$F = \frac{\dot{Q}/(U_{ov} A)}{\Delta T_{lm}}$	



MTD CORRECTION FACTOR		
6 SHELL PASSES	12 OR MORE TUBE PASSES	
$P = \frac{t_2 - t_1}{T_1 - t_1}$	$R = \frac{T_1 - T_2}{t_2 - t_1}$	$F = \frac{\dot{Q}/(U_{ov} A)}{\Delta T_{lm}}$



	MTD CORRECTION FACTOR	
	1 DIVIDED FLOW SHELL PASS	2 OR MORE TUBE PASSES
	$P = \frac{t_2 - t_1}{T_1 - t_1}$	$R = \frac{T_1 - T_2}{t_2 - t_1}$ $F = \frac{\dot{Q}/(U_{ov} A)}{\Delta T_{lm}}$



5 CHEMICAL REACTION ENGINEERING

5.1 Symbols and Definitions

Symbols

Symbol	Description	Units (U.S.)	Units (SI)
C_A or $[A]$	Concentration of component A	$\frac{\text{lb mole}}{\text{ft}^3}$	$\frac{\text{mol}}{\text{liter}}$
$\bar{C}_p$	Average heat capacity	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}}$
F_A	Molar feed of A	$\frac{\text{lb mole}}{\text{sec}}$	$\frac{\text{mol}}{\text{s}}$
$\Delta \hat{g}_r$	Gibbs free energy of reaction (molar)	$\frac{\text{Btu}}{\text{lb mole}}$	$\frac{\text{J}}{\text{mol}}$
$\Delta \hat{h}_r$	Heat of reaction	$\frac{\text{Btu}}{\text{lb mole}}$	$\frac{\text{J}}{\text{mol}}$
K	Equilibrium constant	varies	varies
k	Reaction rate constant	$\frac{\left(\frac{\text{lb mole}}{\text{ft}^3}\right)^{(1-n)}}{\text{sec}}$	$\frac{\left(\frac{\text{mol}}{\text{liter}}\right)^{(1-n)}}{\text{s}}$
M	Molar ratio of initial reactant concentrations (weighted by the stoichiometric constants)	dimensionless	
m	Mass of reactor contents	lbm	kg
$\dot{m}$	Mass flow rate of feed	$\frac{\text{lbm}}{\text{sec}}$	$\frac{\text{kg}}{\text{s}}$
n	Moles of reactant or product	lb mole	g mol
n	Reaction order	dimensionless	
P	Pressure (P_A = partial pressure of A)	$\frac{\text{lbf}}{\text{in}^2}$	Pa

Symbols (con't)

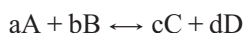
Symbol	Description	Units (U.S.)	Units (SI/metric)
q	Heat transfer	$\frac{\text{Btu}}{\text{sec}}$	$\frac{\text{J}}{\text{s}}$
r_A	Rate of reaction – based on component A	$\frac{\text{lb mole}}{\text{ft}^3\text{-sec}}$	$\frac{\text{g mol}}{\text{L} \cdot \text{s}}$
S_{AB}	Selectivity to A relative to B	dimensionless	
SV	Space velocity = $\frac{1}{\text{space time}}$	$\frac{1}{\text{sec}}$	$\frac{1}{\text{s}}$
T	Temperature	$^{\circ}\text{F or } ^{\circ}\text{R}$	$^{\circ}\text{C or K}$
T_o	Feed temperature	$^{\circ}\text{F or } ^{\circ}\text{R}$	$^{\circ}\text{C or K}$
t	Time	sec	s
θ_A	Fraction of surface covered by adsorbed species A	dimensionless	
V	Reactor volume	ft^3	L
X_A	Fractional conversion of component A	dimensionless	
Y_A	Yield of A relative to reactant use	dimensionless	
ε_A	Fractional volume change at full conversion of A	dimensionless	
τ	Space time = $\frac{1}{\text{space velocity}}$	sec	s

5.2 Fundamentals

5.2.1 Reaction Rate

5.2.1.1 Rate Constant

A chemical reaction may be expressed by the general equation:



The *rate of reaction* of any component is defined as the number of moles of that component formed per unit time per unit volume:

$$-r_A = \frac{1}{V} \frac{dn_A}{dt} \quad (\text{negative because A is consumed})$$

$$-r_A = -\frac{dC_A}{dt} \quad \text{if } V \text{ is constant}$$

The rate of reaction is frequently expressed as

$$-r_A = k f(C_A, C_B, \dots)$$

The fractional conversion X_A is defined as the moles of A reacted per mole of A fed:

$$X_A = \frac{C_{A0} - C_A}{C_{A0}} = 1 - \frac{C_A}{C_{A0}} \quad \text{if } V \text{ is constant}$$

5.2.1.2 Order of Reaction

If $-r_A = k C_A^x C_B^y$, then the reaction is x order with respect to A and y order with respect to B.

The overall order is $n = x + y$.

5.2.1.3 Temperature Dependence (Arrhenius Equations)

The *Arrhenius equation* gives the dependence of k on temperature:

$$k = A e^{\frac{-E_a}{RT}}$$

where

A = pre-exponential or frequency factor

E_a = activation energy $\left(\frac{\text{J}}{\text{mol}} \text{ or } \frac{\text{cal}}{\text{mol}} \right)$

R = universal gas constant

For values of rate constant k_i at two temperatures T_i :

$$E_a = \frac{RT_1 T_2}{(T_1 - T_2)} \ln \left(\frac{k_1}{k_2} \right) \quad \text{or} \quad \ln \left(\frac{k_1}{k_2} \right) = \frac{E_a}{R} \frac{T_1 - T_2}{T_1 T_2}$$

5.2.1.4 Half-Life

The half-life of a reaction, $t_{\frac{1}{2}}$, is the batch time required to reach 50% conversion.

For $-r_A = -\frac{dC_A}{dt} = k C_A^n$ $t_{\frac{1}{2}}$ occurs when $C_A = \frac{1}{2} C_{A0}$

For $n = 1$ (first order) $t_{\frac{1}{2}} = \frac{\ln 2}{k}$

For $n \neq 1$ $t_{\frac{1}{2}} = \frac{2^{n-1} - 1}{(n-1) k C_{A0}^{(n-1)}}$

5.2.2 Rate Equations in Differential Form for Irreversible Reactions

5.2.2.1 Zero-Order ($A \rightarrow R$)

$$-r_A = -\frac{dC_A}{dt} = C_{A0} \frac{dX_A}{dt} = k \quad \text{and} \quad \frac{dX_A}{dt} = \frac{k}{C_{A0}}$$

5.2.2.2 First-Order ($A \rightarrow R$)

$$-r_A = -\frac{dC_A}{dt} = C_{A0} \frac{dX_A}{dt} = k C_A \quad \text{and} \quad \frac{dX_A}{dt} = \frac{k C_A}{C_{A0}} = k(1 - X_A)$$

5.2.2.3 Second-Order ($2A \rightarrow R$), One Reactant

$$-r_A = -\frac{dC_A}{dt} = C_{A0} \frac{dX_A}{dt} = k C_A^2 \quad \text{and} \quad \frac{dX_A}{dt} = \frac{k C_A^2}{C_{A0}} = k C_{A0} (1 - X_A)^2$$

5.2.2.4 Second-Order ($A + bB \rightarrow R$), Two Reactants

$$-r_A = -\frac{dC_A}{dt} = k C_A C_B = k b C_{A0}^2 (1 - X_A)(M - X_A) \quad \text{when } M = \frac{C_{B0}}{b C_{A0}} \neq 1$$

and

$$-r_A = k b C_{A0}^2 (1 - X_A)^2 \quad \text{when } M = 1$$

Integrated forms of these equations for constant- and variable-volume batch, plug flow, and CSTR reactors are included in Integrated Reaction Equations for Irreversible Reactions section in this chapter.

5.2.3 Yield and Selectivity

Yield Y is defined as the molar ratio of the desired product formed to the reactant that is consumed.

Selectivity S is defined as the molar ratio of the desired product to undesired product.

5.2.3.1 Two Irreversible Reactions in Parallel



$$-r_A = -\frac{dC_A}{dt} = k_D C_A^x + k_U C_A^y$$

$$r_D = \frac{dC_D}{dt} = k_D C_A^x$$

$$r_U = \frac{dC_U}{dt} = k_U C_A^y$$

$$Y_D = \text{instantaneous fractional yield of D} = \frac{dC_D}{-dC_A}$$

$$\bar{Y}_D = \text{overall fractional yield of D} = \frac{N_D}{N_{A0} - N_A}$$

where N_A and N_D are the final values measured at the reactor outlet

$$\bar{S}_{DU} = \text{overall selectivity to D over U} = \frac{N_D}{N_U}$$

where N_D and N_U are the final values measured at the reactor outlet

5.2.3.2 Two First-Order Irreversible Reactions in Series



$$-r_A = -\frac{dC_A}{dt} = k_D C_A$$

$$r_D = \frac{dC_D}{dt} = k_D C_A - k_U C_D$$

$$r_U = \frac{dC_U}{dt} = k_U C_D$$

The yield and selectivity definitions for series reactions are identical to those for parallel reactions, and the equations for overall yield and selectivity are the same as those in the previous subsection.

The maximum concentration of D in a plug flow reactor is

$$\frac{C_{D, \max}}{C_{A0}} = \left(\frac{k_D}{k_U} \right)^{\frac{k_U}{k_U - k_D}} \quad \text{at time } \tau_{\max} = \frac{1}{k_{\log \text{ mean}}} = \frac{\ln \left(\frac{k_U}{k_D} \right)}{(k_U - k_D)}$$

The maximum concentration of D in a CSTR is

$$\frac{C_{D, \max}}{C_{A0}} = \frac{1}{\left[\left(\frac{k_U}{k_D} \right)^{\frac{1}{2}} + 1 \right]^2} \quad \text{at time } \tau_{\max} = \frac{1}{\sqrt{k_D k_U}}$$

5.2.4 Pressure Dependence (Gas Phase Reactions)

All of the equations in the previous sections of this chapter can be written in terms of pressure where

$$C_A = \frac{P_A}{RT}$$

5.3 Reactor Equations

5.3.1 Types of Reactors

For flow reactors, space time τ is defined as the reactor volume divided by the inlet volumetric feed rate. Space velocity SV is the reciprocal of space time, that is, $SV = 1/\tau$.

5.3.1.1 Batch Reactor

Constant Volume

For a well-mixed, constant-volume batch reactor:

$$-r_A = -\frac{dC_A}{dt} = C_{A0} \frac{dX_A}{dt} \quad \text{and} \quad t = C_{A0} \int_0^{X_A} \frac{dX_A}{-r_A}$$

Variable Volume

For a well-mixed, variable-volume batch reactor:

$$-r_A = \frac{C_{A0}}{(1 + \varepsilon_A X_A)} \frac{dX_A}{dt} \quad \text{and} \quad t = C_{A0} \int_0^{X_A} \frac{dX_A}{(-r_A)(1 + \varepsilon_A X_A)}$$

where ε_A = fractional volume change at full conversion of A

$$C_A = C_{A0} \left(\frac{1 - X_A}{1 + \varepsilon_A X_A} \right) \quad \text{or} \quad X_A = \frac{1 - C_A/C_{A0}}{1 + \varepsilon_A X_A}$$

5.3.1.2 Plug Flow Reactor

For a plug flow reactor, for all values of ε_A :

$$\tau = \frac{C_{A0} V}{F_{A0}} = C_{A0} \int_0^{X_A} \frac{dX_A}{(-r_A)}$$

where F_{A0} = moles of A fed per unit time

For a constant volume plug flow reactor ($\varepsilon_A = 0$):

$$\tau = - \int_{C_{A0}}^{C_A} \frac{dC_A}{-r_A}$$

5.3.1.3 Continuous Stirred Tank Reactor (CSTR)

For a well-mixed CSTR for all values of ε_A :

$$\tau = \frac{C_{A0}V}{F_{A0}} = \frac{C_{A0}X_A}{(-r_A)}$$

where $-r_A$ is evaluated at exit stream conditions

For a constant volume CSTR ($\varepsilon_A = 0$):

$$\tau = \frac{C_{A0} - C_A}{(-r_A)}$$

5.3.1.4 Equal-Sized Reactors in Series

With a first-order reaction $A \rightarrow R$, with no change in volume:

$$\tau_{N\text{-reactors}} = N \tau_{\text{individual}}$$

$$\tau_{N\text{-reactors}} = \frac{N}{k} \left[\left(\frac{C_{A0}}{C_{AN}} \right)^{\frac{1}{N}} - 1 \right] \quad \text{or} \quad \frac{C_{A0}}{C_{AN}} = \left(1 + \frac{k \tau_N}{N} \right)^N$$

where

N = number of CSTRs (equal volume) in series

C_{AN} = concentration of A leaving the N th CSTR

N plug flow reactors in series gives the same conversion as a single plug flow reactor with the same total volume.

5.3.1.5 Equal-Sized Reactors in Parallel

N identical reactors in parallel give the same conversion as a single reactor of the same total volume. (This is equally true for both plug flow reactors and CSTRs.)

5.3.1.6 Plug Flow Reactors With Recycle

First Order ($\varepsilon_A = 0$)

$$\frac{k \tau}{R+1} = \ln \frac{C_{A0} + R C_A}{(R+1) C_A}$$

Second Order ($\varepsilon_A = 0$)

$$\frac{k C_{A0} \tau}{R+1} = \frac{C_{A0}(C_{A0} - C_A)}{C_A(C_{A0} + R C_A)}$$

where R = recycle ratio, defined as the fraction of the reactor outlet stream that is recycled

Relationship Between Overall Conversion and Single-Pass Conversion

$$X_{As} = \frac{X_{A0}}{1 + R(1 - X_{A0})}$$

5.3.2 Integrated Reactor Equations for Irreversible Reactions

5.3.2.1 Zero-Order Reactions ($A \rightarrow R, -r_A = k$)

Constant Volume

Batch reactor:

$$k t = C_{A0} X_A = C_{A0} - C_A$$

Plug flow reactor or CSTR:

$$k \tau = C_{A0} X_A = C_{A0} - C_A$$

Variable Volume

$$V = V_0(1 + \varepsilon_A X_A), \quad \Delta V = V_0 \varepsilon_A X_A$$

Batch reactor:

$$k t = \frac{C_{A0}}{\varepsilon_A} \ln(1 + \varepsilon_A X_A) = \frac{C_{A0}}{\varepsilon_A} \ln \frac{V}{V_0}$$

Plug flow reactor or CSTR:

$$k \tau = C_{A0} X_A$$

5.3.2.2 First-Order Reactions ($A \rightarrow R, -r_A = k C_A$)

Constant Volume

Batch reactor:

$$k t = \ln \frac{C_{A0}}{C_A} = \ln \frac{1}{1 - X_A} = -\ln(1 - X_A)$$

Plug flow reactor:

$$k \tau = \ln \frac{C_{A0}}{C_A} = \ln \frac{1}{1 - X_A} = -\ln(1 - X_A)$$

CSTR:

$$k \tau = \frac{C_{A0} - C_A}{C_A} = \frac{X_A}{1 - X_A}$$

Variable Volume

$$V = V_0(1 + \varepsilon_A X_A), \quad \Delta V = V_0 \varepsilon_A X_A$$

Batch reactor:

$$k t = \ln \frac{1}{1 - X_A} = -\ln(1 - X_A) = -\ln\left(1 - \frac{\Delta V}{\varepsilon_A V_0}\right)$$

Plug flow reactor:

$$k \tau = -(1 + \varepsilon_A) \ln(1 - X_A) - \varepsilon_A X_A$$

CSTR:

$$k \tau = \frac{X_A(1 + \varepsilon_A X_A)}{1 - X_A}$$

5.3.2.3 Second-Order Reactions ($2A \rightarrow R$, $-r_A = k C_A^2$), One Reactant

Constant Volume

Batch reactor:

$$k t = \frac{1}{C_A} - \frac{1}{C_{A0}} = \frac{X_A}{C_{A0}(1 - X_A)} \quad \text{or} \quad \frac{C_A}{C_{A0}} = \frac{1}{1 + k t C_{A0}}$$

Plug flow reactor:

$$k \tau = \frac{1}{C_A} - \frac{1}{C_{A0}} = \frac{X_A}{C_{A0}(1 - X_A)} \quad \text{or} \quad \frac{C_A}{C_{A0}} = \frac{1}{1 + k \tau C_{A0}}$$

CSTR:

$$k \tau = \frac{C_{A0} - C_A}{C_A^2} = \frac{X_A}{C_{A0}(1 - X_A)^2}$$

Variable Volume

$$V = V_0(1 + \varepsilon_A X_A), \quad \Delta V = V_0 \varepsilon_A X_A$$

Batch reactor:

$$k t = \frac{1}{C_{A0}} \left[\frac{(1 + \varepsilon_A) X_A}{1 - X_A} + \varepsilon_A \ln(1 - X_A) \right]$$

Plug flow reactor:

$$k \tau = \frac{1}{C_{A0}} \left[2\varepsilon_A(1 + \varepsilon_A) \ln(1 - X_A) + \varepsilon_A^2 X_A + (\varepsilon_A + 1)^2 \frac{X_A}{1 - X_A} \right]$$

CSTR:

$$k \tau = \frac{X_A(1 + \varepsilon_A X_A)^2}{C_{A0}(1 - X_A)^2}$$

5.3.2.4 Second-Order Reactions ($A + bB \rightarrow R$, $-r_A = k C_A C_B$), Two Reactants

Constant Volume

Batch reactor:

$$k t b C_{A0}(M - 1) = \ln \frac{C_B}{M C_A} = \ln \frac{M - X_A}{M(1 - X_A)} \quad \text{when } M = \frac{C_{B0}}{b C_{A0}} \neq 1$$

$$k t C_{B0} = k t b C_{A0} = \frac{C_{A0} - C_A}{C_A} = \frac{X_A}{1 - X_A} \quad \text{when } M = 1$$

Plug flow reactor:

$$k t b C_{A0}(M - 1) = \ln \frac{C_B}{M C_A} = \ln \frac{M - X_A}{M(1 - X_A)} \quad \text{when } M = \frac{C_{B0}}{b C_{A0}} \neq 1$$

$$k \tau C_{B0} = k \tau b C_{A0} = \frac{C_{A0} - C_A}{C_A} = \frac{X_A}{1 - X_A} \quad \text{when } M = 1$$

CSTR:

$$k\tau = \frac{C_{A0} - C_A}{bC_A[C_{A0}(M-1) + C_A]} = \frac{X_A}{bC_{A0}(1-X_A)(M-X_A)} \quad \text{when } M = \frac{C_{B0}}{bC_{A0}} \neq 1$$

$$k\tau = \frac{C_{A0} - C_A}{bC_A^2} = \frac{X_A}{bC_{A0}(1-X_A)^2} \quad \text{when } M = 1$$

5.3.3 Complex Reactions

5.3.3.1 First-Order Reversible Reactions ($A \xrightleftharpoons[k_2]{k_1} R$)

$$-r_A = -\frac{dC_A}{dt} = k_1 C_A - k_2 C_R$$

$$K_C = \frac{k_1}{k_2} = \frac{C_{R_{eq}}}{C_{A_{eq}}} \quad \text{and} \quad M = \frac{C_{R0}}{C_{A0}}$$

$$\frac{dX_A}{dt} = \frac{k_1(M+1)}{M+X_{A_{eq}}}(X_{A_{eq}} - X_A)$$

$$-\ln\left(1 - \frac{X_A}{X_{A_{eq}}}\right) = -\ln\frac{C_A - C_{A_{eq}}}{C_{A0} - C_{A_{eq}}} = \frac{(M+1)}{(M+X_{A_{eq}})}k_1 t$$

At equilibrium, when $X_A = X_{A_{eq}}$, then $-\ln(0) \rightarrow \infty$ and $t \rightarrow \infty$.

5.3.3.2 Reactions of Shifting Order

From zero order at high C_A to first order at low C_A :

$$-r_A = \frac{k_1 C_A}{1 + k_2 C_A}$$

$$\ln\left(\frac{C_{A0}}{C_A}\right) + k_2(C_{A0} - C_A) = k_1 t$$

$$\frac{\ln\left(\frac{C_{A0}}{C_A}\right)}{C_{A0} - C_A} = -k_2 + \frac{k_1 t}{C_{A0} - C_A}$$

where

$$\frac{k_1}{k_2} = \text{zero-order rate constant}$$

$$k_1 = \text{first-order rate constant}$$

This form of the rate equation is used for elementary enzyme-catalyzed reactions and for elementary surface-catalyzed reactions in batch reactor. For plug flow reactor replace time, t , with space time, τ . The equation assumes a constant density system.

5.4 Catalytic Reactors

Source: Missen, Ronald W., Charles A. Mims, and Bradley A. Saville, *Introduction to Chemical Reaction Engineering and Kinetics*, New York: John Wiley & Sons, Inc., 1999, pp. 191–192.

5.4.1 Key Assumptions

- Catalyst surface contains a fixed number of sites.
- All the catalytic sites are identical.
- Reactivities of the sites depend only on temperature. They do not depend on the nature or amounts of other materials present on the surface during the reaction.

5.4.2 Surface Reaction Steps

1. Unimolecular surface reaction: $A \cdot s \rightarrow B \cdot s$

$A \cdot S$ is a surface-bound species involving A and site S .

Rate is given by: $(-r_A) = k\theta_A$

2. Bimolecular surface reaction: $A \cdot s + B \cdot s \rightarrow C \cdot s + s$

Rate: $(-r_A) = k\theta_A \theta_B$

3. Eley-Rideal reaction: $A \cdot s + B \rightarrow C + s$

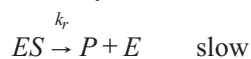
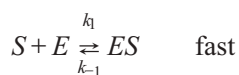
B is a gas-phase species that reacts directly with an adsorbed intermediate.

Rate: $(-r_A) = k\theta_A C_B$

C_B is the gas-phase concentration of B .

5.4.3 Enzyme Kinetics: Michaelis-Menten

Source: Missen, Ronald W., Charles A. Mims, and Bradley A. Saville, *Introduction to Chemical Reaction Engineering and Kinetics*, New York: John Wiley & Sons, Inc., 1999.



where

S = substrate

E = enzyme

ES = enzyme-substrate complex

P = product

5.4.3.1 Michaelis-Menten Model

Material balance on total enzyme: $C_E + C_{ES} = C_{E0}$

Concentration of complex: $C_{ES} = \frac{k_1 C_S C_E}{k_{-1}} = \frac{k_1 C_S C_{E0}}{k_1 C_S + k_{-1}} = \frac{C_S C_{E0}}{\frac{k_{-1}}{k_1} + C_S}$

Define Michaelis constant: $K_M = \frac{k_{-1}}{k_1}$

Rate of production of product P : $r_P = k_r C_{ES} = \frac{k_r C_{E0} C_S}{K_M + C_S}$

Initial rate: $r_{P0} = (-r_{S0}) = \frac{k_r C_{E0} C_{S0}}{K_M + C_{S0}}$

Limiting Cases

Low C_{S0}	$C_{S0} \ll K_M$	$r_{P0} = (-r_{S0}) = \frac{k_r C_{E0} C_{S0}}{K_M}$
High C_{S0}	$C_{S0} \gg K_M$	$r_{P0, \max} = k_r C_{E0}$ maximum rate
Intermediate	$C_{S0} = K_M$	$r_{P0} = \frac{1}{2} k_r C_{E0} = \frac{1}{2} r_{P0, \max}$

Michaelis-Menten Equation

Standard form: $r_P = \frac{r_{P0, \max} C_S}{K_M + C_S}$

Initial rate: $r_{P0} = \frac{r_{P0, \max} C_{S0}}{K_M + C_{S0}}$

5.4.3.2 Estimation of K_M and $V_{\max}$

Linearized Form

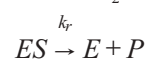
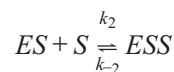
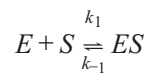
Lineweaver-Burk Plot $\frac{1}{r_{P0}} = \frac{1}{r_{P0, \max}} + \frac{K_M}{r_{P0, \max}} \frac{1}{C_{S0}}$ Intercept = $\frac{1}{r_{P0, \max}}$ Slope = $\frac{K_M}{r_{P0, \max}}$

Linearized Form of Integrated Michaelis-Menten Equation

Constant-volume batch reactor:

$$\frac{\ln\left(\frac{C_S}{C_{S0}}\right)}{C_{S0} - C_S} = \frac{1}{K_M} - \frac{r_{P0, \max}}{K_M} \frac{t}{C_{S0} - C_S}$$

5.4.3.3 Single-Substrate Inhibition



Rate Law $r_P = \frac{k_r C_{E0} C_S}{K_M + C_S + \frac{C_S^2}{K_2}} = \frac{r_{P0, \max} C_S}{K_M + C_S + \frac{C_S^2}{K_2}}$ $K_2 = \frac{k_{-2}}{k_2}$

Inhibition occurs due to the term $\frac{C_S^2}{K_2}$ in the denominator.

Maximum Rate

Occurs at $C_S = (K_M K_2)^{\frac{1}{2}}$

$$r_{P0, \max, \text{apparent}} = \frac{k_r C_{E0}}{1 + 2\left(\frac{K_M}{K_2}\right)^{\frac{1}{2}}} = \frac{r_{P0, \max}}{1 + 2\left(\frac{K_M}{K_2}\right)^{\frac{1}{2}}}$$

The maximum rate from the inhibited reaction is lower than $r_{P0, \max}$ for the uninhibited reaction.

5.5 Heat Effects in Reactors

The reactor design equations in the previous sections assume isothermal operation. For non-isothermal operation, both material and energy balance equations are required.

5.5.1 Batch Reactor

Energy Balance

$$\overline{m} c_p \frac{dT}{dt} = V(-\Delta \hat{h}_r)(r_A) + q$$

endothermic reactions q is positive

exothermic reactions q is negative

adiabatic conditions q is zero

5.5.2 Plug-Flow Reactor

$$\dot{m} c_p \frac{dT}{dt} = V(-\Delta \hat{h}_r)(r_A) + q$$

endothermic reactions q is positive

exothermic reactions q is negative

adiabatic conditions q is zero

5.5.3 Continuous Stirred Tank Reactor

$$\dot{m} c_p (T_o - T) = V(-\Delta \hat{h}_r)(r_A) + q$$

endothermic reactions q is positive

exothermic reactions q is negative

adiabatic conditions q is zero



6 FLUID MECHANICS

6.1 Symbols and Definitions

Symbols

Symbol	Description	Units (U.S.)	Units (SI)
A	Area	ft ²	m ²
Ar	Archimedes diameter	dimensionless	
C	Fitting characteristic	dimensionless	
C_D	Drag coefficient	dimensionless	
C_v	Valve flow coefficient	dimensionless	
c_p	Specific heat (constant pressure)	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}}$
c_v	Specific heat (constant volume)	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}}$
D	Diameter	ft <i>or</i> in.	m
D_H	Hydraulic diameter	ft <i>or</i> in.	m
d	Diameter (minor)	ft <i>or</i> in.	m
$\mathbf{F}$	Force	lbf	N
f	Friction factor (Darcy-Weisbach)	dimensionless	
f_{Fanning}	Fanning friction factor	dimensionless	
H	Total head	ft	m
h	Height	ft	m
h_f	Head loss	ft	m
$h_{f, \text{fitting}}$	Head loss in fitting	ft	m
h_L	Head loss (general)	ft	m
K	Loss coefficient	dimensionless	
KE	Kinetic energy	Btu	J

Symbols (con't)

Symbol	Description	Units (U.S.)	Units (SI)
k	Ratios of specific heats (c_p/c_v)	dimensionless	
L	Length or thickness	ft <i>or</i> in.	m
MW	Molecular weight	$\frac{\text{lb}}{\text{lb mole}}$	$\frac{\text{kg}}{\text{kmol}}$
Ma	Mach number	dimensionless	
m	Mass	lbm	kg
m	Apparent Viscosity		$\text{Pa} \cdot \text{s}^n$
$\dot{m}$	Mass flow rate	$\frac{\text{lbm}}{\text{hr}}$	$\frac{\text{kg}}{\text{s}}$
n	Specific exponent	dimensionless	
N_s	Specific speed	rpm	rpm
$NPSH_a$	Net positive suction head available	ft	m
$NPSH_r$	Net positive suction head required	ft	m
P	Pressure	$\frac{\text{lbf}}{\text{ft}^2}$ <i>or</i> psi	Pa
P	Wetted perimeter	ft	m
PE	Potential energy	Btu	J
P_{vap}	Vapor pressure	psi	Pa
R	Radius	ft <i>or</i> in.	m
R	Universal gas constant	$\frac{\text{Btu}}{\text{lb mole} \cdot ^\circ\text{R}}$ <i>or</i> $\frac{\text{psi} \cdot \text{ft}^3}{\text{lb mole} \cdot ^\circ\text{R}}$	$\frac{\text{J}}{\text{mol} \cdot \text{K}}$
Re	Reynolds number	dimensionless	
r	Radius	ft <i>or</i> in.	m
S	Rotational speed	rpm	rpm
SG	Specific gravity	dimensionless	
T	Temperature	$^\circ\text{F}$ <i>or</i> $^\circ\text{R}$	$^\circ\text{C}$ <i>or</i> K
t	Time	hr <i>or</i> min <i>or</i> sec	s
u	Velocity	$\frac{\text{ft}}{\text{sec}}$	$\frac{\text{m}}{\text{s}}$
V	Volume	ft^3	m^3
$\dot{V}$	Volumetric flow rate	$\frac{\text{ft}^3}{\text{sec}}$	$\frac{\text{m}^3}{\text{s}}$
W	Work	ft-lbf	$\text{N} \cdot \text{m}$
$\dot{W}$	Power	hp	W
X	Distance	ft <i>or</i> in.	m
x	Length, distance, or position	ft <i>or</i> in.	m
y	Length	ft <i>or</i> in.	m
Y	Expansion factor	dimensionless	
z	Length or elevation difference	ft <i>or</i> in.	m
α	Angle	radian	radian

Symbols (con't)

Symbol	Description	Units (U.S.)	Units (SI)
β	Ratio of small to large diameter	dimensionless	
γ	Surface tension	$\frac{\text{lbf}}{\text{ft}}$	$\frac{\text{N}}{\text{m}} = \frac{\text{kg}}{\text{s}^2}$
δ	Thickness of a film	ft	m
ε	Absolute roughness	ft	m
ϵ	Porosity, void fraction, or volume fraction ($0 < \epsilon < 1$)	dimensionless	
η	Efficiency	dimensionless	
η	Fluid viscosity	$\frac{\text{lbm}}{\text{ft} \cdot \text{sec}}$	$\frac{\text{N} \cdot \text{s}}{\text{m}^2}$
θ	Angle	radian	radian
μ	Dynamic viscosity	cP or $\frac{\text{lbm}}{\text{ft} \cdot \text{sec}}$	Pa $\cdot$ s or $\frac{\text{kg}}{\text{s} \cdot \text{m}}$
μ_{∞}	Infinite, plastic, or high shear viscosity	cP or $\frac{\text{lbm}}{\text{ft} \cdot \text{sec}}$	Pa $\cdot$ s or $\frac{\text{kg}}{\text{s} \cdot \text{m}}$
ν	Kinematic viscosity	$\frac{\text{ft}^2}{\text{hr}}$	$\frac{\text{m}^2}{\text{s}}$
ρ	Density	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{kg}}{\text{m}^3}$
ρ_f	Density of fluid	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{kg}}{\text{m}^3}$
ρ_p	Density of particles	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{kg}}{\text{m}^3}$
τ	Stress	$\frac{\text{lbf}}{\text{ft}^2}$	Pa
τ_t	Shear stress	$\frac{\text{lbf}}{\text{ft}^2}$	Pa
τ_0	Yield stress of fluid	$\frac{\text{lbf}}{\text{ft}^2}$	Pa
Φ	Sphericity of particle ($0 < \Phi \leq 1$, where $\Phi = 1$ is a perfect sphere)	dimensionless	

Physical Constants

Symbol	Value	Units	Description
g	32.174	$\frac{\text{ft}}{\text{sec}^2}$	Gravitational acceleration (Earth)
	9.8067	$\frac{\text{m}}{\text{s}^2}$	
g_c	32.174	$\frac{\text{lbf} \cdot \text{ft}}{\text{lbf} \cdot \text{sec}^2}$	Gravitational conversion factor
	1	$\frac{\text{kg} \cdot \text{m}}{\text{N} \cdot \text{s}^2} = 1$	

6.2 Fundamentals of Fluid Mechanics

6.2.1 Mechanical Energy Balance

6.2.1.1 Conservation of Mass

Conservation of mass for flow from Point 1 to Point 2 is

$$\dot{m}_1 = \dot{m}_2$$

The continuity equation is

$$\rho_1 A_1 u_1 = \rho_2 A_2 u_2$$

For an incompressible fluid, $\rho_1 = \rho_2$, therefore:

$$A_1 u_1 = A_2 u_2 \quad \text{and} \quad \dot{V}_1 = \dot{V}_2$$

6.2.1.2 The Bernoulli Equation

The Bernoulli equation states, in energy per unit mass $\frac{\text{ft} \cdot \text{lbf}}{32.2 \text{ lbm}} = \frac{\text{ft}^2}{\text{sec}^2}$ or $\frac{\text{N} \cdot \text{m}}{\text{kg}} = \frac{\text{m}^2}{\text{s}^2}$,

$$\frac{P g_c}{\rho} + \frac{u^2}{2} + g z = \text{constant}$$

For one-dimensional flows (with uniform velocity profiles) through conduits with flow from Point 1 to Point 2, expressed in:

Energy Per Unit Mass (Energy Basis)

$$\frac{P_1 g_c}{\rho} + \frac{u_1^2}{2} + g z_1 + g_c w_{\text{in}} = \frac{P_2 g_c}{\rho} + \frac{u_2^2}{2} + g z_2 + \text{loss}$$

where

w_{in} = net shaft work in = power/mass flow rate

Energy Per Unit Volume (Pressure Basis)

$$P_1 + \frac{u_1^2 \rho}{2 g_c} + \frac{\rho g z_1}{g_c} + \rho w_{\text{in}} = P_2 + \frac{u_2^2 \rho}{2 g_c} + \frac{\rho g z_2}{g_c} + \rho(\text{loss})$$

Height of Fluid (Head Basis)

$$\frac{P_1 g_c}{\rho g} + \frac{u_1^2}{2g} + z_1 + h_s = \frac{P_2 g_c}{\rho g} + \frac{u_2^2}{2g} + z_2 + h_L$$

where

h_s = shaft work head

h_L = head loss

6.2.1.3 Energy Line and Hydraulic Grade Line

Energy Line (or Energy Grade Line)

The *energy line (EL)* represents the total head available to a fluid and can be expressed as:

For inviscid incompressible flow:

$$EL = \frac{P g_c}{\rho g} + \frac{u^2}{2g} + z = \text{constant along a streamline}$$

For incompressible flow with losses:

$$EL = \frac{P g_c}{\rho g} + \frac{u^2}{2g} + z - h_L$$

Hydraulic Grade Line (or Hydraulic Gradient Line)

The *hydraulic grade line* (HGL) represents the total head available to a fluid, minus the velocity head, and can be expressed as:

For inviscid incompressible flow:

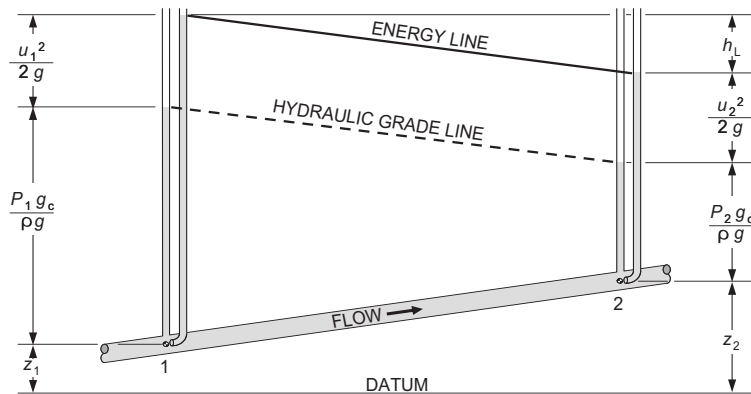
$$HGL = \frac{P g_c}{\rho g} + z$$

For incompressible flow with losses:

$$HGL = \frac{P g_c}{\rho g} + z - h_L$$

Note: The energy or hydraulic grade lines do not represent "sources" or "sinks" of energy such as the effects of pumps or turbines.

Energy Line and Hydraulic Grade Line for Incompressible Fluid Between Two Points (With Losses)



6.2.1.4 The Impulse-Momentum Principle

The resultant force in a given direction acting on a fluid equals the rate of change of momentum of the fluid, where

$$\sum F = \sum \dot{V}_2 \rho_2 u_2 - \sum \dot{V}_1 \rho_1 u_1$$

$$\sum F = \text{result of all external forces acting on the control volume}$$

$$\sum \dot{V}_1 \rho_1 u_1 = \text{rate of momentum of the fluid flow entering the control volume in the same direction as the force}$$

$$\sum \dot{V}_2 \rho_2 u_2 = \text{rate of momentum of the fluid flow leaving the control volume in the same direction as the force}$$

6.2.2 Viscosity and Fluid Properties

6.2.2.1 Hydrostatic Head, Stress, Pressure, and Viscosity

Definitions:

Hydrostatic head is

$$P = \frac{\rho gh}{g_c}$$

Stress is

$$\tau = \lim_{(\Delta A \rightarrow 0)} \frac{\Delta F}{\Delta A}$$

where τ = surface stress at a point

Pressure is

$$P = -\tau_n$$

where τ_n = stress normal at a point

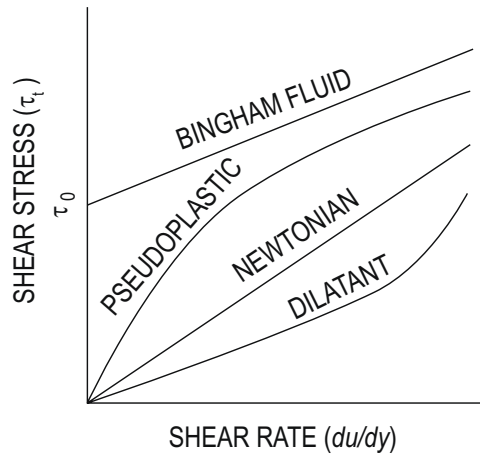
Newton's Law of Viscosity relates shear stress (τ_t = stress tangential to the boundary) to the velocity gradient or shear rate (du/dy), using a constant of proportionality known as the *dynamic (absolute) viscosity* (μ) of the fluid:

$$\tau_t = \mu \frac{du}{dy}$$

Kinematic viscosity is

$$\nu = \frac{\mu}{\rho}$$

6.2.2.2 Fluid Types and Characteristics



Classifications of Fluids

Fluid Classification	Fluid Type	Behavior	Examples
Time-Independent Viscosity	Newtonian	Viscosity is constant. $\tau_t = \mu \frac{du}{dy}$ The term μ is reserved for Newtonian fluids.	Water, light oil, blood plasma
	Pseudoplastic (shear thinning)	Apparent viscosity (m) decreases with increased shear stress. $\tau_t = m \left(\frac{du}{dy} \right)^n$ n = power law index, $n < 1$ m is also known as the <i>consistency coefficient</i> or <i>consistency index</i>	Molasses, latex paint, whole blood
	Dilatant (shear thickening)	Apparent viscosity (m) increases with increased shear stress. $\tau_t = m \left(\frac{du}{dy} \right)^n$ n = power law index, $n > 1$	Corn starch suspensions
Time-Dependent Viscosity	Thixotropic	Apparent viscosity (m) decreases with duration of stress.	Yogurt, plastisols
	Rheopectic	Apparent viscosity (m) increases with duration of stress.	Gypsum paste, kaolin clay suspensions
Viscoplastic	Bingham fluid	Behaves as a rigid body until a minimum stress (yield stress) is applied, then reacts as a Newtonian fluid at shear stresses above the yield stress. $\tau_t = \tau_0 + \eta \frac{du}{dy}$ η = fluid viscosity τ_0 = yield stress	Mayonnaise, river mud, slurries
Viscoelastic	Kelvin material Maxwell material	The materials exhibit both viscous and elastic characteristics during deformation under stress.	Silicone putty

6.2.2.3 Surface Tension and Capillary Rise

Surface tension γ is the force per unit contact length

$$\gamma = \frac{F}{L}$$

where

F = surface force at the interface

L = length of interface

The *capillary rise*, h , is approximated by

$$h = \left(\frac{4\gamma g_c \cos \alpha}{\rho g d} \right)$$

where

h = height of the liquid in the vertical tube

α = angle made by the liquid with the wetted tube wall

d = the diameter of the capillary tube

6.2.3 Velocity

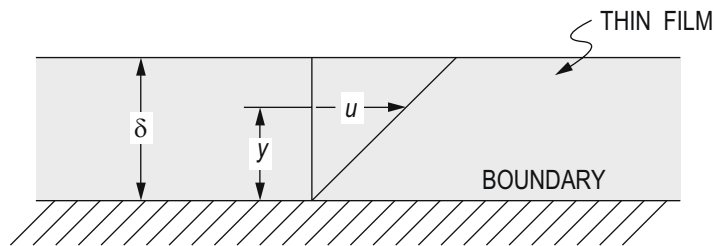
Velocity is defined as the rate of change of position with respect to time

$$u = \frac{dx}{dt}$$

where x = position

Velocity of a Newtonian fluid in a thin film is

$$u(y) = u \frac{y}{\delta} \quad \frac{du}{dy} = \frac{u}{\delta}$$



The velocity distribution for *laminar flow* in circular tubes or between planes is

$$u(r) = u_{\max} \left[1 - \left(\frac{r}{R} \right)^2 \right]$$

where r = distance from the centerline

R = radius of the tube or half the distance between the parallel planes

u = local velocity at r

$u_{\max}$ = velocity at the centerline of the duct

$\bar{u}$ = average velocity in the duct

	Flow Conditions		
	Fully turbulent flow	Circular tubes in laminar flow	Parallel planes in laminar flow
$\frac{u_{\max}}{\bar{u}} =$	1.18	2	1.5

The *shear stress distribution* is

$$\frac{\tau}{\tau_w} = \frac{r}{R}$$

where τ and τ_w = shear stresses at radii r and R , respectively

6.2.4 Reynolds Number

Dimensionless number describing flow behavior with the general definition:

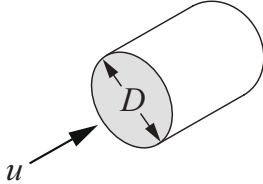
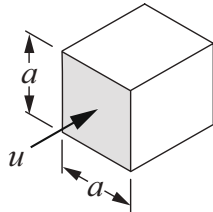
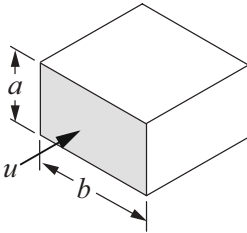
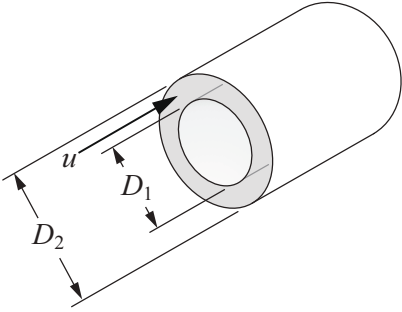
$$Re = \frac{\text{inertial forces}}{\text{viscous forces}}$$

6.2.4.1 Hydraulic Diameter

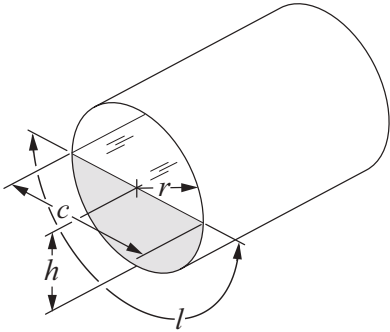
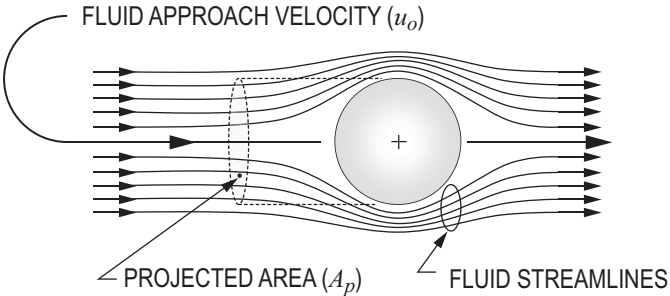
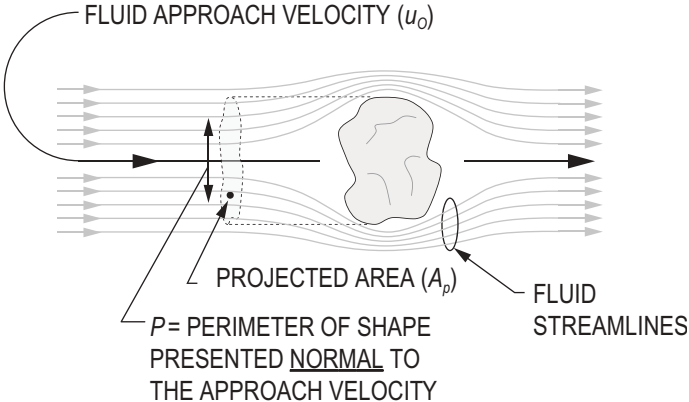
D_H = hydraulic diameter (also known as the *characteristic length*)

$$D_H = 4 \times \frac{\text{cross sectional area}}{\text{wetted perimeter}} = \frac{4A}{P}$$

Hydraulic Diameters for Various Flow Configurations

Flow Configuration	Diagram	Hydraulic Diameter $D_H =$
Through a circular tube		$D = \text{inside diameter}$
Through a square duct		a
Through a rectangular duct		$\frac{2ab}{a+b}$
Through a circular annulus		$D_2 - D_1$

Hydraulic Diameters for Various Flow Configurations (cont'd)

Flow Configuration	Diagram	Hydraulic Diameter $D_H =$
Through a partially filled pipe (tube)		$\frac{2[r l - c(r - h)]}{l}$ where $c = 2\sqrt{h(2r - h)}$
Around a sphere (or sphere through a fluid)		Sphere diameter
Around any object (or an any object through a fluid)		$\frac{4A_p}{P}$

6.2.4.2 Newtonian Fluid

$$Re = \frac{D_H u \rho}{\mu}$$

where u = approach velocity

Various Forms of Reynolds Numbers and Their Units in Circular Conduits (Pipes)

Reynolds Number Form	Diameter D	Fluid Velocity u	Fluid Density ρ	Fluid Viscosity μ	Volumetric Flow rate $\dot{V}$	Mass Flow rate $\dot{m}$	Kinematic Viscosity ν
$\frac{Du\rho}{\mu}$	ft	$\frac{\text{ft}}{\text{sec}}$	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{lbm}}{\text{ft}\cdot\text{sec}}$			
$\frac{Du\rho}{\mu}$	m	$\frac{\text{m}}{\text{s}}$	$\frac{\text{kg}}{\text{m}^3}$	$\text{Pa}\cdot\text{s}$ or $\frac{\text{N}\cdot\text{s}}{\text{m}^2}$ or $\frac{\text{kg}}{\text{m}\cdot\text{s}}$			
$\frac{Du\rho}{32.2\mu}$	ft	$\frac{\text{ft}}{\text{sec}}$	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{lb}\cdot\text{sec}}{\text{ft}^2}$			
$123.9 \frac{Du\rho}{\mu}$	in.	$\frac{\text{ft}}{\text{sec}}$	$\frac{\text{lbm}}{\text{ft}^3}$	cP			
$22,700 \frac{\dot{V}\rho}{D\mu}$	in.		$\frac{\text{lbm}}{\text{ft}^3}$	cP	$\frac{\text{ft}^3}{\text{sec}}$		
$50.6 \frac{\dot{V}\rho}{D\mu}$	in.		$\frac{\text{lbm}}{\text{ft}^3}$	cP	gpm		
$6.31 \frac{\dot{m}}{D\mu}$	in.			cP		$\frac{\text{lbm}}{\text{hr}}$	
$35.42 \frac{\dot{V}\rho}{D\mu}$	in.		$\frac{\text{lbm}}{\text{ft}^3}$	cP	$\frac{\text{barrels}}{\text{hr}}$		
$\frac{Du}{\nu}$	ft	$\frac{\text{ft}}{\text{sec}}$					$\frac{\text{ft}^2}{\text{sec}}$
$\frac{Du}{\nu}$	m	m/s					$\frac{\text{m}^2}{\text{s}}$
$\frac{Du}{12\nu}$	in.	$\frac{\text{ft}}{\text{sec}}$					$\frac{\text{ft}^2}{\text{sec}}$
$7740 \frac{Du}{\nu}$	in.	$\frac{\text{ft}}{\text{sec}}$					cS
$1,419,000 \frac{\dot{V}}{D\nu}$	in.				$\frac{\text{ft}^3}{\text{sec}}$		cS
$3160 \frac{\dot{V}}{D\nu}$	in.				gpm		cS

6.2.4.3 Power Law Fluid

$$Re^x = \frac{(D^n u^{(2-n)} \rho)}{\left(K \left(\frac{(3n+1)}{4n} \right)^n 8^{(n-1)} \right)}$$

where

n = power law index

K = consistency index

6.2.4.4 Bingham Fluid

Bingham fluid flow through a pipe:

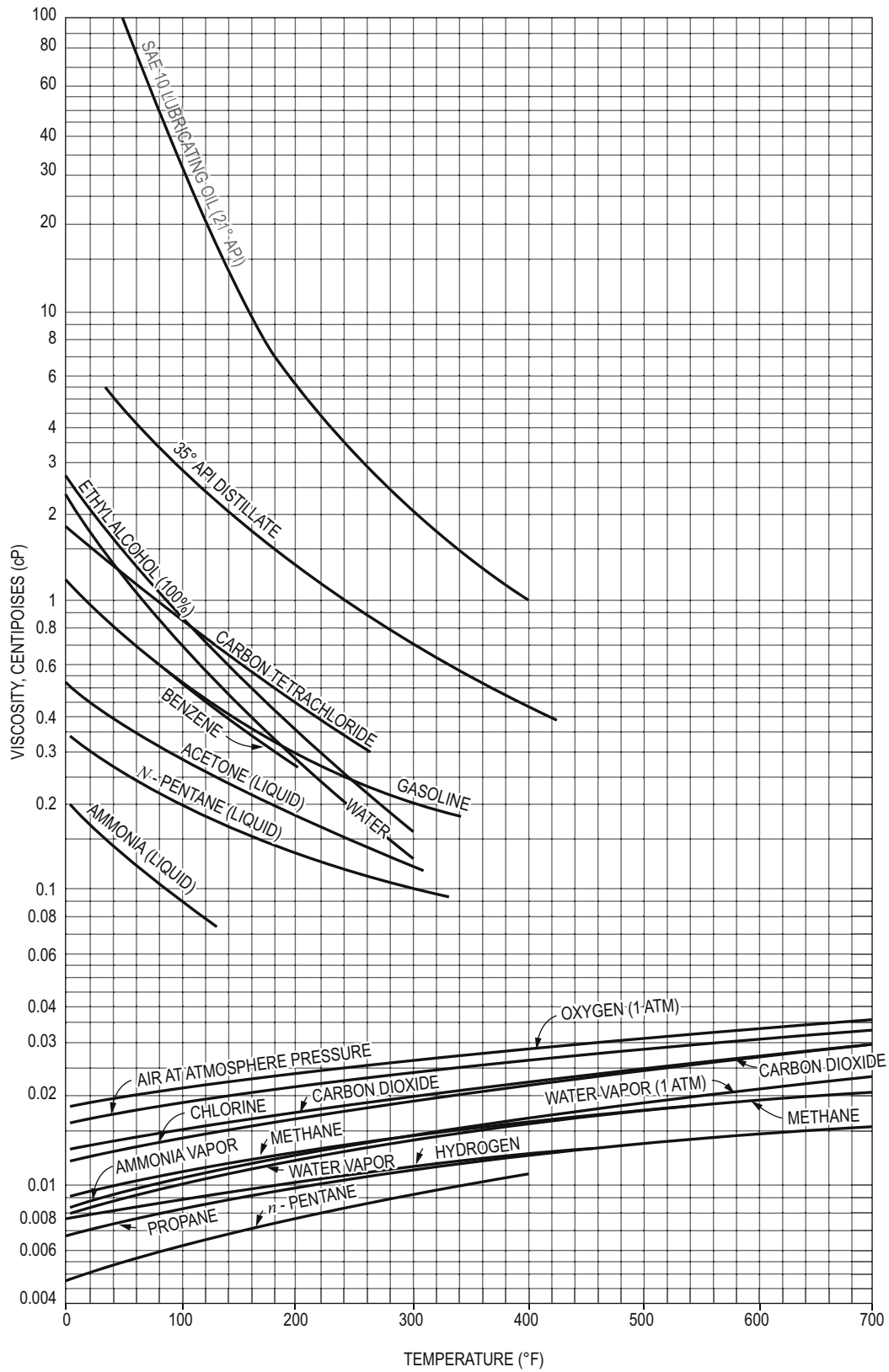
$$Re_{BP} = \frac{4 \dot{V} \rho}{\pi D \mu_{\infty} \left(1 + \frac{\pi D^3 \tau_0 g_c}{24 \dot{V} \mu_{\infty}} \right)}$$

where

μ_{∞} = infinite viscosity, or plastic viscosity, or high shear limiting viscosity

τ_0 = yield stress of the fluid

Viscosity as a Function of Temperature for a Variety of Gases and Liquids



Source: Brown, G. G., et. al., *Unit Operations*, New York: John Wiley & Sons, Inc., 1951.

6.2.4.5 Critical Reynolds Number

The *critical Reynolds number* (Re_c) is the minimum Reynolds number at which flow is expected to become turbulent, as shown in the following table:

Flow Regime	Re_c
Flow through a pipe	2100
Flow around a sphere	10
Circular flow (rotating cylinder, Taylor-Couette flow)	$\sqrt{\frac{1708r_1}{h}}$ where the inner cylinder has a diameter (r_1) and height (h)

6.2.5 Friction

6.2.5.1 Absolute Roughness and Relative Roughness

Relative roughness is $\frac{\epsilon}{D}$.

Absolute Roughness or Specific Roughness (ϵ) of Various Pipes

Material	ϵ			
	ft	in.	m	mm
PVC and plastic pipes	0.0000033	0.00004	1.0E-06	0.001
Copper, lead, brass, aluminum (new)	0.000005	0.00006	1.5E-06	0.0015
Stainless steel	0.00005	0.0006	1.5E-05	0.015
Steel commercial pipe	0.0002	0.0024	6.0E-05	0.06
Asphalted cast iron	0.0004	0.0048	1.2E-04	0.12
Galvanized iron	0.0005	0.006	1.5E-04	0.15
Smoothed cement	0.001	0.012	3.0E-04	0.3
New cast iron	0.0016	0.019	5.0E-04	0.5
Well-planed wood	0.0016	0.019	5.0E-04	0.5
Ordinary concrete	0.0026	0.031	8.0E-04	0.8
Worn cast iron	0.004	0.048	1.2E-03	1.2
Coarse concrete	0.0065	0.078	2.0E-03	2.0
Ordinary wood	0.002	0.024	6.1E-04	0.6

6.2.5.2 Friction Factors for Laminar Flow

For laminar flow ($Re < 2100$)

$$f = \frac{64}{Re}$$

6.2.5.3 Friction Factors for Turbulent Flow

The *Colebrook equation*

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{\epsilon}{D} + \frac{2.51}{Re \sqrt{f}} \right)$$

The *Haaland equation* is an empirical approximation of the friction factor that does not require iteration,

$$\frac{1}{\sqrt{f}} = -1.8 \log_{10} \left[\frac{6.9}{Re} + \left(\frac{\varepsilon}{3.7D} \right)^{\frac{10}{9}} \right]$$

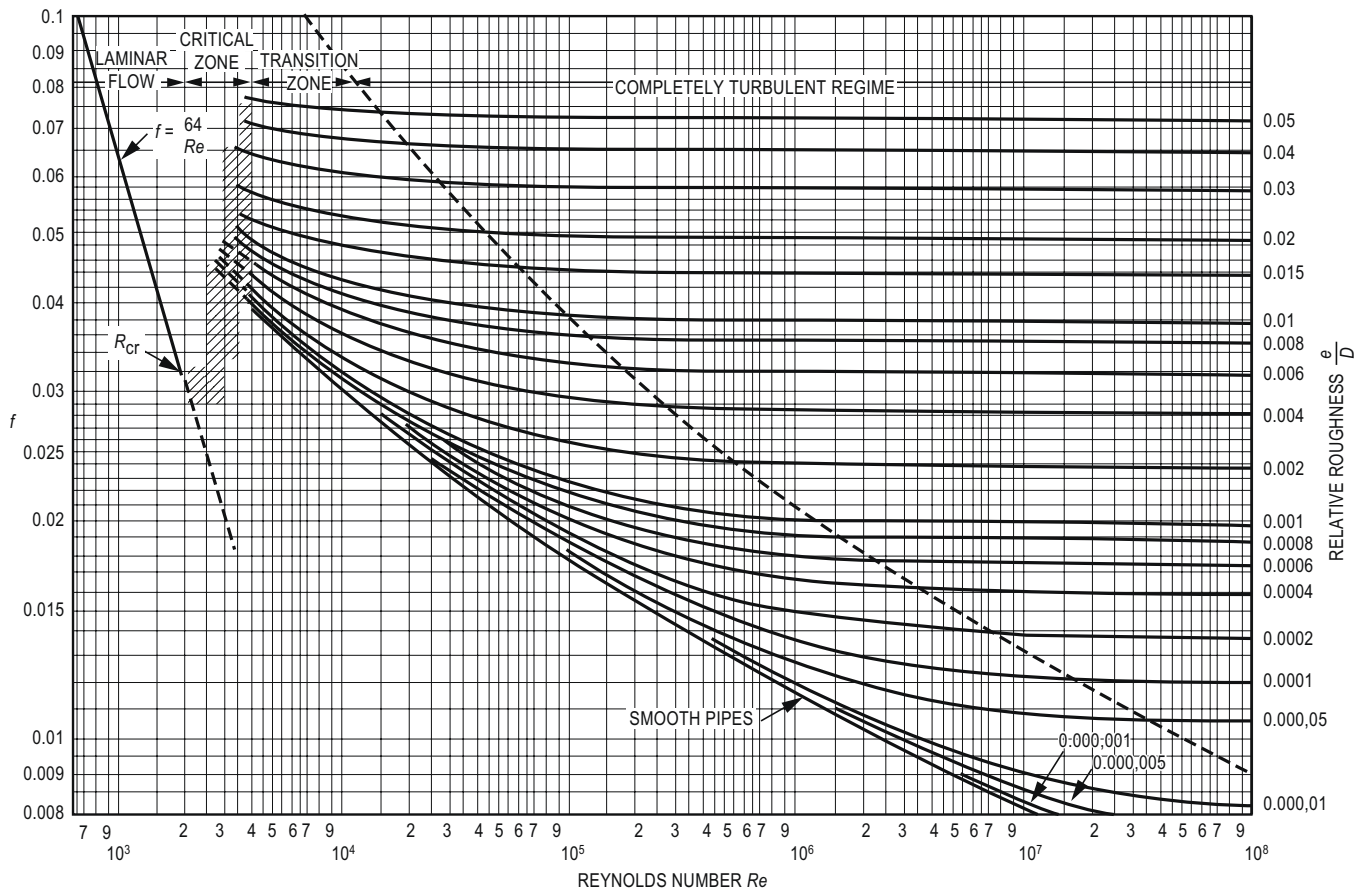
for the following conditions

$$4 \times 10^4 \leq Re \leq 10^8 \text{ and } 0 \leq \frac{\varepsilon}{D} \leq 0.05$$

For fully turbulent flow

$$\frac{1}{\sqrt{f}} = 1.74 - 2 \log_{10} \left(\frac{2\varepsilon}{D} \right)$$

Friction Factor Chart



MOODY DIAGRAM. (FROM L.F. MOODY, TRANS. ASME, VOL. 66, 1944.)

6.2.6 Pressure Drop for Laminar Flow

The *Hagen-Poiseuille equation* for $\dot{V}$ in terms of the pressure drop ΔP_f is

$$\dot{V} = \frac{\pi R^4 \Delta P_f}{8\mu L} = \frac{\pi D^4 \Delta P_f}{128\mu L}$$

This relation is valid only for flow in the laminar region.

6.2.7 Pressure Drop for Turbulent Flow

6.2.7.1 Head Loss in Pipe or Conduit

The *Darcy-Weisbach equation* is

$$h_L = f \frac{L}{D} \frac{u^2}{2g} = K \frac{u^2}{2g}$$

where

f = friction factor

D = inside diameter of the pipe or hydraulic diameter (D_H) of conduit

L = length over which the pressure drop occurs

$f \frac{L}{D} = K$ = the loss coefficient

The total loss coefficient for a system is

$$K = \sum K_i$$

where K_i = the loss coefficient for individual fittings, valves, and other components

Changes in K for different pipe internal diameter are

$$K_a = K_b \left(\frac{D_a}{D_b} \right)^4$$

An alternative formulation is

$$h_L = \frac{2f_{\text{Fanning}} Lu^2}{Dg}$$

where the Fanning friction factor is

$$f_{\text{Fanning}} = \frac{f}{4}$$

Loss Coefficients and Equivalent Lengths for Fittings and Valves

Fitting			Equivalent Length*	Loss Coefficient	
				$K = \frac{K_1}{Re} + K_\infty \left(1 + \frac{1}{\text{ID}_{\text{inches}}} \right)$	
			$\frac{L}{D}$	K_1	K_∞
Elbows	90°	Standard $\left(\frac{r}{d} = 1 \right)$, threaded	35	800	0.40
		Standard $\left(\frac{r}{d} = 1 \right)$, flanged or welded	20	800	0.25
		Long radius $\left(\frac{r}{d} = 1.5 \right)$	16	800	0.20
		Mitered	100	1000	1.15
	45°	Standard $\left(\frac{r}{d} = 1 \right)$, threaded	16	500	0.20
		Long radius $\left(\frac{r}{d} = 1.5 \right)$	13	500	0.15
		Mitered, 1 weld (45°)	20	500	0.25
	180°	Standard $\left(\frac{r}{d} = 1 \right)$, threaded	60	1000	0.70
		Standard $\left(\frac{r}{d} = 1 \right)$, flanged or welded	30	1000	0.35
		Long radius $\left(\frac{r}{d} = 1.5 \right)$	25	1000	0.30
Tees	Used as elbows	Standard, threaded	60	500	0.70
		Long radius, threaded	35	800	0.40
		Standard, flanged or welded	65	800	0.80
		Stub-in branch	85	1000	1.00
	Run through	Threaded	10	200	0.10
		Flanged or welded	40	150	0.50
		Stub-in branch	5	100	0.05

Loss Coefficients and Equivalent Lengths for Fittings and Valves (cont'd)

Fitting			Equivalent Length*	Loss Coefficient	
				$K = \frac{K_1}{Re} + K_\infty \left(1 + \frac{1}{ID_{\text{inches}}}\right)$	
			$\frac{L}{D}$	K_1	K_∞
Valves	Gate, ball, or plug	Full line size $\left(\frac{D_{\text{opening}}}{D_{\text{pipe}}} = 1.0\right)$	10	300	0.10
		Reduced trim $\left(\frac{D_{\text{opening}}}{D_{\text{pipe}}} = 0.9\right)$	12	500	0.15
		Reduced trim $\left(\frac{D_{\text{opening}}}{D_{\text{pipe}}} = 0.8\right)$	20	1000	0.25
	Globe	Standard	330	1500	4.00
		Angle or Y type	165	1000	2.00
	Diaphragm	Fully open	165	1000	2.00
	Butterfly	Full open	20	800	0.25
	Check	Lift	830	2000	10.00
		Swing	125	1500	1.50
		Tilting disk	40	1000	0.50

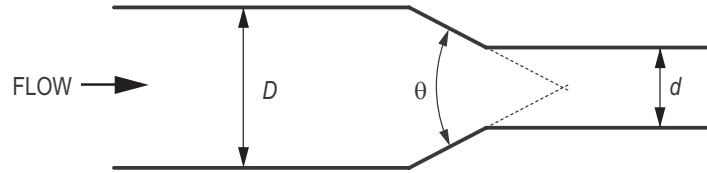
* Approximated from the loss coefficient equation using friction factors for fully turbulent flow for pipe sizes 1" through 24"

6.2.7.2 Loss Coefficients for Contraction and Expansion

Notes:

1. Reynolds Number (Re) and friction factor (f) are based on inlet velocity.
2. $\beta = \frac{d}{D}$

Contraction:



When $\theta < 45^\circ$ and

$$Re < 2500, \text{ then } K = 1.6 \left(1.2 + \frac{160}{Re} \right) \left(\frac{1}{\beta^4} - 1 \right) \sin \left(\frac{\theta}{2} \right)$$

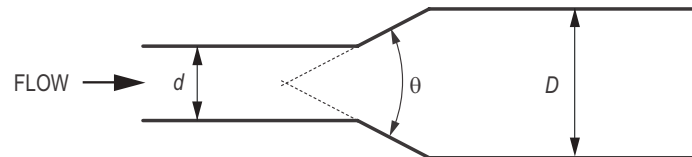
$$Re > 2500, \text{ then } K = 1.6 (0.6 + 1.92f) \left(\frac{1 - \beta^2}{\beta^4} \right) \sin \frac{\theta}{2}$$

When $\theta > 45^\circ$ and

$$Re < 2500, \text{ then } K = 1.6 \left(1.2 + \frac{160}{Re} \right) \left(\frac{1}{\beta^4} - 1 \right) \left[\sin \left(\frac{\theta}{2} \right) \right]^{\frac{1}{2}}$$

$$Re > 2500, \text{ then } K = (0.6 + 0.48f) \left(\frac{1 - \beta^2}{\beta^4} \right) \left[\sin \left(\frac{\theta}{2} \right) \right]^{\frac{1}{2}}$$

Expansion:



When $\theta < 45^\circ$ and

$$Re < 4000, \text{ then } K = 5.2 (1 - \beta^4) \sin \left(\frac{\theta}{2} \right)$$

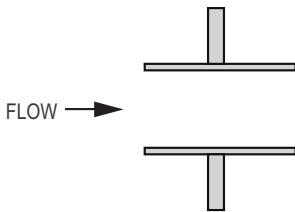
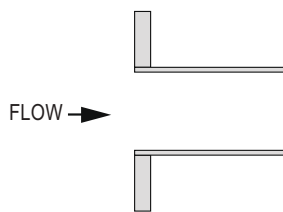
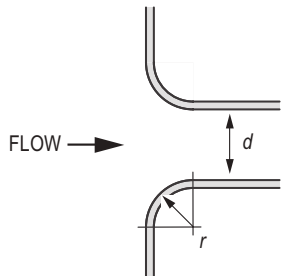
$$Re > 4000, \text{ then } K = 2.6 (1 + 3.2f) (1 - \beta^4) \sin \left(\frac{\theta}{2} \right)$$

When $\theta > 45^\circ$ and

$$Re < 4000, \text{ then } K = 2 (1 - \beta^4)$$

$$Re > 4000, \text{ then } K = (1 + 3.2f) (1 - \beta^4)^2$$

6.2.7.3 Loss Coefficients for Pipe Entrance and Exit

Loss Coefficients															
Fitting	Type	Configuration	Loss Coefficient												
			$K = \frac{K_1}{Re} + K_\infty \left(1 + \frac{1}{ID_{inches}}\right)$												
			K_1	K_∞											
Entrance	Inward projecting or reentrant		160	1.0											
	Sharp-edged		160	0.5											
	Rounded		160	<table><tr><th>r/d</th><th>K_∞</th></tr><tr><td>0.02</td><td>0.28</td></tr><tr><td>0.04</td><td>0.24</td></tr><tr><td>0.06</td><td>0.15</td></tr><tr><td>0.10</td><td>0.09</td></tr><tr><td>0.15 & up</td><td>0.04</td></tr></table>	r/d	K_∞	0.02	0.28	0.04	0.24	0.06	0.15	0.10	0.09	0.15 & up
r/d	K_∞														
0.02	0.28														
0.04	0.24														
0.06	0.15														
0.10	0.09														
0.15 & up	0.04														
Exit	All geometries		0.0	1.0											

6.2.7.4 Valve Flow Coefficient (C_v)

Valve flow coefficient (C_v) is a value of the relationship between the pressure drop across a valve and the corresponding flow rate:

$$C_v = \dot{V} \sqrt{\frac{SG}{\Delta P}}$$

Also:

$$C_v = \frac{ad^2}{\sqrt{K}}$$

where

$$a = \text{constant, } 29.9 \frac{\text{gpm}}{\text{in}^2 \sqrt{\text{psi}}} \quad \text{or} \quad 0.0352 \frac{\text{m}^3/\text{s}}{\text{m}^2 \sqrt{\text{Pa}}}$$

d = effective diameter of the valve, in inches or meters

K = loss coefficient

Note: Values of C_v are not interchangeable between unit systems.

The estimated flow rate with a known K value is

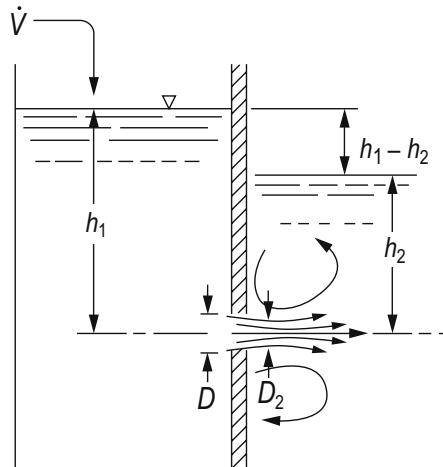
$$\dot{V}_{\text{gpm}} = \frac{ad^2}{\sqrt{K}} \sqrt{\frac{\Delta P}{SG}}$$

where ΔP = pressure drop (psi or Pa)

6.2.8 Flow Through an Orifice

6.2.8.1 Submerged Orifice

Submerged Orifice Operating Under Steady-Flow Conditions



$$\dot{V} = A_2 u_2 = CA \sqrt{2g(h_1 - h_2)}$$

where

u_2 = velocity of fluid exiting the orifice

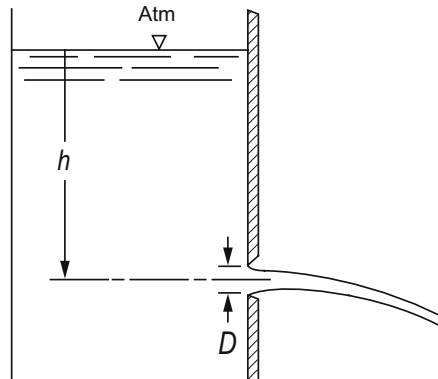
A = cross-sectional area at diameter D

A_2 = vena contracta cross-sectional area at diameter D_2

C = coefficient of discharge

6.2.8.2 Orifice Discharging Freely into Atmosphere

Orifice Discharging Into Atmosphere



Torricelli's equation is

$$u = \sqrt{2gh}$$

$$\dot{V} = CA\sqrt{2gh}$$

where

h = distance from the liquid surface to the centerline of the orifice opening

A = cross-sectional area at diameter D

C = coefficient of discharge

6.2.9 Particle Flow

The force exerted by a fluid that opposes the weight of an immersed object (buoyant force) can be expressed in terms of differential densities:

$$F_G = \frac{(\rho_p - \rho_f)gV_p}{g_c}$$

where

F_G = buoyant force

ρ_p = particle density

ρ_f = fluid density

V_p = volume of particle

The force exerted by a fluid flowing past a solid body (drag force) can be expressed in terms of a drag coefficient (C_D):

$$F_D = \frac{C_D \rho_f u_\infty^2 A_p}{2g_c}$$

where

F_D = drag force

u_∞ = approach velocity

A_p = the projected area of object with axes perpendicular to the flow

6.2.9.1 Stokes Law or Stokes Flow

For a sphere moving through a fluid at $Re \ll 1$:

$$C_D = \frac{24}{Re}$$

where

$$Re = \frac{D_p u_\infty \rho}{\mu}$$

D_p = the particle diameter

In *Stokes flow*, viscosity can be determined using:

$$\mu = \frac{D_p^2 g (\rho_p - \rho_f)}{18u_t}$$

where u_t = terminal (or settling) velocity of particle

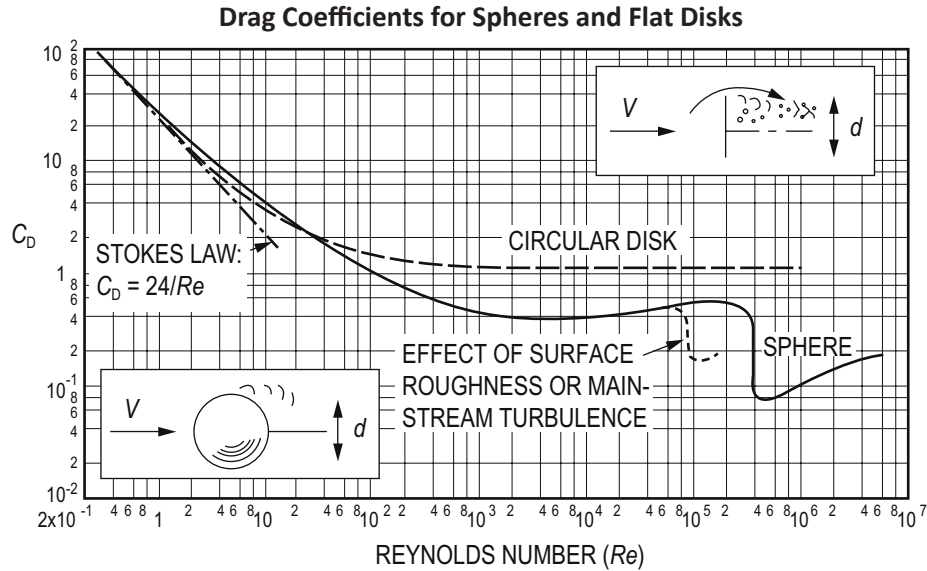
Drag Coefficients

For spheres in a flowing fluid with Reynolds numbers ($1 < Re < 2 \times 10^5$), the *Dallavalle equation* applies:

$$C_D = \left(0.632 + \frac{4.8}{\sqrt{Re}} \right)^2$$

For cylinders in a flowing fluid with Reynolds numbers ($1 < Re < 2 \times 10^5$) and with the axis normal to the flow, this equation applies:

$$C_D = \left(1.05 + \frac{1.9}{\sqrt{Re}} \right)^2$$



6.2.9.2 Terminal Velocity (u_t)

For a sphere of diameter D_p , the following equation applies for any Reynolds number (Newton's Law of falling particles):

$$u_t = \sqrt{\frac{4g D_p (\rho_{\text{sphere}} - \rho_f)}{3\rho_f C_D}}$$

For a small sphere of diameter D_p , following Stokes Law:

$$u_t = \frac{D_p^2 g (\rho_{\text{sphere}} - \rho_f)}{18\mu}$$

6.2.9.3 Reynolds Numbers for Particles in a Fluid

Reynolds number when particle velocity (u_t) is unknown and D_p , ρ_p , ρ_f , and μ are known:

$$Re = \left[(14.42 + 1.827 \sqrt{Ar})^{\frac{1}{2}} - 3.798 \right]^2$$

where the Archimedes number (Ar) is:

$$Ar = \frac{D_p^3 \rho_f g (\rho_p - \rho_f)}{\mu^2}$$

Reynolds number when particle diameter (D_p) is unknown and u_t , ρ_p , ρ_f , and μ are known:

$$\frac{1}{\sqrt{Re}} = \left(0.00433 + 0.203 \sqrt{\frac{C_D}{Re}} \right)^{\frac{1}{2}} - 0.0658$$

where $\frac{C_D}{Re} = \frac{4\mu g(\rho_p - \rho_f)}{3\rho_f^2 u_t^3}$

Reynolds number when fluid viscosity (μ) is unknown and D_p , u_t , ρ_p , and ρ_f are known:

$$Re = \left(\frac{4.8}{\sqrt{C_D} - 0.632} \right)^2$$

Use known quantities to solve for C_D .

6.2.9.4 Flow Through Porous Media and Packed Beds

A porous, fixed bed of solid particles can be characterized by:

L = length of particle bed

d_s = average particle diameter (diameter of a sphere with the same volume of the particle)

Φ = sphericity of particle (0–1)

ϵ = porosity or void fraction of the particle bed (dimensionless)

Porosity (ϵ) or void fraction:

$$\epsilon = \frac{(\text{Total volume} - \text{Volume of solids})}{\text{Total volume}} = \frac{1 - A_{\text{solid}}}{A} = \frac{A_{\text{voids}}}{A}$$

where

A_{solid} = area of the solid phase in a cross-section of area A

A_{voids} = void area in a cross-section of area A

Interstitial velocity (actual velocity of fluid within the pores or voids):

$$u_i = \frac{\dot{V}}{\epsilon A} = \frac{u}{\epsilon}$$

where u = approach velocity (or superficial velocity)

Sphericity of a particle (shape factor):

$$\Phi = \frac{\text{surface area of sphere with same volume as particle}}{\text{surface area of particle}}$$

Friction loss through porous media:

$$h_f = \frac{3}{4} \left(f \frac{L}{d_s} \right) \frac{u^2}{g} \left(\frac{(1 - \epsilon)}{\epsilon^3} \right)$$

Reynolds number for flow through porous media:

$$Re = \frac{2}{3} \frac{d_s \mu \rho}{\mu} \left(\frac{1}{(1 - \epsilon)} \right)$$

Use the *Ergun equation* to estimate the pressure loss through a packed bed (ΔP) under laminar and turbulent conditions:

$$\frac{\Delta P}{L} = \frac{(150\mu u)}{(\Phi^2 d_s^2)} \frac{(1-\epsilon)^2}{\epsilon^3} + \frac{(1.75\rho u^2)}{(\Phi d_s)} \left(\frac{(1-\epsilon)}{\epsilon^3} \right)$$

Typical Shape Factors

Particle	Φ
Spheres	1.00
Torus	0.89
Ideal cylinder ($h = d$)	0.87
Octahedron	0.85
Cube	0.81
Sand (average)	0.75
Cylinder ($h = 5d$)	0.70
Cylinder ($h = 10d$)	0.58
Tetrahedron	0.67
Berl saddles	0.30–0.37
Raschig rings	0.26–0.53

6.2.9.5 Fluidization

For a fluid passing vertically through a bed of particles, ΔP increases as fluid velocity u increases. The net upward force F_B on the bed is

$$F_B = A\Delta P$$

where A = cross-sectional area of the bed

At fluidization, net upward force (*fluid drag force*) equals the weight of the bed ($F_B = W_B$), while the fluid velocity above the bed is less than the terminal velocity of the particles (u_t).

The Reynolds number for a fluidized bed can be approximated by:

$$Re = \sqrt{C_1 + C_2 Ar} - C_1$$

where

Ar = Archimedes number

$$C_1 = \frac{180(1-\epsilon)}{3.5}$$

$$C_2 = \frac{\epsilon^3}{1.75}$$

where ϵ = minimum bed void fraction (porosity) at the point of fluidization

The minimum bed void fraction for bed height H at the first indication of fluidization is

$$\epsilon = 1 - \frac{m_{\text{particles}}}{H A \rho_p}$$

The minimum fluidization velocity is

$$u_{mf} = \frac{(\rho_p - \rho_f)gd_{\text{particles}}^2}{150\mu} \frac{\epsilon^3}{1 - \epsilon}$$

note:

1. $u_{\text{superficial}} = u_{mf}$ is the incipient fluidization.
2. For large particles, $d_{\text{particles}} \geq 1 \text{ mm}$, inertial effects are important. Use the Ergun equation.

The maximum fluidization velocity that avoids entrainment is

$$u_{\text{settling}} = \frac{(\rho_p - \rho_f)gd_{\text{particles}}^2}{18\mu} \quad (\text{Stokes})$$

$$\frac{u_{\text{settling}}}{u_{mf}} = \left(\frac{25}{3}\right) \frac{1 - \epsilon}{\epsilon^3} \text{ for common operating condition } u = 30 u_{mf}$$

6.2.10 Open-Channel Flow

6.2.10.1 Specific Energy (or Specific Heat)

$$E = \frac{u^2}{2g} + y$$

where

E = specific energy (or head)

u = fluid velocity

y = depth of liquid

Critical Depth: The depth of flow for a given discharge where the specific energy is at q minimum.

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$$

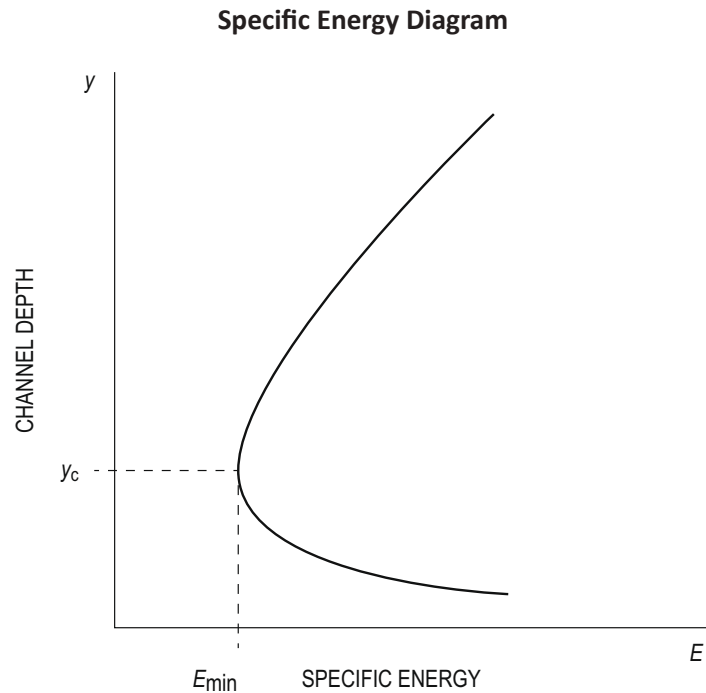
where

y_c = critical depth

q = unit discharge $\left(\frac{\dot{V}}{B}\right)$

$\dot{V}$ = total discharge, volumetric flow rate

B = channel width



6.2.10.2 Froude Number

$$Fr = \frac{u^2}{g y_h} = \sqrt{\frac{\dot{V} B}{g A^3}}$$

where

$$y_h = \text{hydraulic depth} = \frac{A}{B}$$

A = cross-sectional area of flow

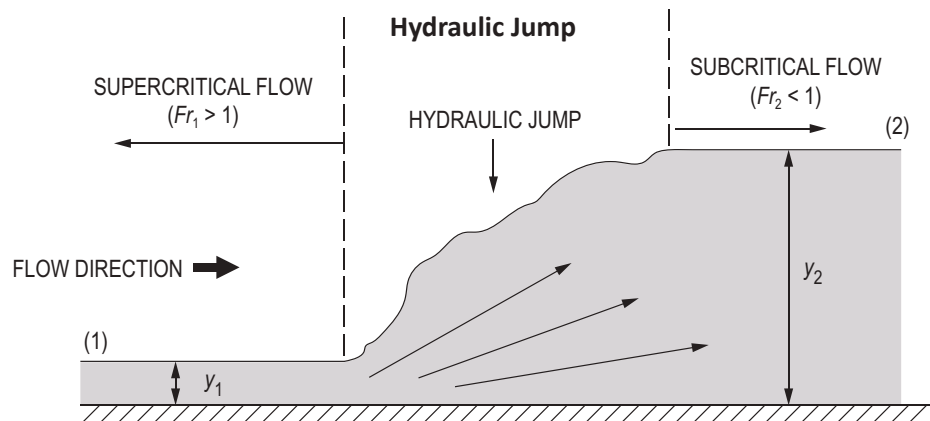
B = channel width

Supercritical flow: $Fr > 1$

Subcritical flow: $Fr < 1$

Critical flow: $Fr = 1$

6.2.10.3 Hydraulic Jump



$$\frac{y_2}{y_1} = \frac{1}{2} \left(-1 + \sqrt{1 + 8Fr_1^2} \right)$$

where

y_1 = flow depth at upstream supercritical flow location

y_2 = flow depth at downstream subcritical flow location

Fr_1 = Froude number at upstream supercritical flow location

Fr_2 = Froude number at downstream subcritical flow location

6.2.10.4 Manning Equation

$$\dot{v} = \frac{\kappa}{n} A R_H^{\frac{2}{3}} S^{\frac{1}{2}}$$

where

$\dot{v}$ = discharge volumetric flow rate

κ = 1.0 for SI units; 1.49 for U.S. units

A = cross-sectional area of flow

R_H = hydraulic radius

S = slope of hydraulic surface

n = Manning's roughness coefficient

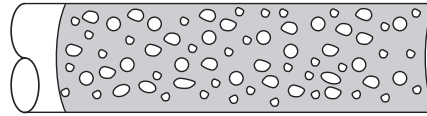
Manning's Roughness Coefficients

Material	n
Cast iron pipe	0.013
Wrought iron pipe	0.015
Riveted steel pipe	0.016
Corrugated storm pipe	0.024
Glass	0.010
Vitrified sewer pipe	0.014
Concrete pipe	0.013
Excavated canal—earth, uniform	0.023
Natural channel—uniform cross-section	0.050

6.2.11 Two-Phase Flow

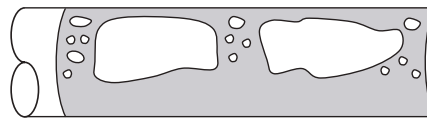
6.2.11.1 Flow Patterns

Bubble or Froth Flow: Bubbles of gas are dispersed throughout the liquid. Gas bubbles move at roughly the same velocity as the liquid.



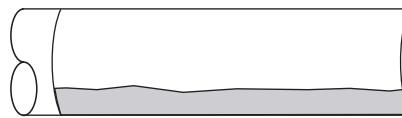
BUBBLE FLOW

Plug Flow: Alternate plugs of liquid and gas move along the upper portion of the pipe, with mostly liquid moving along the lower portion.



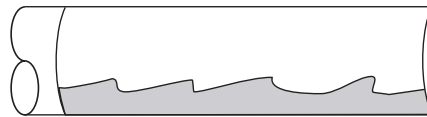
PLUG FLOW

Stratified Flow: Gas flow moves on top and over the liquid forming a distinct, relatively smooth, liquid-gas interface.



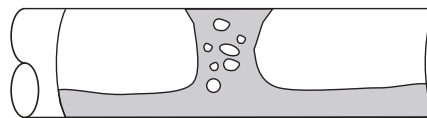
STRATIFIED FLOW

Wave Flow: Similar to stratified flow, the fast-moving gas flow creates waves in the liquid phase.



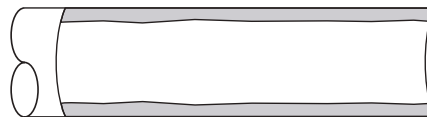
WAVE FLOW

Slug Flow: High-velocity gas picks up waves to form frothy slugs of liquid. These slugs move at higher velocity than the bulk liquid phase and can create vibrations that can damage equipment.



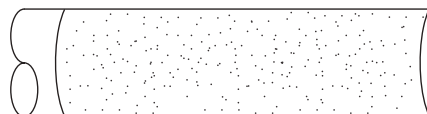
SLUG FLOW

Annular Flow: As gas velocity increases, liquid forms around the inside of the pipe wall, with the high-velocity gas flowing through the center.



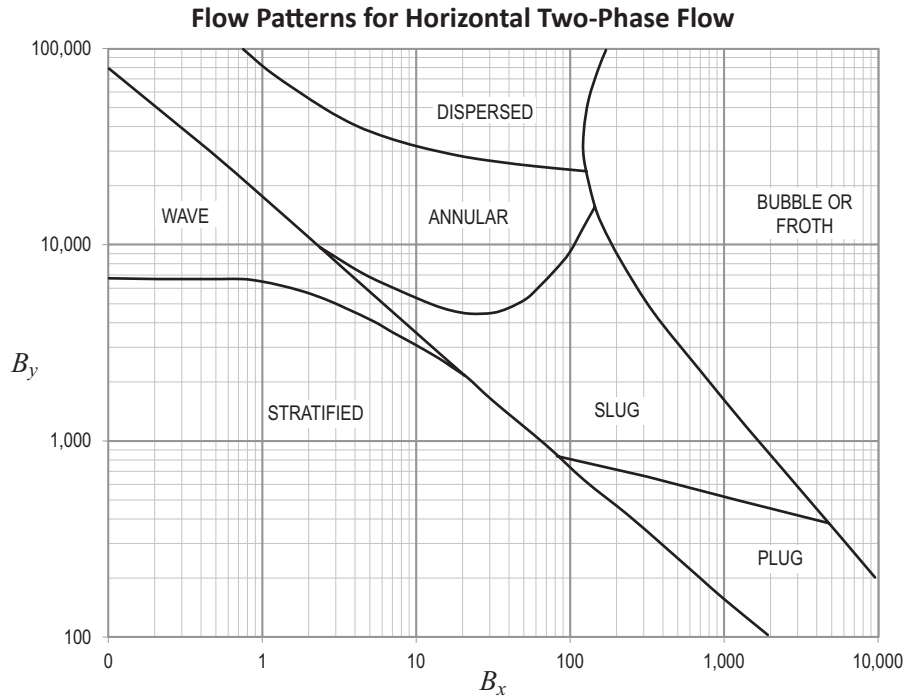
ANNULAR FLOW

Dispersed Flow (or Spray Flow or Mist Flow): Liquid is entrained as fine droplets in the gas phase.



DISPERSED FLOW

6.2.11.2 Flow Regimes



Source: Baker, Ovid, *Oil and Gas Journal*, Nov. 10, 1958.

Baker parameters for the previous chart:

$$B_x = 531 \left(\frac{\dot{m}_L}{\dot{m}_G} \right) \left[\frac{(\rho_L \rho_G)^{\frac{1}{2}}}{\rho_L^{\frac{2}{3}}} \right] \left(\frac{\mu_L^{\frac{1}{3}}}{\gamma_L} \right)$$

$$B_y = 2.16 \left(\frac{\dot{m}_G}{A} \right) \left(\frac{1}{(\rho_L \rho_G)^{\frac{1}{2}}} \right)$$

where

A = internal pipe cross-sectional area, ft^2

$\dot{m}_G$ = gas flow rate, $\frac{\text{lbm}}{\text{hr}}$

$\dot{m}_L$ = liquid flow rate, $\frac{\text{lbm}}{\text{hr}}$

ρ_L = liquid density, $\frac{\text{lbm}}{\text{ft}^3}$

ρ_G = gas density, $\frac{\text{lbm}}{\text{ft}^3}$

μ_L = liquid viscosity, cP

γ_L = liquid surface tension, $\frac{\text{dyn}}{\text{cm}}$

6.2.12 Compressible Flow

6.2.12.1 Isentropic Flow Relationships

In an ideal gas for an *isentropic process*, the following relationships exist between static properties at any two points in the flow:

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1} \right)^{\frac{k}{k-1}} = \left(\frac{\rho_2}{\rho_1} \right)^k$$

where $k = \text{ratio of specific heats} = \frac{c_p}{c_v}$

The stagnation temperature T_0 at a point in the flow is related to the static temperature:

$$T_0 = T + \frac{u^2}{2c_p}$$

Energy relation between two points is

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

The relationship between the static and stagnation properties (T_0 , P_0 , and ρ_0) at any point in the flow can be expressed as a function of the *Mach number* (Ma):

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} Ma^2$$

$$\frac{P_0}{P} = \left(\frac{T_0}{T} \right)^{\frac{k}{k-1}} = \left(1 + \frac{k-1}{2} Ma^2 \right)^{\frac{k}{k-1}}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T} \right)^{\frac{k}{k-1}} = \left(1 + \frac{k-1}{2} Ma^2 \right)^{\frac{1}{k-1}}$$

Compressible flows are often accelerated or decelerated through a nozzle or diffuser. For subsonic flows, the velocity decreases as the flow cross-sectional area increases and vice versa. For supersonic flows, the velocity increases as the flow cross-sectional area increases and decreases as the flow cross-sectional area decreases. The point at which the Mach number is sonic is called the *throat*; its area is represented by the variable A^* .

The following area ratio holds for any Mach number:

$$\frac{A}{A^*} = \frac{1}{Ma} \left[\frac{1 + \frac{1}{2}(k-1) Ma^2}{\frac{1}{2}(k+1)} \right]^{\frac{(k+1)}{2(k-1)}}$$

where

A = area

A^* = area at the sonic point ($Ma = 1.0$)

In an ideal gas, sonic velocity is:

$$u_{\text{sound}} = (kRT)^{1/2}$$

when $Ma = 1.0$

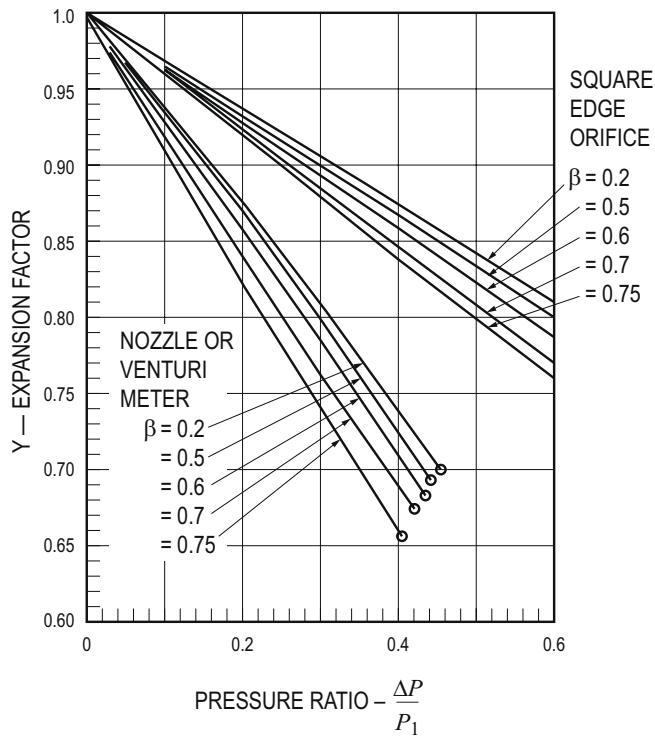
6.2.12.2 Simplified Isothermal Equation

$$\dot{m} = \sqrt{\left(\frac{\rho g_c A^2}{\frac{fL}{D} + 2 \ln\left(\frac{P_1}{P_2}\right)} \right) \left(\frac{P_1^2 - P_2^2}{P_1} \right)}$$

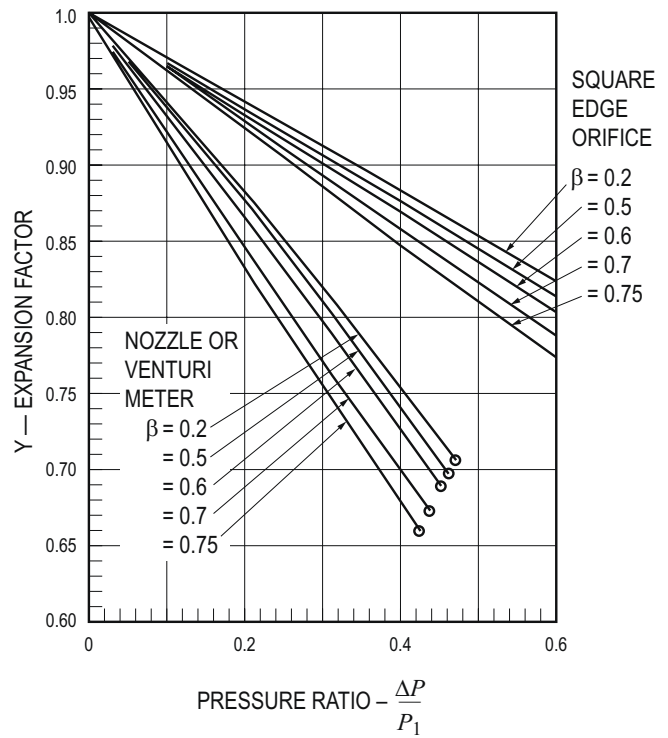
6.2.12.3 Net Expansion Factors For Gases

Expansion Factors for Compressible Flow Through Orifices and Nozzles

$k = 1.3$ approximately [CO₂, SO₂, H₂O (steam), H₂S, NH₃, N₂O, Cl₂, CH₄, C₂H₂, and C₂H₄]



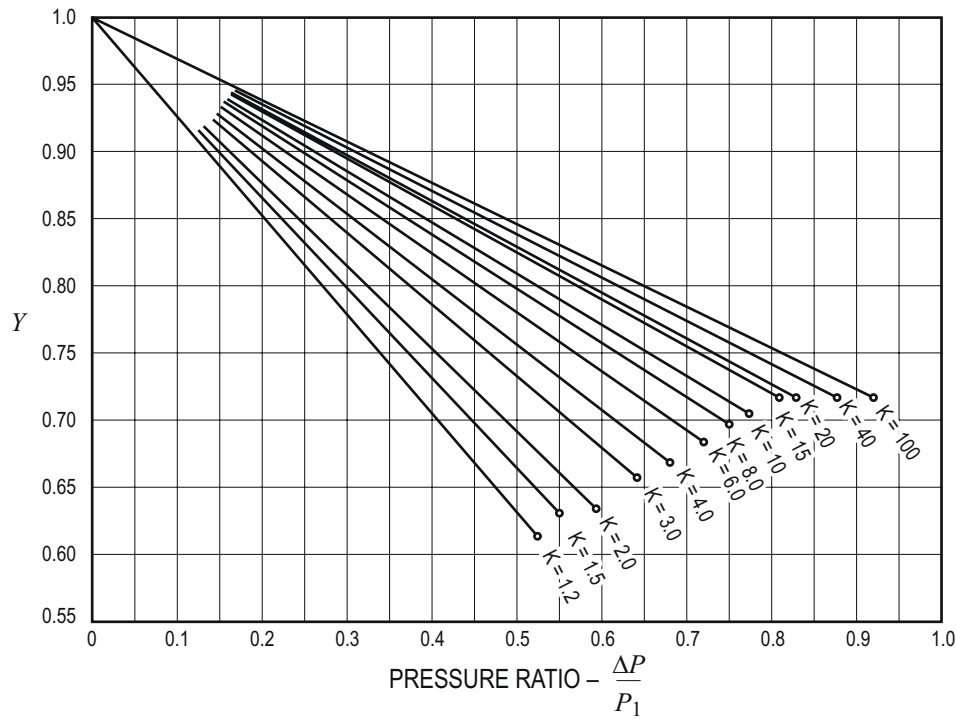
$k = 1.4$ approximately [Air, H₂, O₂, N₂, CO, NO, and HCl]



where P_1 = absolute upstream pressure

Expansion Factors for Compressible Flow Through Pipes

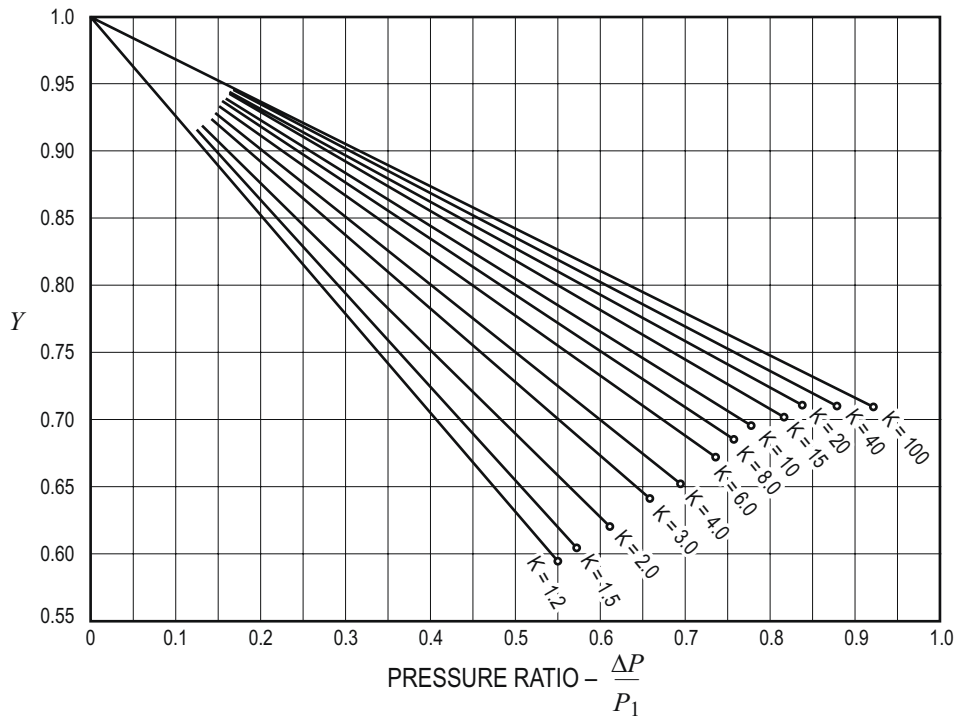
$k = 1.3$ approximately [CO_2 , SO_2 , H_2O (steam), H_2S , NH_3 , N_2O , Cl_2 , CH_4 , C_2H_2 , and C_2H_4]



LIMITING FACTORS
FOR SONIC VELOCITY
 $k = 1.3$

K	$\frac{\Delta P}{P_1}$	Y
1.2	.525	.612
1.5	.550	.631
2.0	.593	.635
3	.642	.658
4	.678	.670
6	.722	.685
8	.750	.698
10	.773	.705
15	.807	.718
20	.831	.718
40	.877	.718
100	.920	.718

$k = 1.4$ approximately [Air, H_2 , O_2 , N_2 , CO , NO , HCl]



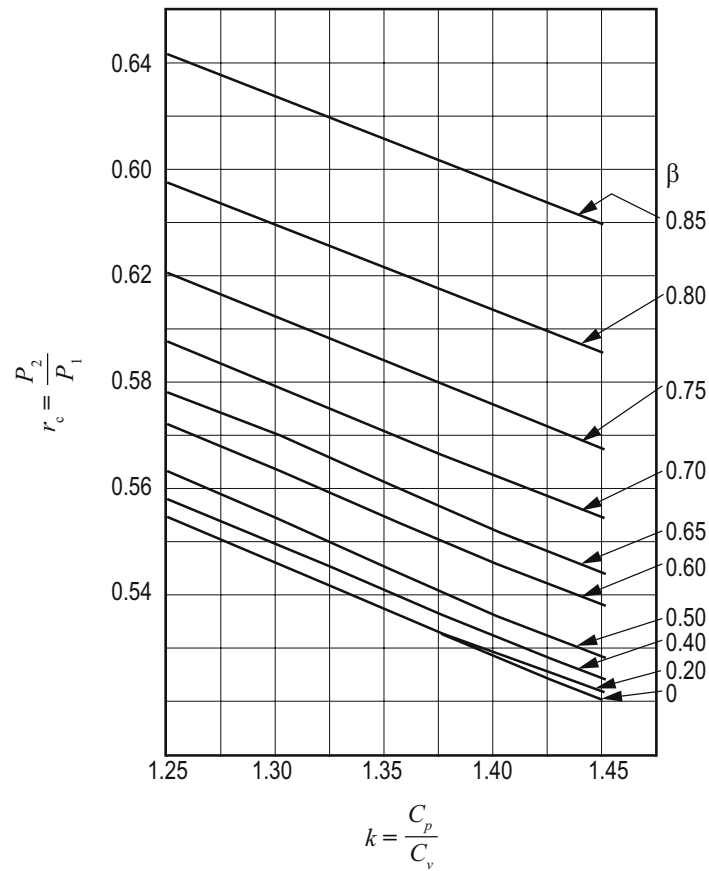
LIMITING FACTORS
FOR SONIC VELOCITY
 $k = 1.4$

K	$\frac{\Delta P}{P_1}$	Y
1.2	.552	.588
1.5	.576	.606
2.0	.612	.622
3	.662	.639
4	.697	.649
6	.737	.671
8	.762	.685
10	.784	.695
15	.818	.702
20	.839	.710
40	.883	.710
100	.926	.710

where P_1 = absolute upstream pressure

6.2.12.4 Critical Pressure Ratio, r_c , for Compressible Flow

Critical Pressure Ratio Through Nozzles and Venturi Tubes (Only)



where P_1 and P_2 = absolute pressures upstream and downstream of the nozzle or venturi tube, respectively

6.2.12.5 Choked Flow

Choked flow is a limiting condition where the mass flow will not increase with a further decrease in the downstream pressure environment while upstream pressure is fixed. Choked flow occurs when the Mach number is 1.0 at the minimum cross-section area.

Mass flow rate of gas at choked flow:

$$\dot{m} = C_d A \sqrt{k \rho_1 P_1 g_c \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}}}$$

where

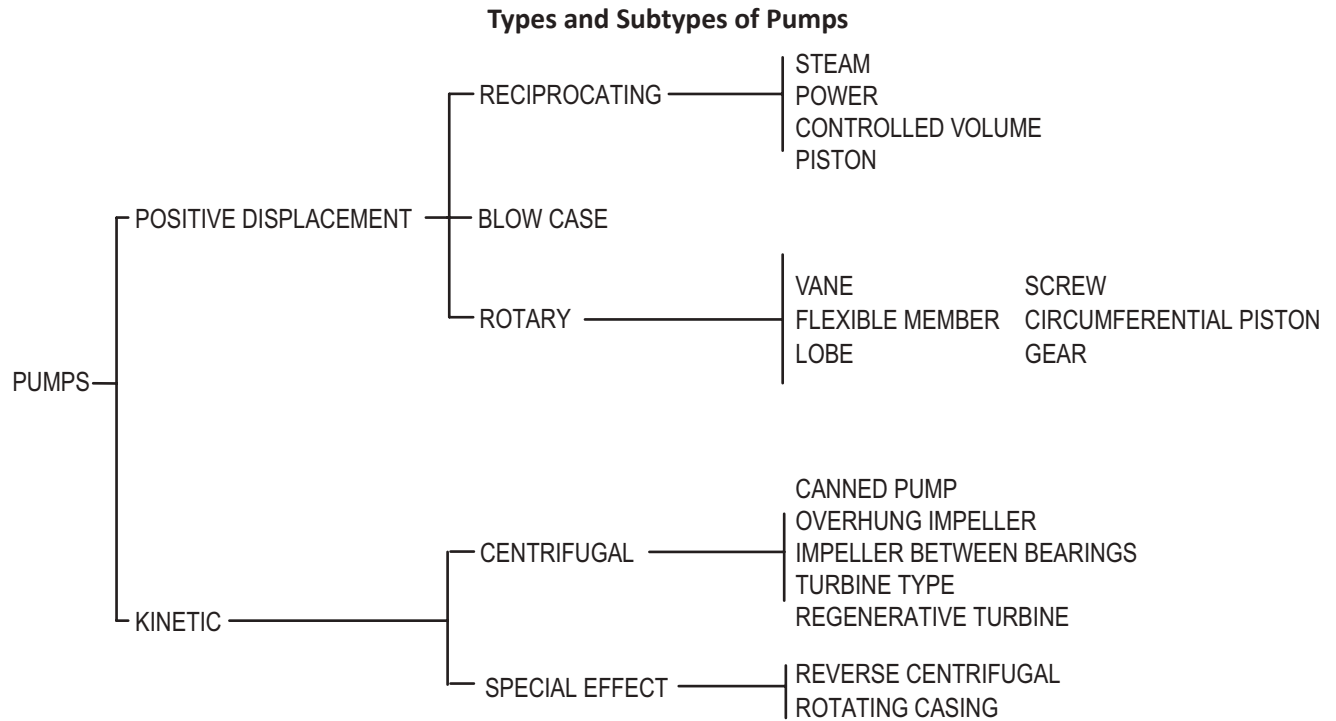
C_d = discharge coefficient of restriction

ρ_1 = density of gas before restriction

P_1 = pressure of gas before restriction (absolute)

6.3 Applications of Fluid Mechanics

6.3.1 Pumps



6.3.1.1 Affinity Laws for Pumps, Fans, and Compressors

For small changes in impeller diameter (changes not to exceed 20%):

$$\frac{D_1}{D_2} = \frac{\dot{V}_1}{\dot{V}_2} = \frac{\sqrt{H_1}}{\sqrt{H_2}} \quad \text{and} \quad \frac{BP_1}{BP_2} = \frac{D_1^3}{D_2^3}$$

For variations in speed (constant impeller diameter):

$$\frac{S_1}{S_2} = \frac{\dot{V}_1}{\dot{V}_2} = \frac{\sqrt{H_1}}{\sqrt{H_2}} \quad \text{and} \quad \frac{BP_1}{BP_2} = \frac{S_1^3}{S_2^3}$$

where

BP = brake power

D = impeller or wheel diameter

H = head (height of fluid)

$\dot{V}$ = volumetric capacity

S = speed (rpm)

6.3.1.2 Pump Similitude

Predicting Performance of Homologous Pumps

Volume capacity estimate:

$$\frac{\dot{V}_1}{\dot{V}_2} = \frac{S_1}{S_2} \left(\frac{D_1}{D_2} \right)^3 = \left(\frac{D_1}{D_2} \right)^2 \left(\frac{H_1}{H_2} \right)^{0.5}$$

Pressure or head estimate:

$$\frac{H_1}{H_2} = \left(\frac{S_1}{S_2} \right)^2 \left(\frac{D_1}{D_2} \right)^2$$

Brake power estimate:

$$\frac{BP_1}{BP_2} = \frac{\rho_1}{\rho_2} \left(\frac{S_1}{S_2} \right)^3 \left(\frac{D_1}{D_2} \right)^5 = \frac{\rho_1}{\rho_2} \left(\frac{D_1}{D_2} \right)^2 \left(\frac{H_1}{H_2} \right)^{1.5}$$

Impeller or wheel speed estimate:

$$\frac{S_1}{S_2} = \frac{D_2}{D_1} \left(\frac{H_1}{H_2} \right)^{0.5} = \left(\frac{\dot{V}_2}{\dot{V}_1} \right)^{0.5} \left(\frac{H_1}{H_2} \right)^{0.75}$$

6.3.1.3 Pump Head

Pump head (H_p) is a variation of the head-basis Bernoulli equation:

$$H_p = \frac{(P_d - P_s)g_c}{\rho g} + \frac{(u_d^2 - u_s^2)}{2g} + (z_d - z_s) + h_f$$

where

P_s = suction pressure at suction reference point (absolute)

P_d = discharge pressure at discharge reference point (absolute)

u_s = velocity at the pump suction

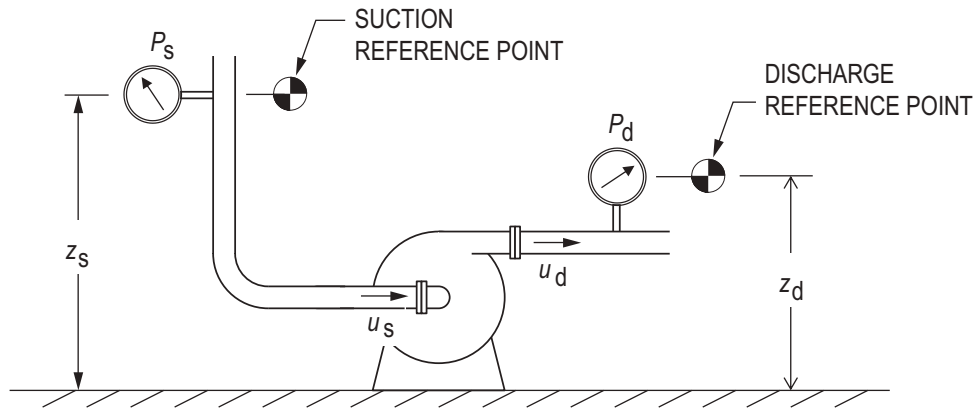
u_d = velocity at the pump discharge

z_s = elevation at the suction reference point

z_d = elevation at the discharge reference point

h_f = friction loss in the pipe between the reference points

Centrifugal Pump



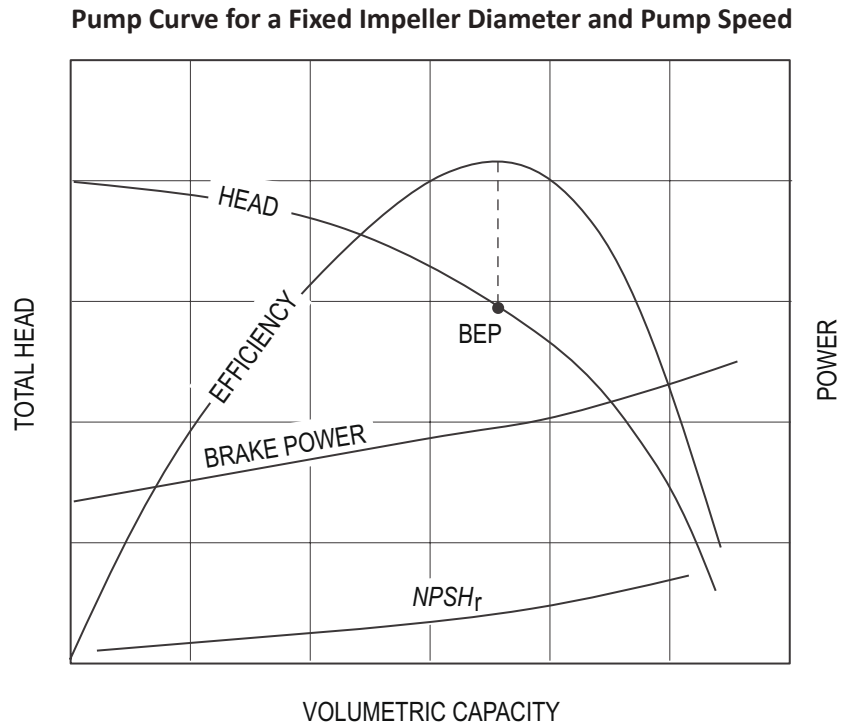
Pump Head in Common Units

Pump Head Calculations

Component	U.S. Units	SI Units
	$H_p = \frac{2.31(P_d - P_s)}{SG} + \frac{(u_d^2 - u_s^2)}{2g} + (z_d - z_s) + h_f$	$H_p = \frac{(P_d - P_s)}{\rho g} + \frac{(u_d^2 - u_s^2)}{2g} + (z_d - z_s) + h_f$
H_p	ft	m
P	psi	Pa
u	$\frac{\text{ft}}{\text{sec}}$	$\frac{\text{m}}{\text{s}}$
z	ft	m
g	$32.2 \frac{\text{ft}}{\text{sec}^2}$	$9.81 \frac{\text{m}}{\text{s}^2}$
h_f	ft	m
ρ	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{kg}}{\text{m}^3}$

6.3.1.4 Pump Curve

A pump curve, head-capacity curve, or H-Q curve is provided by pump manufacturers.



where BEP = best efficiency point (sometimes called best operating point, or BOP)

6.3.1.5 Net-Positive Suction Head (NPSH)

NPSH: Total suction head *minus* the vapor pressure of the liquid being pumped (units are in height of liquid (absolute) and the referenced datum is the suction nozzle.)

$NPSH_a$: Net-positive suction head available to the pump

$NPSH_r$: Net-positive suction head required by the pump (provided by the pump manufacturer)

For suction lift:

$$NPSH_a = h_a - h_{vap} - h_{st} - h_L$$

For flooded suction:

$$NPSH_a = h_a - h_{vap} + h_{st} - h_L$$

where

h_a = absolute pressure (in height of liquid) on the surface of the liquid supply level

h_{vap} = vapor pressure (in height of liquid) of the liquid at the temperature being pumped

h_{st} = static height of liquid supply, either above or below the pump centerline or impeller eye

h_L = suction line losses in height of liquid

6.3.1.6 Pump Power

Power required to move the fluid, or *water power (WP)*:

$$\text{U.S. units} \quad WP(\text{horsepower}) = \frac{\text{Flow rate}(\text{gpm}) \times H(\text{ft}) \times \rho \left(\frac{\text{lbm}}{\text{ft}^3} \right)}{246,780}$$

$$\text{Metric units} \quad WP(\text{watts}) = \text{Flow rate} \left(\frac{\text{m}^3}{\text{s}} \right) \times H(\text{m}) \times \rho \left(\frac{\text{kg}}{\text{m}^3} \right) \times g \left(\frac{\text{m}}{\text{s}^2} \right)$$

Power required at the pump shaft, or *brake power (BP)*:

$$BP = \frac{WP}{\eta_{\text{pump}}}$$

Power required by the pump driver, or *supplied power (SP)*:

$$SP = \frac{WP}{\eta_{\text{pump}} \eta_{\text{driver}} \eta_{\text{transmission}}}$$

6.3.1.7 Temperature Rise in a Centrifugal Pump

$$\Delta T = \frac{BP(1 - \eta_{\text{pump}})}{c_p \dot{V} \rho}$$

6.3.1.8 Specific Speed (N_s) at the BEP

$$N_s = \frac{S \dot{V}^{0.5}}{H^{0.75}}$$

where head (H) and flow rate ($\dot{V}$) are taken at the *BEP*

6.3.1.9 Suction-Specific Speed (N_s) at the BEP

$$N_s = \frac{S \dot{V}^{0.5}}{(NPSH_r)^{0.75}}$$

6.3.1.10 System Curves

System curves are developed from different flow rates through a given system, using the Bernoulli equation.

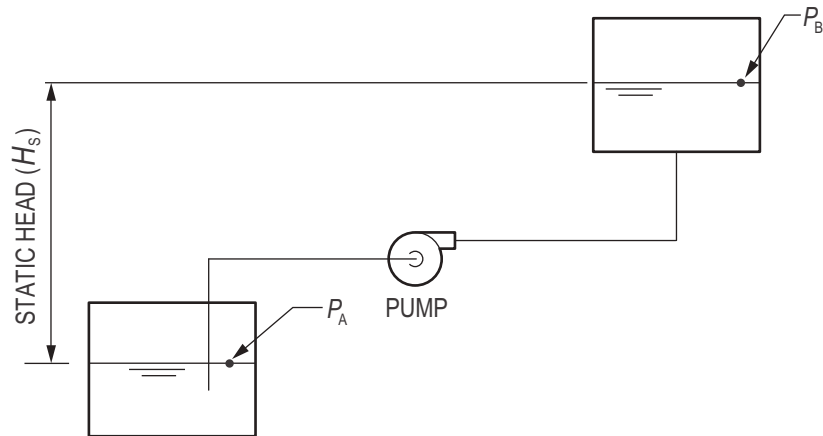
Note: The velocity head terms are usually omitted because the changes in $\frac{u^2}{2g}$ are negligible.

$$H_s = \text{pressure head} + \text{static head} (h_s) + \text{pipe losses}^* (h_f)$$

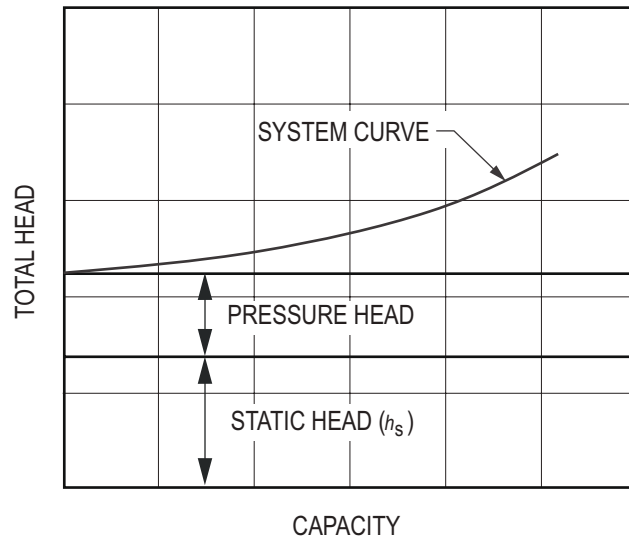
*Include friction, entrance, and exit losses:

$$H_s = \frac{(P_B - P_A) g_c}{\rho g} + h_s + h_f$$

Simple Pumping System



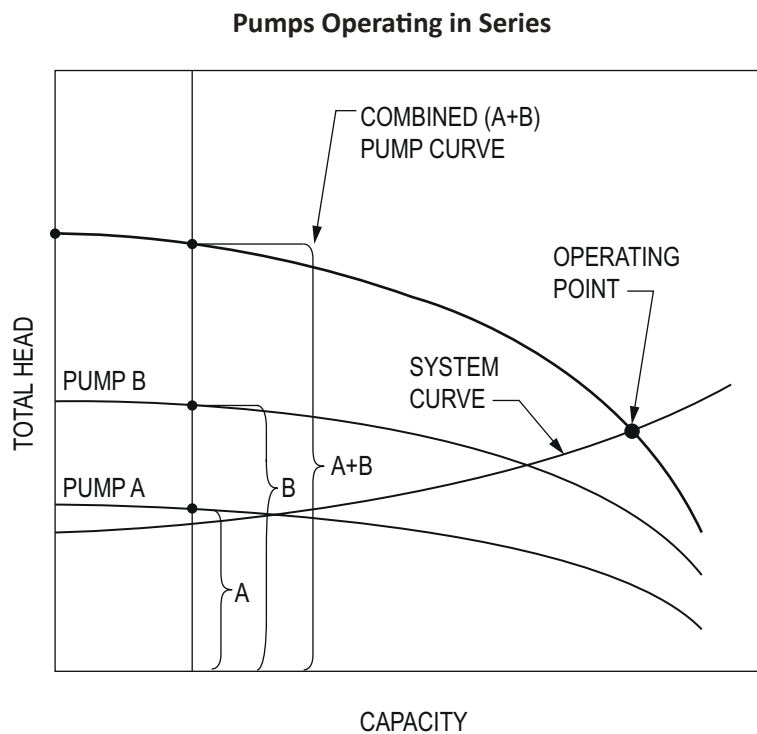
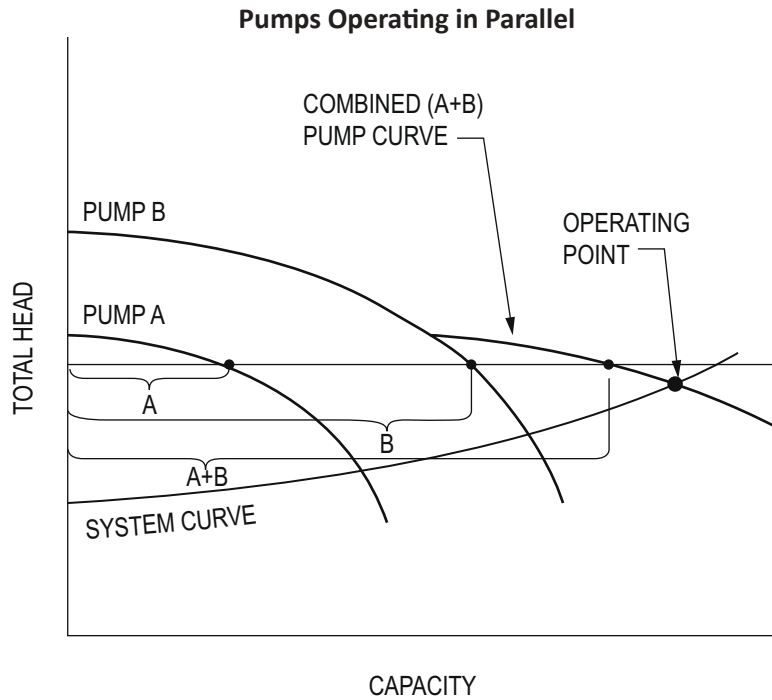
System Curve Plot



6.3.1.11 Pumps in Parallel and Series

Operating point: Centrifugal pumps operate at the intersection of the pump curve and the system curve.

For pumps in parallel, capacities are added horizontally. For pumps in series, heads are added vertically:



6.3.2 Fans, Blowers, Compressors

Fans and Blowers

Typical *backward curved fans*:

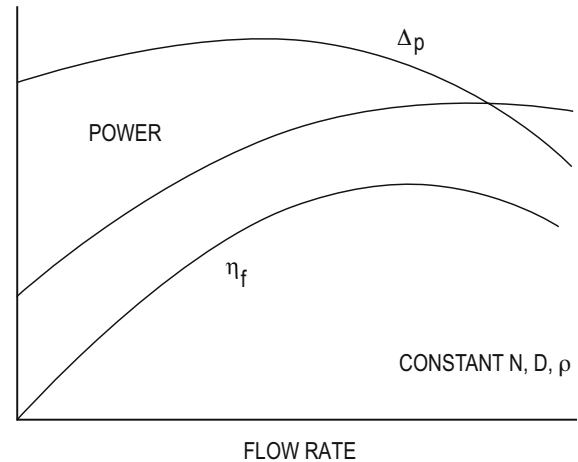
$$\dot{W} = \frac{\Delta P \dot{V}}{\eta_f}$$

where

$\dot{W}$ = fan power

ΔP = pressure rise

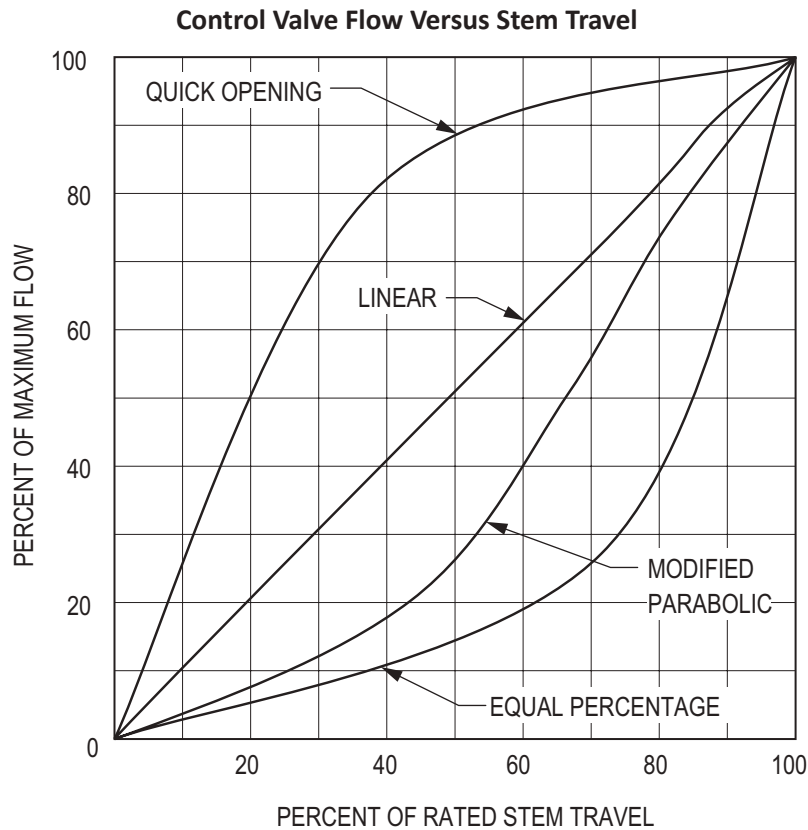
η_f = fan efficiency



6.3.3 Control Valves

6.3.3.1 Control Valve Flow Characteristics

Flow characteristic of a control valve: The relationship between valve capacity and valve stem travel (or valve lift).



Linear: Flow capacity increases linearly with stem travel.

Equal Percentage: Flow capacity increases exponentially with stem travel. Equal increments of stem travel produce equal percentage changes in the existing C_v .

Modified Parabolic: Valve characteristic is approximately midway between linear and equal-percentage characteristics. It provides fine throttling at low flow capacities and approximately linear characteristics at higher flow capacities.

Quick Opening: Provides large changes in flow for very small changes in early stem travel.

6.3.3.2 Control Valve Sizing (Traditional Method)

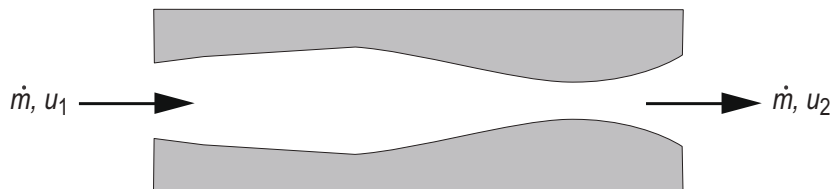
Control Valve Sizing Equations for Liquids (Incompressible Flow)

Equation	Use	Notes
$\dot{V} = C_V \sqrt{\frac{\Delta P}{SG}}$	Basic sizing equation; does not consider viscosity effects or valve recovery capabilities	C_V is the flow coefficient for a control valve. The value of C_V is dependent on the type of valve and also varies with stem travel or percentage of valve opening. The units and values for the flow coefficient are provided by the manufacturer.
$C_V = \dot{V} \sqrt{\frac{SG}{\Delta P}}$	Flow coefficient	For Newtonian fluids of viscosities similar to water.
$C_{V-Corr} = C_V F_V$	Corrected flow coefficient for viscosity	Use the appropriate F_V to predict pressure drop, select valve size, or predict flow rate.
$\Delta P_{max} = K_m (P_1 - r_C p_v)$	Maximum allowable differential pressure	where: K_m = valve recovery coefficient (provided by manufacturer) P_1 = valve body inlet pressure (absolute) p_v = liquid vapor pressure (absolute) at the valve body inlet temperature r_C = critical pressure ratio The critical pressure ratio is provided by the manufacturer or, in the absence of correlation data, the equation below can be used.
$r_C = 0.96 - 0.28 \sqrt{\frac{p_v}{p_c}}$	Critical pressure ratio (when manufacturer data is not available)	p_c is the critical pressure of the fluid (absolute).
$Re = 17,250 \frac{\dot{V} \rho}{\mu \sqrt{C_V}}$	Control valve Reynolds number	For engineering units only, where $\dot{V}$ is in gpm, ΔP is in psi, μ is in cP, and ρ is in $\frac{\text{lbm}}{\text{ft}^3}$.

6.3.4 Jet Propulsion

The force produced by jetting action is

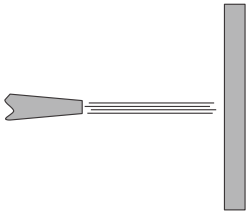
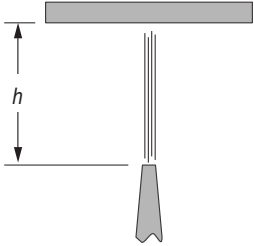
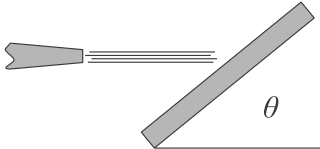
$$\mathbf{F} = \dot{m}(u_2 - u_1)$$



Therefore, according to the conservation of mass:

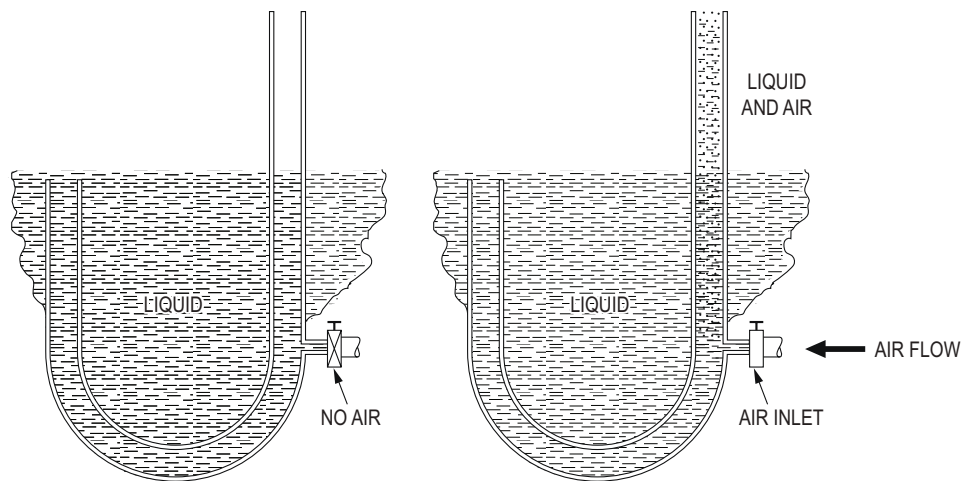
$$\mathbf{F} = \frac{\dot{V}_2 \rho_2 u_2 - \dot{V}_1 \rho_1 u_1}{g_c}$$

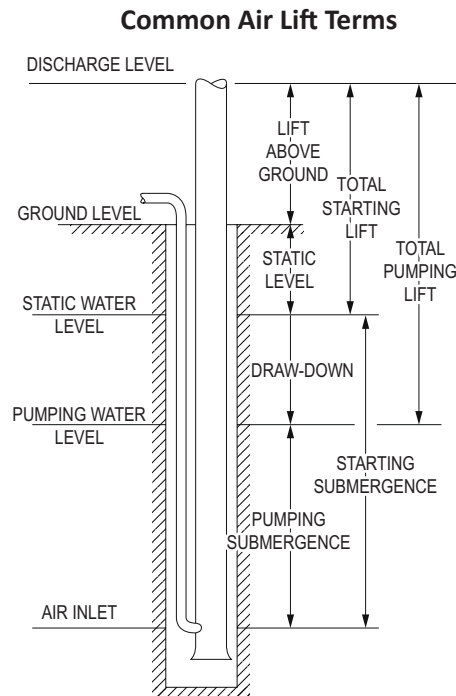
Jet Forces on Plates

Jet on a Vertical Plate	Jet on a Horizontal Plate	Jet on an Inclined Plate
		
$F_x = \frac{-\dot{m} u_{\text{jet}}}{g_c}$	$F_y = \frac{-\dot{m}^3 \sqrt{u_{\text{jet}}^2 - 2g h}}{g_c}$	$F = \frac{-\dot{m} u_{\text{jet}} \sin \theta}{g_c}$

6.3.5 Air Lift

Air Lift Operation





Air lifts are used to pump liquids and mixtures of liquids and solids. The volume of air required to pump is

$$V_a = \frac{L}{C \log_{10} \left(\frac{S+34}{34} \right)}$$

where

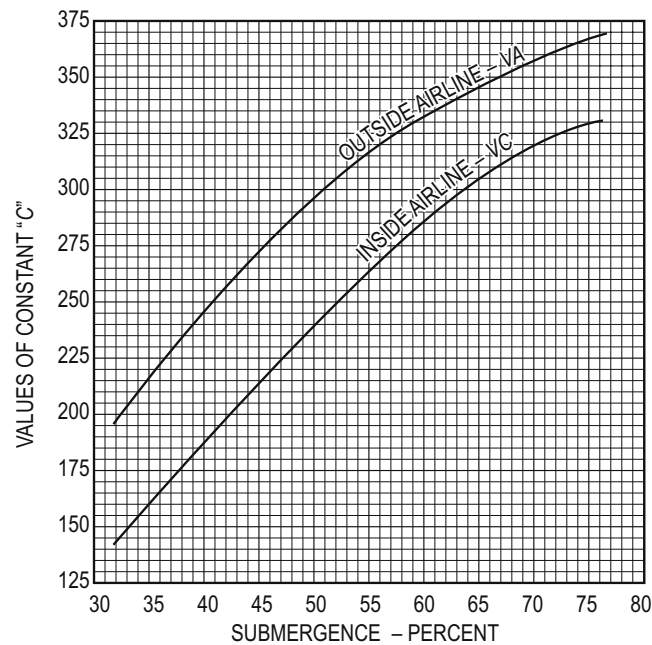
V_a = quantity of free air required per gallon of liquid pumped $\left(\frac{\text{ft}^3}{\text{gallon pumped}} \right)$

C = constant found for outside airline (VA) and inside airline (VC) in figure below

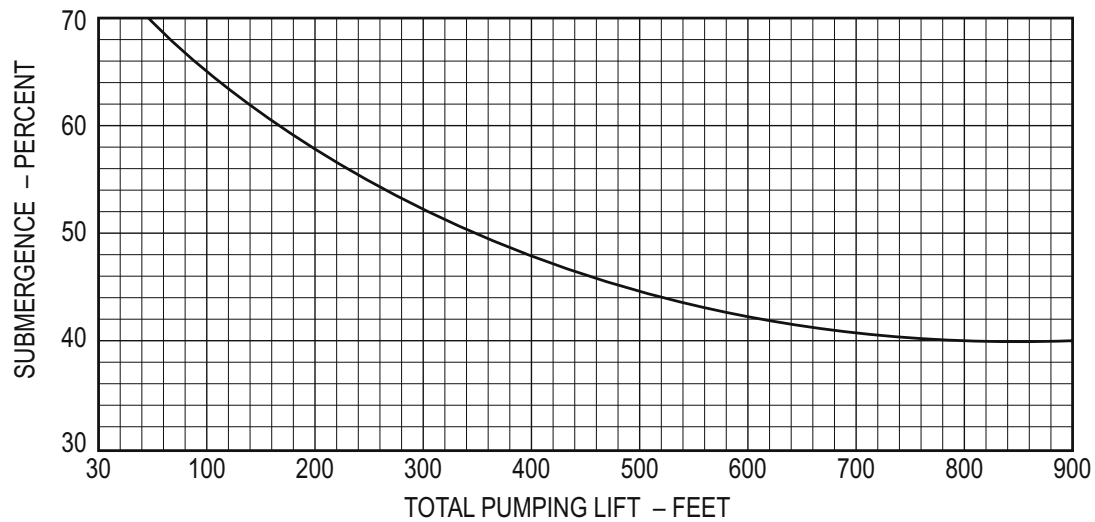
S = pumping submergence (%) in figure below

L = total pumping lift (ft)

Constant in Formula for V_a



Approximate Percent Submergence for Optimum Efficiency

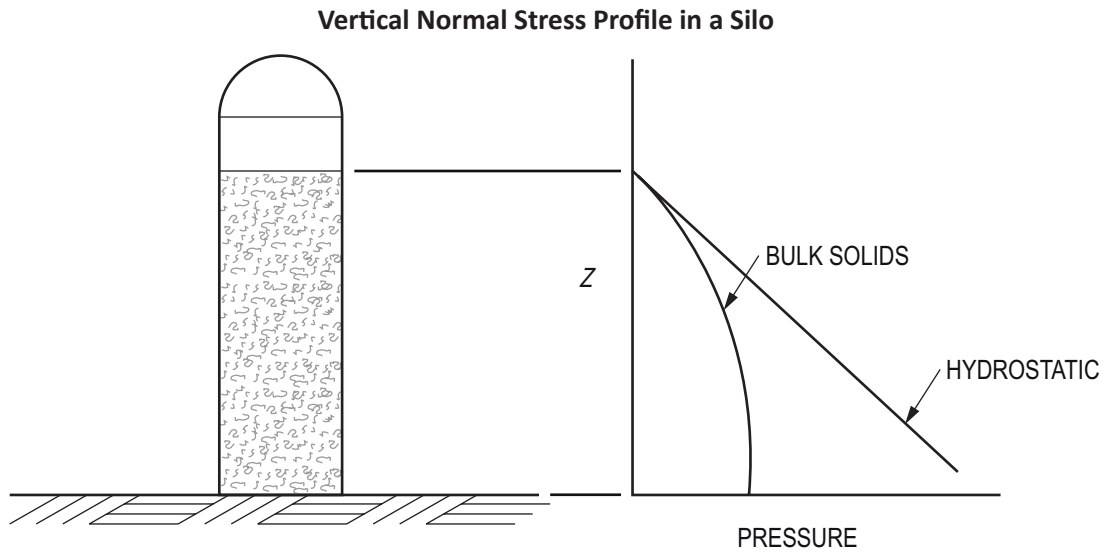


Use for either system with straight or tapered pipe. Graphs only available in U.S. units; SI not available.

Source: Gibbs, C.W., *New Compressed Air and Gas Data*, 2nd ed., Davidson, NC: Ingersoll-Rand Company, 1971, p. 31-3.

6.3.6 Solids Handling

6.3.6.1 Granular Media Storage



Source: Chase, George G., *Solids Notes 10*, Akron: University of Akron.

Compressive normal stress (P_v) in silos can be calculated by the *Janssen equation*:

$$P_v = \frac{\rho g D}{4\mu K g_c} \left[1 - \exp\left(\frac{-4\mu K z}{D}\right) \right]$$

where

ρ = granular bulk density

μ = solids coefficient of friction

D = silo diameter

K = lateral pressure ratio, where $P_w = K P_v$ (Janssen's assumption that vertical normal stress is proportional to the lateral normal stress)

z = bed depth at which pressure is being measured

Sources: Don McGlinchey, editor, *Bulk Solids Handling: Equipment Selection and Operation*, and J.M. Rotter, *Silo and Hopper Design for Strength*, Oxford: Blackwell Publishing Ltd., 2008.

6.3.6.2 Pneumatic Transport

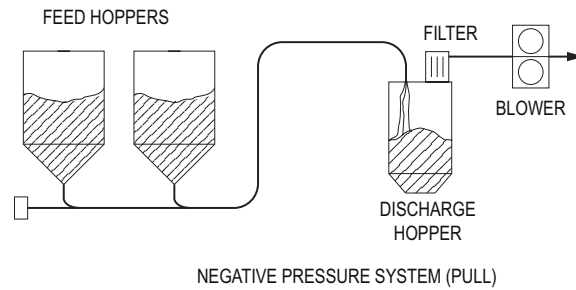
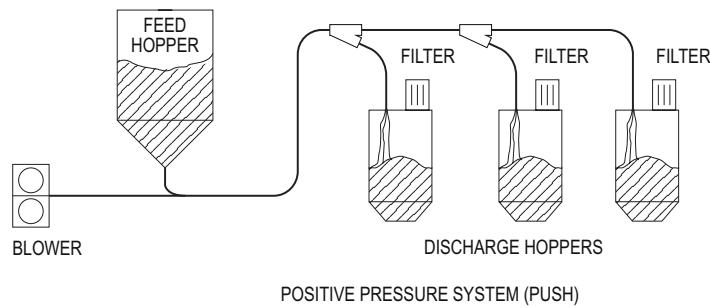
Pneumatic transport (or pneumatic conveying) is using gas to transport particulate solids through a pipeline (such as flour, pulverized coal, powdered clay).

Flow Regimes:

Dilute Phase—Particles are fully suspended at loadings less than 1%.

Dense Phase—Particles are not suspended (or periodically suspended) with loadings greater than 20%.

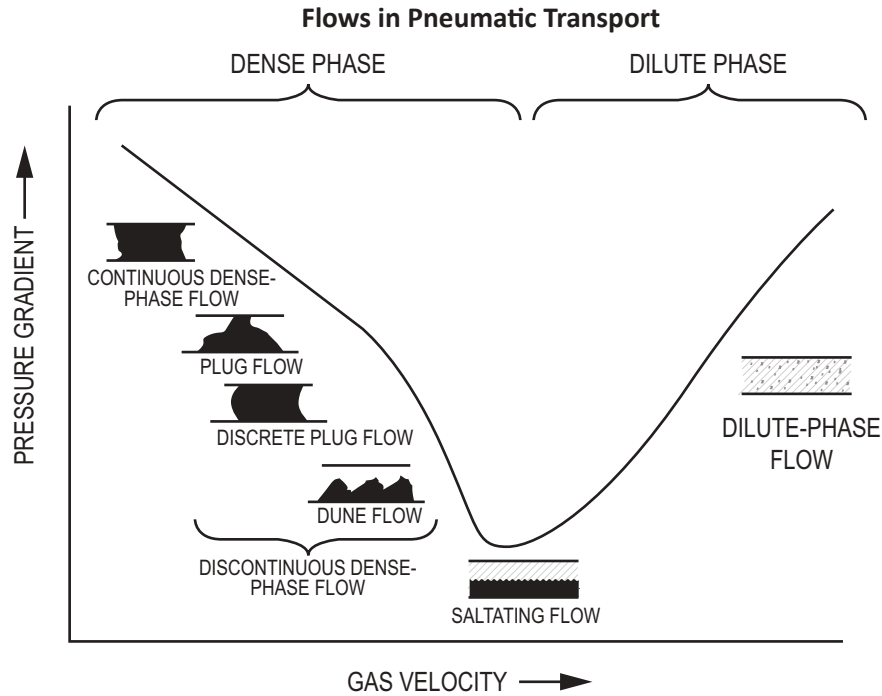
Pressure Systems



A "PUSH-PULL" SYSTEM USES BLOWERS TO SIMULTANEOUSLY
PUSH (POSITIVE PRESSURE) AND PULL THE SOLIDS (NEGATIVE PRESSURE)

Characteristics of Pneumatic Conveying Flow Regimes

Dilute Phase	Dense Phase
High velocity	Low velocity
Particles subject to attrition	Low particle attrition
Low pressure	High pressure
Low cost/simple operation	Complex operation
Low loadings	High solids loading



Definitions

Saltation—Settling of solid particles in the bottom of the pipe during dilute-phase pneumatic transport

Superficial gas velocity ($\bar{u}_g$)—The gas volumetric flow ($\dot{V}_g$) divided by the pipe cross-sectional area (A):

$$\bar{u}_g = \frac{\dot{V}_g}{A}$$

Superficial solids velocity ($\bar{u}_s$)—The solids volumetric flow ($\dot{V}_s$) divided by the pipe cross-sectional area:

$$\bar{u}_s = \frac{\dot{V}_s}{A}$$

where $\dot{V}_s = \frac{\dot{m}_s}{\rho_s}$, with $\dot{m}_s$ and ρ_s as the mass flow rate and density of the solid particles, respectively

Actual gas velocity (u_g):

$$u_g = \frac{\dot{V}_g}{A\epsilon}$$

where ϵ = void fraction

Actual particle velocity (u_s):

$$u_s = \frac{\dot{V}_s}{A(1 - \epsilon)}$$

Relationships

In vertical pipes, the minimum gas velocity ($u_{\min}$) to suspend particles is when the net upward force on the bed provided by the gas equals the net weight of the solids bed. (see the Fluidization section in this chapter)

$$F_B = W_B$$

Practical minimum gas velocity:

$$u = 2 u_{\min} = 2 \sqrt{\frac{4 g D_p \left(\frac{\rho_s}{\rho_g} - 1 \right)}{3 C_D}}$$

where $C_D = \frac{24}{Re}$

Mass flow rate of the solid particles:

$$\dot{m}_s = A u_s (1 - \epsilon) \rho_s$$

Mass flow rate of the gas:

$$\dot{m}_g = A u_g \epsilon \rho_g$$

Solids loading (R):

$$R = \frac{\dot{m}_s}{\dot{m}_g}$$

Concentration (volume fraction) of solids:

$$C_s = \frac{\dot{V}_s}{\dot{V}_s + \dot{V}_g} = \frac{\bar{u}_s}{\bar{u}_s + \bar{u}_g}$$

Dilute-phase pressure drop: The total pressure drop is the sum of the contributions from the carrier-gas pressure drop, acceleration of the solid particles, the friction of the solid particles against the pipe wall and fittings, the lifting of the solid particles through the vertical sections, and miscellaneous factors.

$$\Delta P = \Delta P_{gf} + (\Delta P_{sa} + \Delta P_{sf} + \Delta P_{sb} + \Delta P_{sv}) + \Delta P_{misc}$$

Carrier-gas pressure drop (ΔP_{gf}): For the purpose of this equation, compressible flow equations are not used. Treat the gas as an incompressible fluid:

$$\Delta P_{gf} = \frac{f L u_g^2 \rho_g}{2 g_c D}$$

Acceleration of solids pressure drop (ΔP_{sa}):

$$\Delta P_{sa} = \frac{\dot{m}_s u_s}{A g_c}$$

where A = pipe cross-sectional area

Solids friction in straight pipe pressure drop (ΔP_{sf}):

$$\Delta P_{sf} = \frac{\lambda_s R \rho_g u_g^2 L_{actual}}{2 D g_c}$$

where

λ_s = solids friction factor (if unknown, assume 0.2)

R = solids loading

L_{actual} = actual length of pipe (not equivalent length)

Solids friction in bends pressure drop (ΔP_{sb}):

$$\Delta P_{sb} = L_{eq} \left(\frac{\Delta P_s}{L_{actual}} \right)$$

Vertical lift pressure drop (ΔP_{sv}):

$$\Delta P_{sv} = \frac{R g Z \rho_g u_g}{g_c u_s}$$

where Z = total length of vertical pipe where the flow is upwards

Miscellaneous pressure drop (ΔP_{misc}):

where $\Delta P_{misc} \equiv$ additional pressure drop for other components, interferences, and other special conditions

Saltation velocity (u_{salt}):

$$R = \frac{1}{10^a} \left(\frac{u_{salt}}{\sqrt{g D}} \right)^b \quad (\text{Rizk correlation})$$

where

D = inside diameter of conveying pipe

$a = 1440 D_p + 1.96$ (SI units)

$= 439 D_p + 1.96$ (U.S. units)

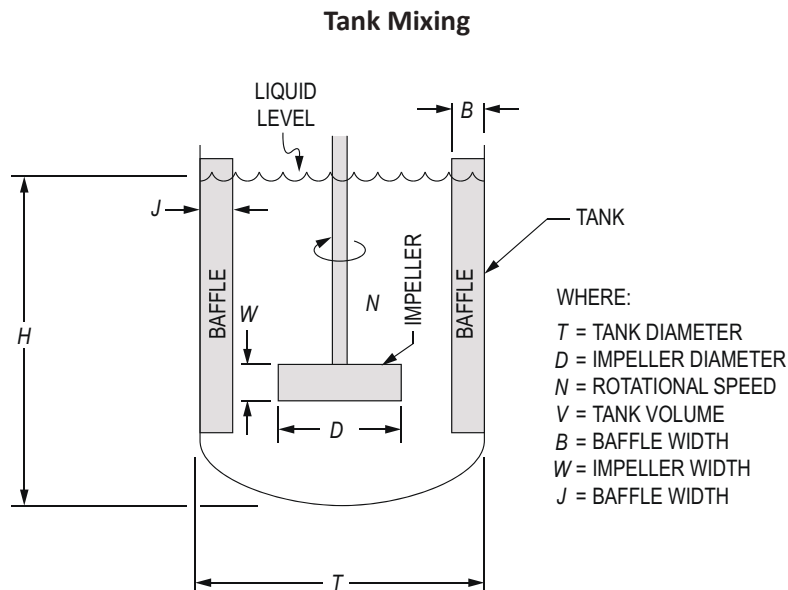
$b = 1100 D_p + 2.5$ (SI units)

$= 325 D_p + 2.5$ (U.S. units)

D_p = mean particle diameter

6.3.7 Mixing

6.3.7.1 Tank Mixing



Impeller Reynolds number

$$Re = \frac{D^2 N \rho}{\mu}$$

Flow Number

$$N_Q = \frac{q}{N D^3}$$

where q = volumetric flow rate through the impeller

Power Number

$$N_P = \frac{P g_c}{N^3 D^5 \rho}$$

where P = impeller power

Ratio of tangential liquid velocity at blade tips to blade tip velocity (K):

$$K = \pi^2 \frac{N_P}{N_Q}$$

Froude number for tank agitation:

$$Fr = \frac{N^2 D}{g}$$

Power function (ϕ) is defined by:

$$\phi = \frac{N_P}{Fr^m}$$

where $m = \frac{a - \log_{10} Re}{b}$

Examples of Mixing Configurations

Configuration (Unbaffled)	a	b
Six-blade turbine (vertical blades)	1.0	40.0
Three-blade propeller (pitch 2:1)	1.7	18.0
Three-blade propeller (pitch 1:1)	2.3	18.0

Power delivered to the liquid by an impeller:

$$P = \frac{\phi Fr^m N^3 D^5 \rho}{g_c}$$

For tanking mixing where the liquid surface has insignificant wave formation, the Froude number is not a factor:

$$P = \frac{\phi N^3 D^5 \rho}{g_c}$$

For $Re < 10$

$$P = \frac{K_L N^2 D^3 \mu}{g_c}$$

where K_L = empirical constant (laminar)

For $Re > 10,000$

$$P = \frac{K_T N^3 D^5 \rho}{g_c}$$

where K_T = empirical constant (fully turbulent)

Values of Constants K_L and K_T for Baffled Tanks

Type of Impeller	K_L	K_T
Propeller, square pitch, 3 blades	41.0	0.32
Propeller, pitch = 2, 3 blades	43.5	1.00
Turbine, 6 flat blades	71.0	6.30
Turbine, 6 curved blades	70.0	4.80
Fan turbine, 6 blades	70.0	1.65
Flat paddle, 2 blades	36.5	1.70
Shrouded turbine, 6 curved blades	97.5	1.08

Note: Table is specific to tank configuration and provided as an example only.

Power required to suspend particles to a maximum height (Z) using a *turbine impeller* is

$$P = g \rho_m V_m u_t (1 - \epsilon_m)^{\frac{2}{3}} \left(\frac{T}{D} \right)^{\frac{1}{2}} e^{4.35\beta}$$

where

$$\beta = \frac{Z-E}{T} - 0.1, \text{ with } E = \text{clearance between impeller and tank floor}$$

ρ_m, V_m = density and volume, respectively, of solid-liquid suspension, not including the clear liquid in zone above height Z (also known as cloud height)

u_t = terminal velocity of particles

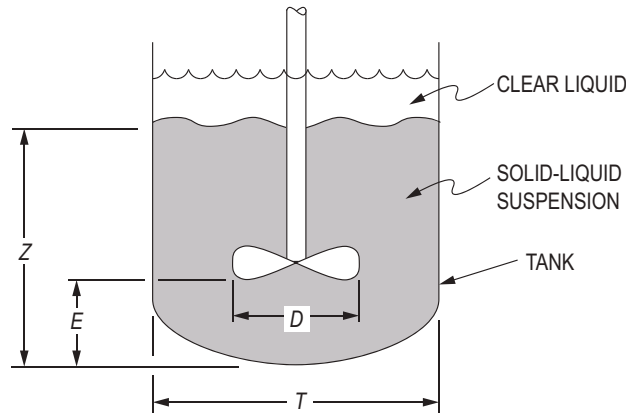
ϵ_m = volume fraction of liquid in zone occupied by suspension

and

$$\frac{1}{\rho_m} = \frac{1}{\rho_{\text{liquid}}} + x_{\text{solids}} \left(\frac{1}{\rho_{\text{solids}}} - \frac{1}{\rho_{\text{liquids}}} \right)$$

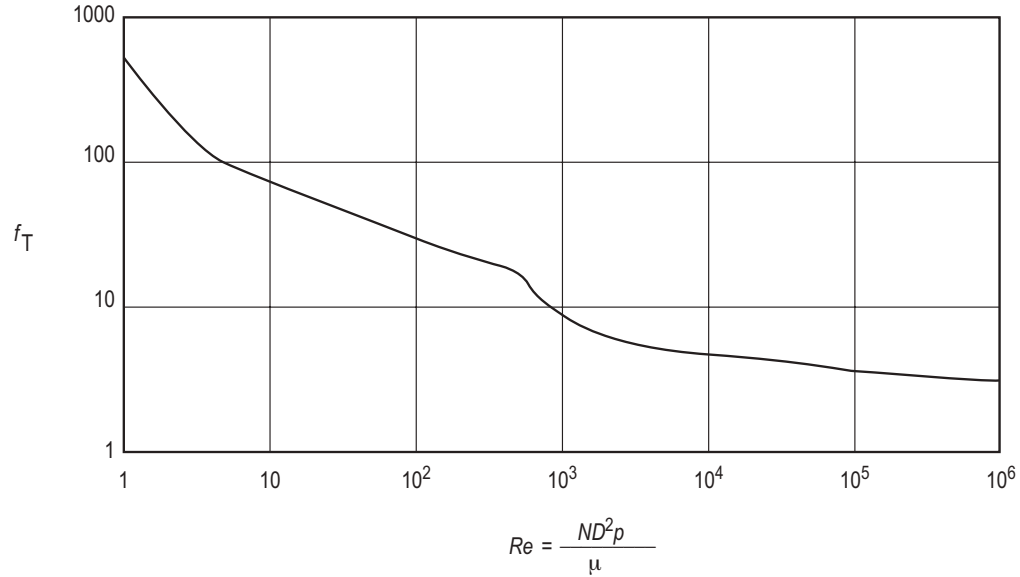
with x_{solids} = mass fraction of the solid particles in the solid-liquid suspension

Suspension of Particles in a Tank



6.3.7.2 Blending of Miscible Liquids in a Tank

Correlation of Blending Times for Miscible Liquids in a Turbine-Agitated, Baffled Vessel



Blending time factor (f_T) (for miscible Newtonian fluids only):

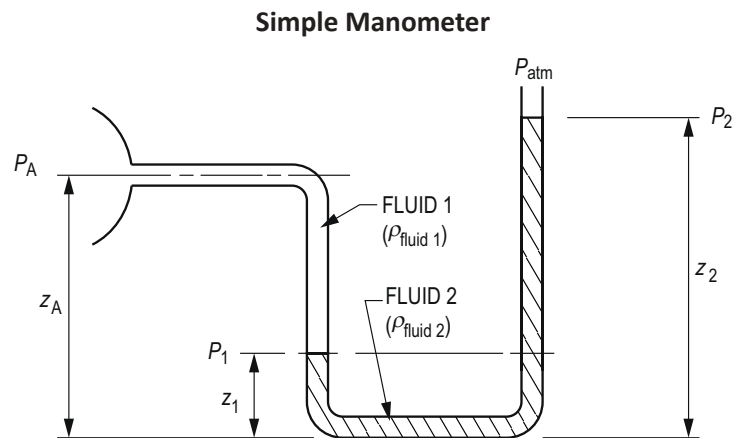
$$f_T = \frac{t(ND^2)^{\frac{2}{3}} g^{\frac{1}{6}} D^{\frac{1}{2}}}{H^{\frac{1}{2}} T^{\frac{3}{2}}}$$

where t = blend time (sec)

6.4 Flow and Pressure Measurement Techniques

6.4.1 Manometers and Barometers

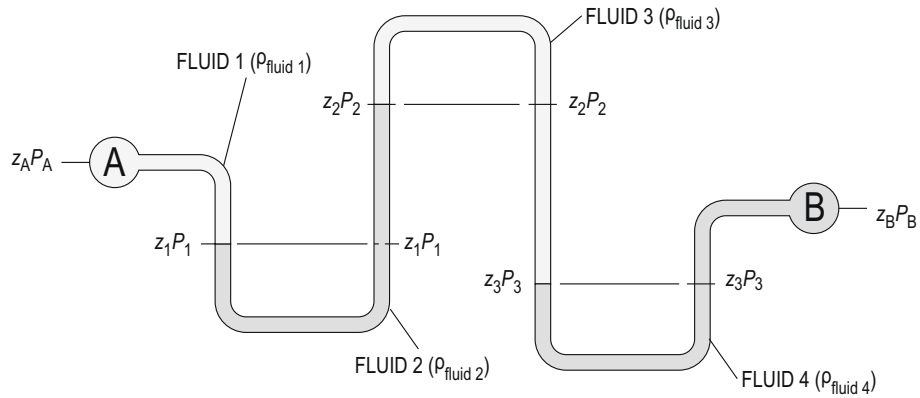
6.4.1.1 Simple Manometer



$$P_A - P_{atm} = P_A - P_2 = \frac{g}{g_c} [\rho_{fluid2} (z_2 - z_1) - \rho_{fluid1} (z_A - z_1)]$$

6.4.1.2 Manometer With Multiple Fluids

Manometer With Multiple Fluids

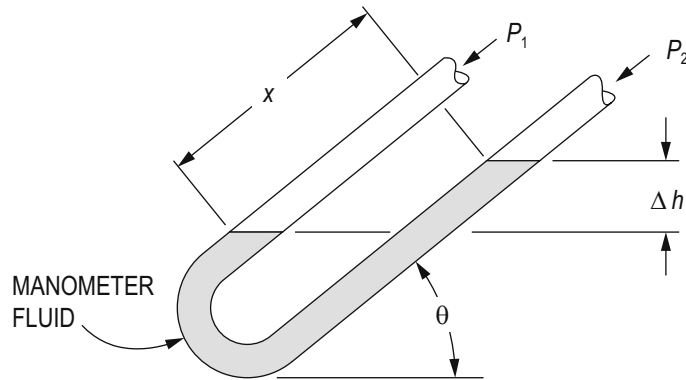


$$P_A - P_B = (P_A - P_1) + (P_1 - P_2) + (P_2 - P_3) + (P_3 - P_B)$$

$$P_A - P_B = \frac{g}{g_c} \left[\rho_{\text{fluid1}} (z_1 - z_A) + \rho_{\text{fluid2}} (z_2 - z_1) + \rho_{\text{fluid3}} (z_3 - z_2) + \rho_{\text{fluid4}} (z_B - z_3) \right]$$

6.4.1.3 Inclined U-Tube Manometer

Inclined U-Tube Manometer



$$P_1 - P_2 = \frac{g}{g_c} \rho_m x \sin \theta = \frac{g}{g_c} \rho_m \Delta h$$

where

x = difference in tube fill length

ρ_m = density of the manometer fluid (densities of the fluids on each side of the manometer are equal)

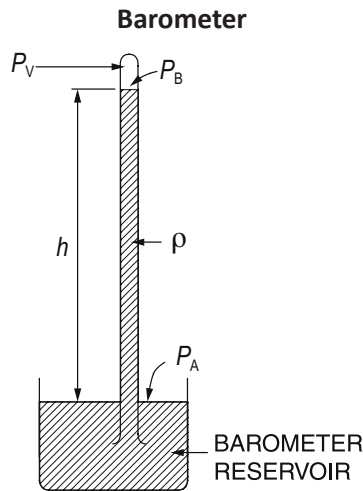
θ = angle of inclination (horizontal = 0°)

6.4.1.4 Barometers

Another device that works on the same principle as the manometer is the simple barometer.

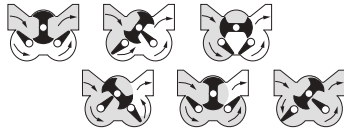
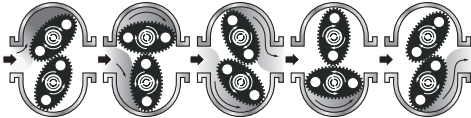
$$P_{\text{atm}} = P_A = P_v + \frac{\rho g h}{g_c} = P_B + \frac{\rho g h}{g_c}$$

where P_v = vapor pressure of the barometer fluid

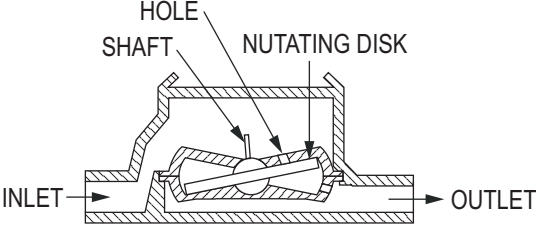
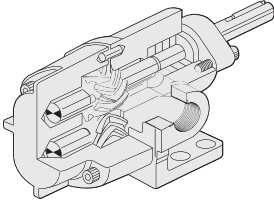
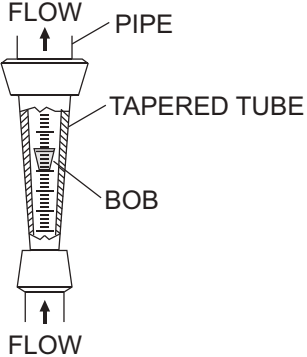


6.4.2 Flow Measurement Devices (Summary)

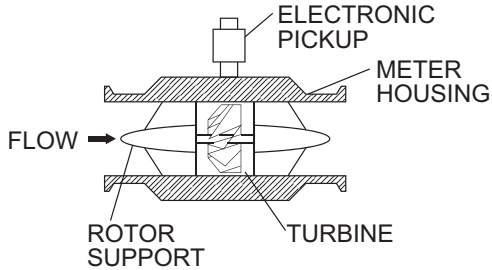
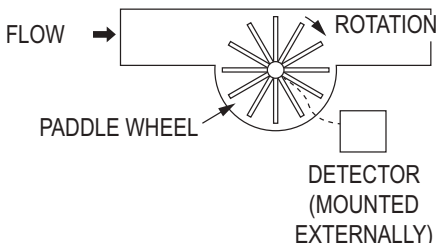
Flow Measurement Devices

Class	Meter Type	Description	Advantages	Drawbacks
Mechanical	Rotary piston	Rotary piston spins within a chamber of known volume. For each rotation, an amount of fluid passes through the piston chamber. The rotations are counted and the flow rate is determined from the rate of rotations. 	- Accurate; suitable for fuel metering - Suitable for low volume metering and laboratory or bench scale testing	- High permanent pressure drop at high flows - Clear liquids only - High cost
	Gear	Two rotating gears with synchronized, close-fitting teeth. A fixed quantity of liquid passes through the meter for each revolution. Permanent magnets in the rotating gears transmit a signal to a transducer for flow measurement.  <i>Operation of an oval gear meter</i>	- Accurate; suitable for fuel metering - Suitable for low volume metering and laboratory or bench scale testing	- High permanent pressure drop at high flows - Clear liquids only - High cost

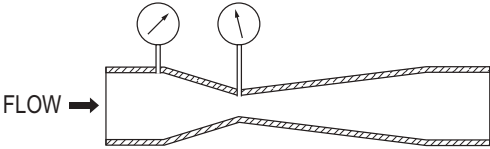
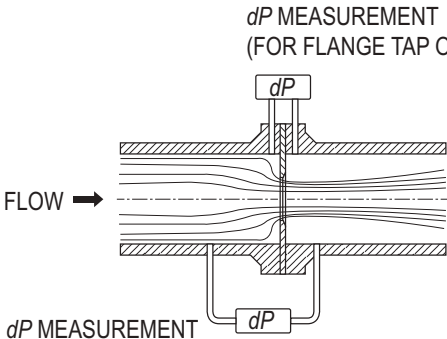
Flow Measurement Devices (cont'd)

Class	Meter Type	Description	Advantages	Drawbacks
Mechanical (cont'd)	Nutating Disk	Also known as a wobbly plate meter. Fluid enters a chamber of known volume. When the chamber is filled, the fluid is released, which causes the disk to perform a nutating action (wobble in a circular path without actually spinning on its axis). The motion is detected by either gearing or magnetic transducers. The flow rate is determined from the rate of motions. 	• Accurate and repeatable; used for water service metering • Good for hot liquids	• Accuracy is adversely affected by viscosities below the meter's designated threshold
	Helical	Counter-rotation of the gears carries known volumes of liquid axially down the length of the gears. The rotation rate is measured using sensors, which in turn correlates to flow rate.  Source: Flowserve Corp., Irving, TX	• Used for heavy and high-viscous liquids • Highest accuracy of any positive displacement flow meter	• Can only measure liquids • Low corrosion allowance • Cannot handle abrasive fluids
	Rotameter (variable area)	Fluid flows upward through a clear tapered tube and suspends a bob. The higher the flow rate, the higher the bob suspends in the tube. The bob is the indicator and the reading is obtained from the scale marked on the tube. 	• Simple operation with few moving parts and no external power source • Inexpensive and widely available • Accurate provided the fluid properties remain unchanged • Resistant to shock and chemical action	• Must be mounted vertically • Changes in fluid properties gives erroneous results • Not suited for large pipes (< 6 inches) • Readout uncertainty near bottom of the scale • Some fluids may obscure reading. • •

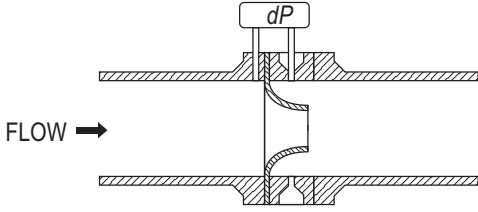
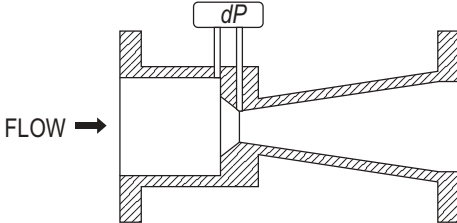
Flow Measurement Devices (cont'd)

Class	Meter Type	Description	Advantages	Drawbacks
Mechanical (cont'd)	Turbine (or Woltmann Type)	Fluid flows past a turbine wheel positioned in the center of the pipe with the shaft in line with the pipe. The rotational speed is proportional to the flow rate. Shaft rotation is detected electronically. 	• Simple and durable structure; can be installed vertically or horizontally • Can be designed to detect flow in either direction • Operates under a wide range of temperatures and pressures • Low pressure drop across the flow meter • Effective in applications with steady, high-speed flows • Can be used for gasses but not suitable for steam	• Cannot tolerate cavitation • Accuracy adversely affected by entrained gas • Sensitive to changes in fluid viscosity • Long straight runs of pipe upstream and downstream of the meter are needed • Bearings are prone to wear (though some are provided "bearingless") • Not suitable for steam
	Paddle Wheel Type	Fluid flows past a paddle wheel positioned off-center of the pipe with the shaft perpendicular with the pipe. The rotational speed is proportional to the flow rate. Shaft rotation is detected electronically.  Other meters in this class: Single Jet Multi Jet Pelton Wheel	• Simple and durable structure; can be installed vertically or horizontally • Easy installation into existing systems for insertion models • Can be designed to detect flow in either direction • Operates under a wide range of temperatures and pressures • Low pressure drop across the flow meter • Effective in applications with steady, high-speed flows	• Requires a full pipe of liquid • Not suitable for steam • Bearings are prone to wear

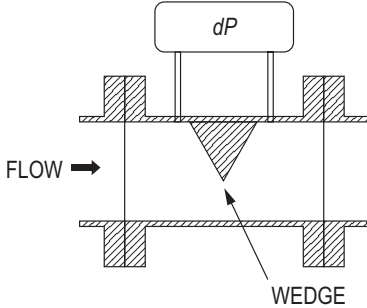
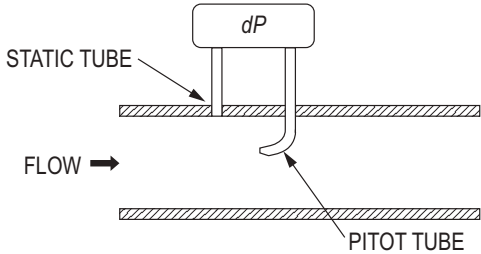
Flow Measurement Devices (cont'd)

Class	Meter Type	Description	Advantages	Drawbacks
Pressure	Venturi	The meter constricts the fluid flow and sensors measure the differential pressure before and within the constriction. The differential pressure is then converted to a corresponding flow rate. PRESSURE MEASUREMENT 	• Highly accurate over a wide range of flows • No moving parts • Low pressure drop	• Flow must be derived from pressure drop • Pipe must be full (mostly used for liquid service) • Occupies space ($\frac{L}{D}$ of approximately 50) • Cannot measure fluids in reverse flow
	Orifice Plate (also square-edge orifice plate)	Flow is restricted using a plate with a hole drilled through it. Sensors measure the differential pressure before and after the meter (two tap configurations are shown). The differential pressure is then converted to a corresponding flow rate. dP MEASUREMENT (FOR FLANGE TAP OPTION)  dP MEASUREMENT (FOR VENA CONTRACTA TAP OPTION) Note: Orifices may be drilled in the middle of the plate (concentric) or off-center (eccentric) to accommodate certain fluid types and flow regimes. Orifices may also be round or segmented.	• Accurate over a wide range of flows, but not suitable for trade use (2–4% of full scale) • No moving parts • Low cost; price does not dramatically increase with pipe size • Low maintenance (orifice plates can be replaced during maintenance operations) • Easy to convert to different applications or fluids by replacing the orifice plate • In common use	• Flow must be derived from pressure drop • Accuracy reduced at low flows • Plate materials prone to wear and corrosion, which adversely effects accuracy • Accuracy effected by high-viscous fluids • Moderate to high permanent pressure drop • Pipe must be full (for liquids)

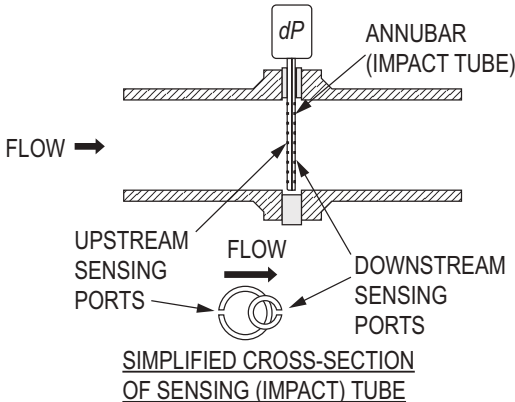
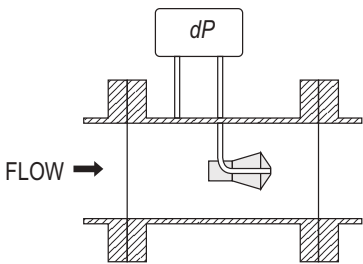
Flow Measurement Devices (cont'd)

Class	Meter Type	Description	Advantages	Drawbacks
Pressure (cont'd)	Nozzle	Similar to a venturi meter, but the inlet section is in the shape of an ellipse and there is no exit section. dP MEASUREMENT 	• More accurate than orifice plates • High flow capacity and high velocity applications • Less susceptible to wear and corrosion than orifice plates • Can operate in higher turbulence • Tolerant of fluids containing suspended solids • Less expensive than the venturi meter • Physically smaller than the venturi meter • Can indicate a reverse-flow condition	• Flow must be derived from pressure drop • More expensive than orifice plates • Takes up slightly more room than orifice plates • Higher permanent pressure drop than venturi meters • Pipe must be full (for liquids)
	Dall Tube	Similar to the venturi meter but more compact at the expense of some loss in accuracy and additional permanent pressure loss. 	• Similar performance as the venturi meter • Shorter length than the venturi meter • Low unrecoverable pressure loss • Accurate to within 1% of full scale	• More expensive than orifice plates or flow nozzle meters • Sensitive to turbulence • More complex to manufacture • Accuracy dependent on actual flow data • Cannot indicate a reverse-flow condition

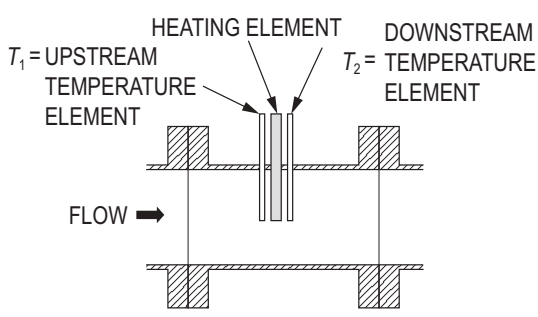
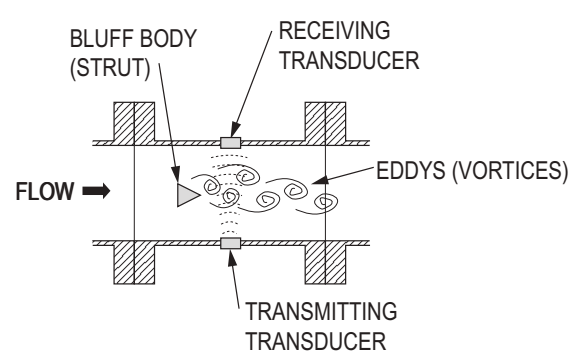
Flow Measurement Devices (cont'd)

Class	Meter Type	Description	Advantages	Drawbacks
Pressure (cont'd)	Wedge	Similar in principle to the orifice meter, a wedge placed in the flow stream creates the differential pressure element. The fluid is forced downward, similar to a segmented orifice plate, but is guided along a sloping wedge shape rather than a sharp edge. The differential pressure is then converted to a corresponding flow rate. 	Well suited for sludge, slurry, or high-viscous fluid service	- Differential pressure to flow rate dependent on empirical data unique to each model and application - High permanent pressure drop
	Pitot Tube	The pitot tube is primarily used for gas or air service. The Pitot tube measures the total pressure (dynamic and static pressures combined). The static tube measures the static pressure only. The difference between the two measurements reveals that the dynamic pressure is converted into the flow rate.  Note: The pitot tube (impact tube) and the static tube are sometimes provided within a single element.	- Essentially no pressure drop - Easy to install and use - Instrument can be removed when not in service - Can be used to measure gas velocities and to establish a velocity profile	- Low accuracy (differential pressure between static and dynamic is small and therefore prone to error) - Accuracy dependent on placement within the flow cross-section - Low rangeability - Requires clean fluids (tube easily plugs)

Flow Measurement Devices (cont'd)

Class	Meter Type	Description	Advantages	Drawbacks
Pressure (cont'd)	Annubar	The annubar or averaging pitot-tube flow meter measures the difference between the total pressure (upstream) and the static pressure (downstream) to derive the flow rate.  <u>SIMPLIFIED CROSS-SECTION OF SENSING (IMPACT) TUBE</u> Note: Temperature elements can be made integral with the impact tube to provide temperature compensation and corrections.	• Accurate (1% of full scale) • Compact design (sensing lines not required)	• Not suitable for dirty or viscous fluids • Element must be centered within the pipe
	Cone (or V-Cone)	A cone is inserted in the flow stream to create a differential pressure similar to a venturi meter or Dall tube meter, which is then correlated to the flow rate. 	• Excellent accuracy (0.5% of full scale) • Suitable for fluids with suspended solids • Compact design (0–2 pipe diameters) • Suitable for gas flow measurement	• Moderate permanent pressure drop • Requires extensive calibration to achieve rated accuracy • Must operate within rated β-ratio range

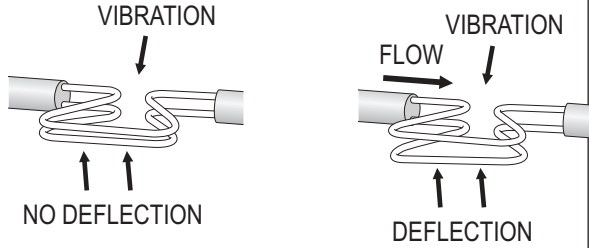
Flow Measurement Devices (cont'd)

Class	Meter Type	Description	Advantages	Drawbacks
Thermal	Thermal Mass Meters	A known amount of heat is applied to the heating element. Some of this heat is lost to the flowing fluid. As flow increases, more heat is lost. The amount of heat lost is sensed using temperature elements (comparing the upstream and downstream values). The fluid flow is derived from the known heat input and the temperature measurements. 	• Used primarily for gas service (stack flow measurement and emissions monitoring) • Low pressure drop • The temperature and heating elements come in a single element assembly for a compact design • Detects low flows (laminar flows) • Can be used as a velocity meter • Results are in true mass flow	• Thermal properties of the gas must be known • Moderate accuracy • Not for steam service
		Vortices (or eddy currents) created by an obstruction are detected by ultrasonic or optical transducers. The rate of vortex formation and subsequent shedding caused by the bluff body or obstruction is proportional to the fluid velocity. 		
Vortex	Vortex Shedding		• Can be used for liquids, gases, and steam • Low wear • Low cost to install and maintain • Low sensitivity to variations in process conditions • Stable long-term accuracy and repeatability • Applicable to a wide range of process temperatures • Available for a wide variety of pipe sizes	• Not suitable for low flow rates • Minimum length of straight pipe is required upstream and downstream of the meter

Flow Measurement Devices (cont'd)

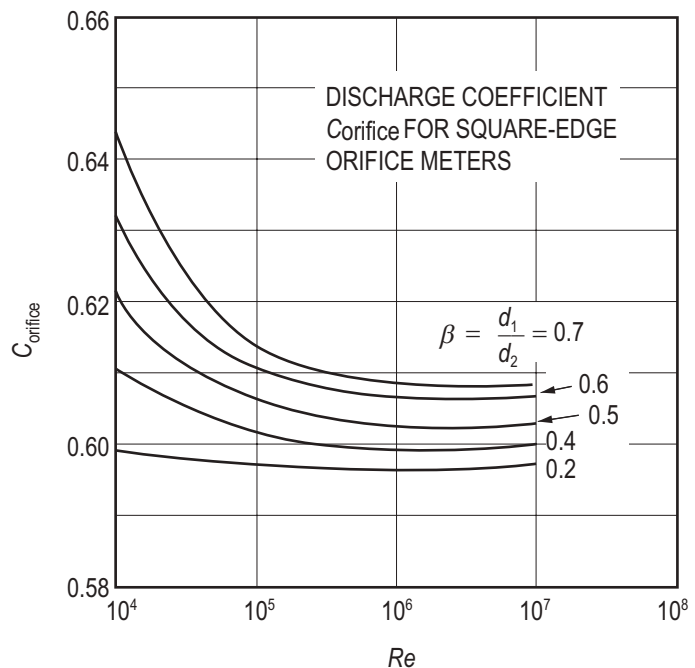
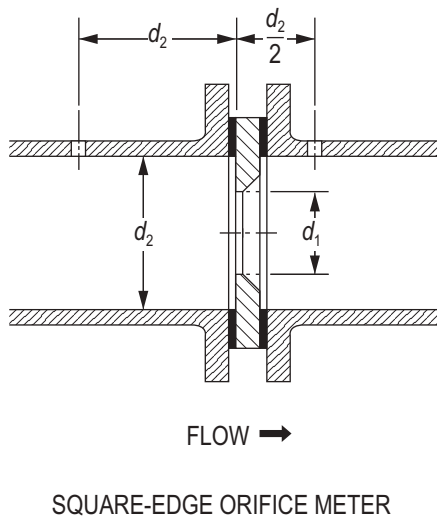
Class	Meter Type	Description	Advantages	Drawbacks
Magnetic	Mag Meter	The operation of a magnetic flow meter or mag meter is based on Faraday's Law, which states that the voltage induced across any conductor as it moves at right angles through a magnetic field is proportional to the velocity of that conductor. $E \propto u \times B \times D$ where E = voltage generated in a conductor u = velocity of the conductor B = magnetic field strength D = length of the conductor The flow meter applies a magnetic field through the entire cross-section of the flow tube. The velocity is then determined by the meter by measuring the magnetic strength.	• Ideal for dirty water or other conductive fluids • Suitable for fluids with two-phase flow • No pressure drop (models are available for full pipe bores) • Accurate • Measures true volumetric flow	• Does not work on nonconductive fluids (e.g., hydrocarbons) • Expensive • Does not correlate to mass flow until fluid or bulk slurry density is known
		For a simple <i>Doppler system</i>, sound waves are used to determine the velocity of a fluid flowing in a pipe. At zero flow, the frequencies of an ultrasonic wave transmitted into a pipe and its reflections from the fluid are the same. At flow, the frequency of the reflected wave is different because of the Doppler effect. As fluid velocity increases, the frequency shift increases linearly. A transmitter evaluates the frequency shift to determine the flow rate. For a <i>Transit time system</i>, ultrasonic waves are sent and received between transducers in both directions in the pipe. At zero flow, it takes the same time to travel upstream and downstream between the transducers. At flow, the upstream wave travels more slowly and takes more time than the downstream wave. As fluid velocity increases, the difference between the upstream and downstream times also increases. A transmitter evaluates the delay times to determine the flow rate. Note: Either method can be deployed as a clamp-on unit (dry) or be installed integral to the fluid (wet).		
Ultrasonic			• Sufficiently accurate for custody transfer • Clamp-on systems suitable for field testing and verification of installed flow meters	• Expensive • Sensitive to stray vibrations • Unwanted attenuation can occur • Fluid must be able to transmit ultrasonic waves

Flow Measurement Devices (cont'd)

Class	Meter Type	Description	Advantages	Drawbacks
Impulse	Coriolis	A Coriolis flow meter uses the natural phenomenon in which an object begins to "drift" as it travels from or toward the center of a rotation occurring in the surrounding environment. Coriolis flow meters generate this effect by diverting the fluid flow through a pair of parallel U-tubes with an induced vibration (by an actuator, not shown) perpendicular to the flow. The vibration simulates a rotation of the pipe and the resulting Coriolis "drift" in the fluid causes the U-tubes to twist and deviate from their parallel alignment. The force producing this deviation is proportional to the mass flow rate through the U-tubes. 	• Suitable for highly viscous fluids • Insensitive to temperature and fluid properties • Measures mass flow rate directly	• Not accurate for gases at low flow rates • High permanent pressure drop

6.4.3 Orifice, Nozzle, and Venturi Meters

6.4.3.1 Square-Edge Orifice Meter (Vena Contracta Taps)



Flow Coefficient (C) and Orifice Loss Coefficient

$$C = \frac{C_{\text{orifice}}}{\sqrt{1 - \beta^4}}$$

Incompressible Flow

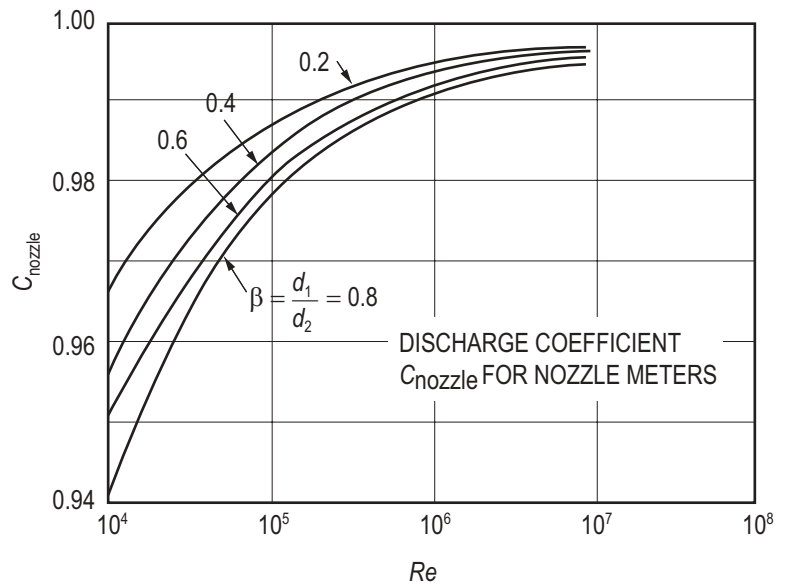
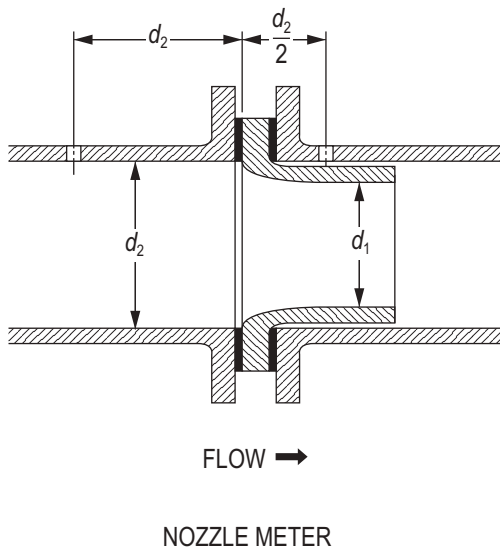
$$\dot{V} = C A_{\text{orifice}} \sqrt{\frac{2 g_c \Delta P}{\rho}}$$

Compressible Flow

$$\dot{V} = Y C A_{\text{orifice}} \sqrt{\frac{2 g_c \Delta P}{\rho}}$$

where Y = expansion factor

6.4.3.2 Flow Nozzle Meter



Flow Coefficient (C)

$$C = \frac{C_{\text{nozzle}}}{\sqrt{1 - \beta^4}}$$

Incompressible Flow

$$\dot{V} = C A_{\text{nozzle}} \sqrt{\frac{2 g_c \Delta P}{\rho}}$$

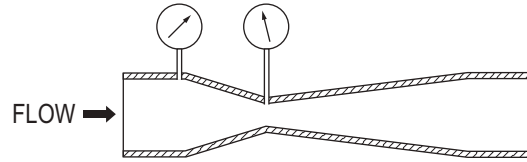
Compressible Flow

$$\dot{V} = Y C A_{\text{nozzle}} \sqrt{\frac{2 g_c \Delta P}{\rho}}$$

where Y = expansion factor

6.4.3.3 Venturi Flow Nozzle Meter

The venturi discharge coefficient is a function of the specific geometry of the meter.



Flow Coefficient (C)

$$C = \frac{C_{\text{venturi}}}{\sqrt{1 - \beta^4}}$$

Incompressible Flow

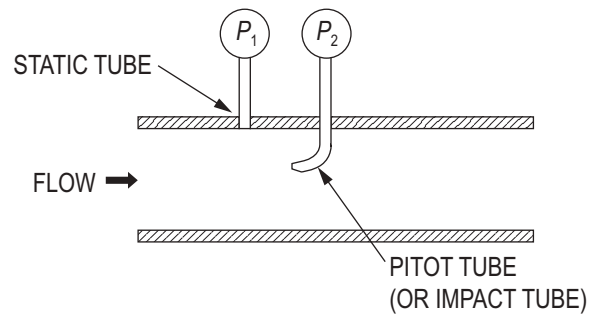
$$\dot{V} = C A_{\text{venturi}} \sqrt{\frac{2 g_c \Delta P}{\rho}}$$

Compressible Flow

$$\dot{V} = Y C A_{\text{venturi}} \sqrt{\frac{2 g_c \Delta P}{\rho}}$$

where Y = expansion factor

6.4.3.4 Pitot Tube Flow Meter



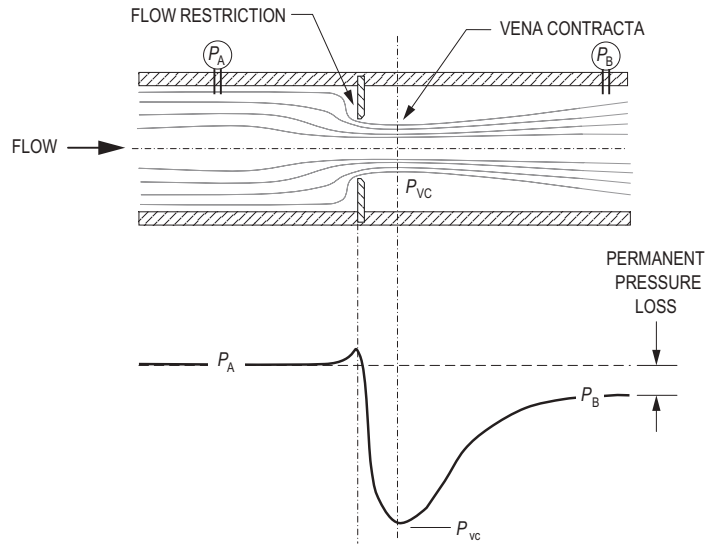
P_1 measures the static pressure. Assuming elevation effects are negligible, P_2 is the stagnation pressure:

$$P_1 + \frac{\rho u^2}{2 g_c}$$

Therefore:

$$u = \sqrt{\frac{2 g_c (P_2 - P_1)}{\rho}}$$

6.4.3.5 Permanent Pressure Loss in Flow Meters



Pressure Loss Across Restrictive Flow Meters: The *permanent pressure loss* (or *nonrecoverable pressure drop*) across a restrictive flow meter (e.g., orifices and nozzles) is the difference between the upstream pressure, P_A , (the static pressure not influenced by the device, or roughly one pipe diameter upstream), and the pressure measured downstream of the device where the static pressure recovery is complete, P_B (approximately six pipe diameters downstream).

For a given measured differential pressure, ΔP (e.g., radius or flange taps for an orifice), the permanent pressure loss can be estimated by:

$$P_A - P_B = \Delta P \left(\frac{\sqrt{1 - \beta^4(1 - C_d^2)} - C_d \beta^2}{\sqrt{1 - \beta^4(1 - C_d^2)} + C_d \beta^2} \right)$$

where

C_d = coefficient of discharge for the device (e.g., C_{orifice} and C_{nozzle})

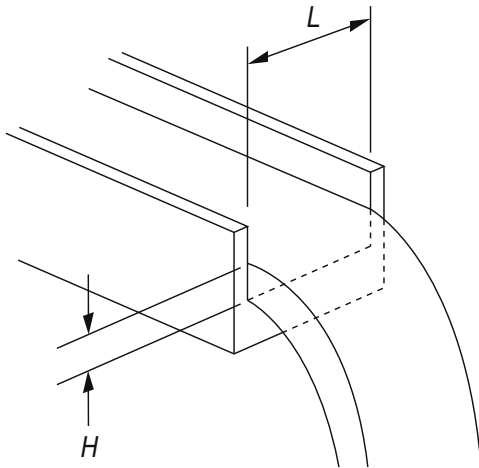
For orifice plates and nozzles, the flow coefficient, K , can be approximated

$$K = \left(\frac{\sqrt{1 - \beta^4(1 - C_d^2)}}{C_d \beta^2} - 1 \right)^2$$

Source: ASME MFC-3M-2004, Measurement of Fluid Flow in Pipes Using Orifice, Nozzle, and Venturi

6.4.3.6 Weir Meters

Rectangular Weir—Suppressed



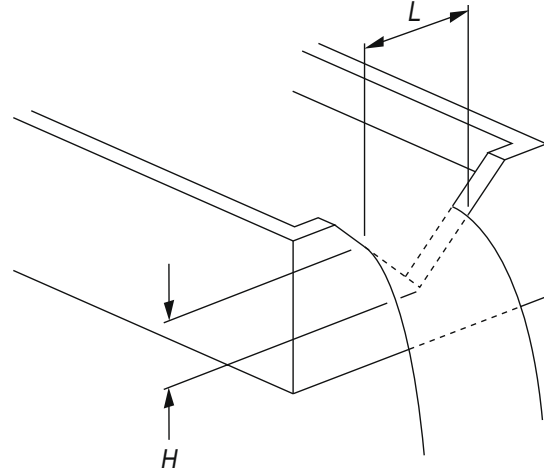
$$\dot{V} = C L H^{\frac{3}{2}}$$

where

$$C = 3.33 \frac{\text{ft}^{0.5}}{\text{sec}}$$

$$C = 1.84 \frac{\text{m}^{0.5}}{\text{s}}$$

V-Notch Weir (90° Notch)



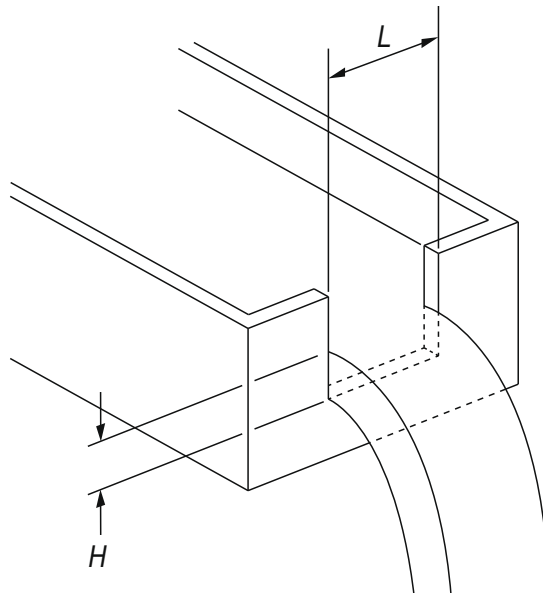
$$\dot{V} = C H^{\frac{5}{2}}$$

where

$$C = 2.5 \frac{\text{ft}^{0.5}}{\text{sec}}$$

$$C = 1.4 \frac{\text{m}^{0.5}}{\text{s}}$$

Rectangular Weir—Contracted



$$\dot{V} = C (L - 0.2H) H^{\frac{3}{2}}$$

where $C = 3.33 \frac{\text{ft}^{0.5}}{\text{sec}}$

$$C = 1.84 \frac{\text{m}^{0.5}}{\text{s}}$$

6.5 Tables

Pipe Dimensions and Weights

Weights are based on carbon steel pipe

Pipe Size	OD	Identification			Wall Thickness		Weight		Inside Diameter	
inches <i>mm</i>	inches <i>mm</i>	Steel		Stainless Steel	inches	<i>mm</i>	<i>lbm</i> <i>ft</i>	<i>kg</i> <i>m</i>	inches	<i>mm</i>
		Iron Pipe	Schedule No.	Schedule						
1/8 6	0.405 10.3	STD XS	10	10S	0.049	1.24	0.19	0.28	0.307	7.82
			40	40S	0.068	1.73	0.24	0.37	0.269	6.84
			80	80S	0.095	2.41	0.31	0.47	0.215	5.84
1/4 8	0.54 13.7	STD XS	10	10S	0.065	1.65	0.33	0.49	0.410	10.40
			40	40S	0.088	2.24	0.43	0.63	0.364	9.22
			80	80S	0.119	3.02	0.54	0.80	0.302	7.66
3/8 10	0.675 17.1	STD XS	10	10S	0.065	1.65	0.42	0.63	0.545	13.80
			40	40S	0.091	2.31	0.57	0.84	0.493	12.48
			80	80S	0.126	3.20	0.74	1.10	0.423	10.70
1/2 15	0.840 21.3	STD XS	5	5S	0.065	1.65	0.54	0.80	0.710	18.00
			10	10S	0.083	2.11	0.67	1.00	0.674	17.08
			40	40S	0.109	2.77	0.85	1.27	0.622	15.76
		XX	80	80S	0.147	3.73	1.09	1.62	0.546	13.84
			160		0.188	4.78	1.31	1.95	0.464	11.74
					0.294	7.47	1.72	2.55	0.252	6.36
3/4 20	1.050 26.7	STD XS	5	5S	0.065	1.65	0.69	1.03	0.920	23.40
			10	10S	0.083	2.11	0.86	1.28	0.884	22.48
			40	40S	0.113	2.87	1.13	1.69	0.824	20.96
		XX	80	80S	0.154	3.91	1.48	2.20	0.742	18.88
			160		0.219	5.56	1.95	2.90	0.612	15.58
					0.308	7.82	2.44	3.64	0.434	11.06
1 25	1.315 33.4	STD XS	5	5S	0.065	1.65	0.87	1.29	1.185	30.10
			10	10S	0.109	2.77	1.41	2.09	1.097	27.86
			40	40S	0.133	3.38	1.68	2.50	1.049	26.64
		XX	80	80S	0.179	4.55	2.17	3.24	0.957	24.30
			160		0.250	6.35	2.85	4.24	0.815	20.70
					0.358	9.09	3.66	5.45	0.599	15.22
1-1/4 32	1.660 42.2	STD XS	5	5S	0.065	1.65	1.11	1.65	1.530	38.90
			10	10S	0.109	2.77	1.81	2.69	1.442	36.66
			40	40S	0.140	3.56	2.27	3.39	1.380	35.08
		XX	80	80S	0.191	4.85	3.00	4.47	1.278	32.50
			160		0.250	6.35	3.77	5.61	1.160	29.50
					0.382	9.70	5.22	7.77	0.896	22.80
1-1/2 40	1.900 48.3	STD XS	5	5S	0.065	1.65	1.28	1.90	1.770	45.00
			10	10S	0.109	2.77	2.09	3.11	1.682	42.76
			40	40S	0.145	3.68	2.72	4.05	1.610	40.94
		XX	80	80S	0.200	5.08	3.63	5.41	1.500	38.14
			160		0.281	7.14	4.86	7.25	1.338	34.02
					0.400	10.15	6.41	9.55	1.100	28.00

Pipe Dimensions and Weights (cont'd)

Weights are based on carbon steel pipe

Pipe Size	OD	Identification			Wall Thickness		Weight		Inside Diameter	
inches <i>mm</i>	inches <i>mm</i>	Steel		Stain- less Steel Schedule	inches	<i>mm</i>	<i>lbm</i> <i>ft</i>	<i>kg</i> <i>m</i>	inches	<i>mm</i>
		Iron Pipe	Schedule No.							
2 50	2.375 60.3	STD XS XX	5	5S	0.065	1.65	1.61	2.39	2.245	57.00
			10	10S	0.109	2.77	2.64	3.93	2.157	54.76
			40	40S	0.154	3.91	3.66	5.44	2.067	52.48
			80	80S	0.218	5.54	5.03	7.48	1.939	49.22
			160		0.344	8.74	7.47	11.11	1.687	42.82
					0.436	11.07	9.04	13.44	1.503	38.16
2-1/2 65	2.875 73	STD XS XX	5	5S	0.083	2.11	2.48	3.69	2.709	68.78
			10	10S	0.120	3.05	3.53	5.26	2.635	66.90
			40	40S	0.203	5.16	5.80	8.63	2.469	62.68
			80	80S	0.276	7.01	7.67	11.41	2.323	58.98
			160		0.375	9.53	10.02	14.92	2.125	53.94
					0.552	14.02	13.71	20.39	1.771	44.96
3 80	3.5 88.9	STD XS XX	5	5S	0.083	2.11	3.03	4.52	3.334	84.68
			10	10S	0.120	3.05	4.34	6.46	3.260	82.80
			40	40S	0.216	5.49	7.58	11.29	3.068	77.92
			80	80S	0.300	7.62	10.26	15.27	2.900	73.66
			160		0.438	11.13	14.34	21.35	2.624	66.64
					0.600	15.24	18.6	27.68	2.300	58.42
3-1/2 90	4 101.6	STD XS XX	5	5S	0.083	2.11	3.48	5.18	3.834	97.38
			10	10S	0.120	3.05	4.98	7.41	3.760	95.50
			40	40S	0.226	5.74	9.12	13.57	3.548	90.12
			80	80S	0.318	8.08	12.52	18.64	3.364	85.44
					0.636	16.15	22.87	34.03	2.728	69.30
4 100	4.5 114.3	STD XS XX	5	5S	0.083	2.11	3.92	5.84	4.334	110.08
			10	10S	0.120	3.05	5.62	8.37	4.260	108.20
			40	40S	0.237	6.02	10.8	16.08	4.026	102.26
			80	80S	0.337	8.56	15.00	22.32	3.826	97.18
			120		0.438	11.13	19.02	28.32	3.624	92.04
			160		0.531	13.49	22.53	33.54	3.438	87.32
					0.674	17.12	27.57	41.03	3.152	80.06
4-1/2 115	5 127	STD	40	40S	0.247	6.27	12.55	18.67	4.506	114.46
		XS	80	80S	0.355	9.02	17.63	26.24	4.290	108.96
		XX			0.710	18.03	32.56	48.45	3.580	90.94
5 125	5.563 141.3	STD XS XX	5	5S	0.109	2.77	6.36	9.46	5.345	135.76
			10	10S	0.134	3.40	7.78	11.56	5.295	134.50
			40	40S	0.258	6.55	14.63	21.77	5.047	128.20
			80	80S	0.375	9.53	20.80	30.97	4.813	122.24
			120		0.500	12.70	27.06	40.28	4.563	115.90
			160		0.625	15.88	32.99	49.12	4.313	109.54
					0.750	19.05	38.59	57.43	4.063	103.20

Pipe Dimensions and Weights (cont'd)

Weights are based on carbon steel pipe

Pipe Size	OD	Identification			Wall Thickness		Weight		Inside Diameter	
inches <i>mm</i>	inches <i>mm</i>	Steel		Stain- less Steel Schedule	inches	<i>mm</i>	<i>lbm</i> <i>ft</i>	<i>kg</i> <i>m</i>	inches	<i>mm</i>
		Iron Pipe	Schedule No.							
6 <i>150</i>	6.625 <i>168.3</i>	STD XS	5	5S	0.109	<i>2.77</i>	7.59	<i>11.31</i>	6.407	<i>162.76</i>
			10	10S	0.134	<i>3.40</i>	9.30	<i>13.83</i>	6.357	<i>161.50</i>
			40	40S	0.280	<i>7.11</i>	18.99	<i>28.26</i>	6.065	<i>154.08</i>
			80	80S	0.432	<i>10.97</i>	28.60	<i>42.56</i>	5.761	<i>146.36</i>
			120		0.562	<i>14.27</i>	36.43	<i>54.21</i>	5.501	<i>139.76</i>
			160		0.719	<i>18.26</i>	45.39	<i>67.57</i>	5.187	<i>131.78</i>
		XX			0.864	<i>21.95</i>	53.21	<i>79.22</i>	4.897	<i>124.40</i>
7 <i>175</i>	7.625 <i>193.7</i>	STD	40	40S	0.301	<i>7.65</i>	23.57	<i>35.10</i>	7.023	<i>178.40</i>
		XS		80S	0.500	<i>12.70</i>	38.08	<i>56.69</i>	6.625	<i>168.30</i>
		XX			0.875	<i>22.23</i>	63.14	<i>94.00</i>	5.875	<i>149.24</i>
8 <i>200</i>	8.625 <i>219.1</i>	STD	5	5S	0.109	<i>2.77</i>	9.92	<i>14.78</i>	8.407	<i>213.56</i>
			10	10S	0.148	<i>3.76</i>	13.41	<i>19.97</i>	8.329	<i>211.58</i>
			20		0.250	<i>6.35</i>	22.38	<i>33.32</i>	8.125	<i>206.40</i>
			30		0.277	<i>7.04</i>	24.72	<i>36.82</i>	8.071	<i>205.02</i>
			40	40S	0.322	<i>8.18</i>	28.58	<i>42.55</i>	7.981	<i>202.74</i>
			60		0.406	<i>10.31</i>	35.67	<i>53.09</i>	7.813	<i>198.48</i>
		XS	80	80S	0.500	<i>12.70</i>	43.43	<i>64.64</i>	7.625	<i>193.70</i>
			100		0.594	<i>15.09</i>	51.00	<i>75.92</i>	7.437	<i>188.92</i>
			120		0.719	<i>18.26</i>	60.77	<i>90.44</i>	7.187	<i>182.58</i>
			140		0.812	<i>20.62</i>	67.82	<i>100.93</i>	7.001	<i>177.86</i>
		XX			0.875	<i>22.23</i>	72.49	<i>107.93</i>	6.875	<i>174.64</i>
					0.906	<i>23.01</i>	74.76	<i>111.27</i>	6.813	<i>173.08</i>
9 <i>225</i>	9.625 <i>244.5</i>	STD		40S	0.342	<i>8.69</i>	33.94	<i>50.54</i>	8.941	<i>227.12</i>
		XS		80S	0.500	<i>12.70</i>	48.77	<i>72.60</i>	8.625	<i>219.10</i>
		XX			0.875	<i>22.23</i>	81.85	<i>121.85</i>	7.875	<i>200.04</i>
10 <i>250</i>	10.75 <i>273</i>	STD	5	5S	0.134	<i>3.40</i>	15.21	<i>22.61</i>	10.482	<i>266.20</i>
			10	10S	0.165	<i>4.19</i>	18.67	<i>27.78</i>	10.420	<i>264.62</i>
			20		0.250	<i>6.35</i>	28.06	<i>41.76</i>	10.250	<i>260.30</i>
			30		0.307	<i>7.80</i>	34.27	<i>51.01</i>	10.136	<i>257.40</i>
			40	40S	0.365	<i>9.27</i>	40.52	<i>60.29</i>	10.020	<i>254.46</i>
			60	80S	0.500	<i>12.70</i>	54.79	<i>81.53</i>	9.750	<i>247.60</i>
		XS	80		0.594	<i>15.09</i>	64.49	<i>95.98</i>	9.562	<i>242.82</i>
			100		0.719	<i>18.26</i>	77.10	<i>114.71</i>	9.312	<i>236.48</i>
			120		0.844	<i>21.44</i>	89.38	<i>133.01</i>	9.062	<i>230.12</i>
			140		1.000	<i>25.40</i>	104.23	<i>155.10</i>	8.750	<i>222.20</i>
		XX	160		1.125	<i>28.58</i>	115.75	<i>172.27</i>	8.500	<i>215.84</i>
11 <i>275</i>	11.75 <i>298.5</i>	STD		40S	0.375	<i>9.53</i>	45.60	<i>67.91</i>	11.000	<i>279.44</i>
		XS		80S	0.500	<i>12.70</i>	60.13	<i>89.51</i>	10.750	<i>273.10</i>
		XX			0.875	<i>22.23</i>	101.72	<i>151.46</i>	10.000	<i>254.04</i>

Pipe Dimensions and Weights (cont'd)

Weights are based on carbon steel pipe

Pipe Size	OD	Identification			Wall Thickness		Weight		Inside Diameter	
inches <i>mm</i>	inches <i>mm</i>	Steel		Stain- less Steel Schedule	inches	<i>mm</i>	<i>lbm</i> <i>ft</i>	<i>kg</i> <i>m</i>	inches	<i>mm</i>
		Iron Pipe	Schedule No.							
12 <i>300</i>	12.75 <i>323.8</i>	STD		5S	0.156	<i>3.96</i>	21.00	<i>31.24</i>	12.438	<i>315.88</i>
				10S	0.180	<i>4.57</i>	24.19	<i>35.98</i>	12.390	<i>314.66</i>
			20		0.250	<i>6.35</i>	33.41	<i>49.71</i>	12.250	<i>311.10</i>
					0.330	<i>8.38</i>	43.81	<i>65.19</i>	12.090	<i>307.04</i>
			40	40S	0.375	<i>9.53</i>	49.61	<i>73.86</i>	12.000	<i>304.74</i>
					0.406	<i>10.31</i>	53.57	<i>79.71</i>	11.938	<i>303.18</i>
		XS		80S	0.500	<i>12.70</i>	65.48	<i>97.44</i>	11.750	<i>298.40</i>
					0.562	<i>14.27</i>	73.22	<i>108.93</i>	11.626	<i>295.26</i>
					0.688	<i>17.48</i>	88.71	<i>132.05</i>	11.374	<i>288.84</i>
					0.844	<i>21.44</i>	107.42	<i>159.87</i>	11.062	<i>280.92</i>
		XX			1.000	<i>25.40</i>	125.61	<i>186.92</i>	10.750	<i>273.00</i>
					1.125	<i>28.58</i>	139.81	<i>208.08</i>	10.500	<i>266.64</i>
					1.312	<i>33.32</i>	160.42	<i>238.69</i>	10.126	<i>257.16</i>
14 <i>350</i>	14 <i>355.6</i>	STD		10S	0.188	<i>4.78</i>	27.76	<i>41.36</i>	13.624	<i>346.04</i>
					0.250	<i>6.35</i>	36.75	<i>54.69</i>	13.500	<i>342.90</i>
			20		0.312	<i>7.92</i>	45.65	<i>67.91</i>	13.376	<i>339.76</i>
					0.375	<i>9.53</i>	54.62	<i>81.33</i>	13.250	<i>336.54</i>
			40	40S	0.438	<i>11.13</i>	63.50	<i>94.55</i>	13.124	<i>333.34</i>
					0.500	<i>12.70</i>	72.16	<i>107.40</i>	13.000	<i>330.20</i>
		XS		80S	0.594	<i>15.09</i>	85.13	<i>126.72</i>	12.812	<i>325.42</i>
					0.750	<i>19.05</i>	106.23	<i>158.11</i>	12.500	<i>317.50</i>
					0.938	<i>23.83</i>	130.98	<i>194.98</i>	12.124	<i>307.94</i>
					1.094	<i>27.79</i>	150.93	<i>224.66</i>	11.812	<i>300.02</i>
					1.250	<i>31.75</i>	170.37	<i>253.58</i>	11.500	<i>292.10</i>
					1.406	<i>35.71</i>	189.29	<i>281.72</i>	11.188	<i>284.18</i>
16 <i>400</i>	16 <i>406.4</i>	STD		10S	0.188	<i>4.78</i>	31.78	<i>47.34</i>	15.624	<i>396.84</i>
					0.250	<i>6.35</i>	42.09	<i>62.65</i>	15.500	<i>393.70</i>
			20		0.312	<i>7.92</i>	52.32	<i>77.83</i>	15.376	<i>390.56</i>
					0.375	<i>9.53</i>	62.64	<i>93.27</i>	15.250	<i>387.34</i>
			40	40S	0.438	<i>11.13</i>	72.16	<i>107.40</i>	15.124	<i>384.10</i>
					0.500	<i>12.70</i>	82.85	<i>123.31</i>	15.000	<i>381.00</i>
		XS		80S	0.656	<i>16.66</i>	107.60	<i>160.13</i>	14.688	<i>373.08</i>
					0.844	<i>21.44</i>	136.74	<i>203.54</i>	14.312	<i>363.52</i>
					1.031	<i>26.19</i>	164.98	<i>245.57</i>	13.938	<i>354.02</i>
					1.219	<i>30.96</i>	192.61	<i>286.66</i>	13.562	<i>344.48</i>
					1.438	<i>36.53</i>	223.85	<i>333.21</i>	13.124	<i>333.34</i>
					1.594	<i>40.49</i>	245.48	<i>365.38</i>	12.812	<i>325.42</i>

Pipe Dimensions and Weights (cont'd)

Weights are based on carbon steel pipe

Pipe Size	OD	Identification			Wall Thickness		Weight		Inside Diameter	
inches <i>mm</i>	inches <i>mm</i>	Steel		Stain- less Steel Schedule	inches	<i>mm</i>	<i>lbm</i> <i>ft</i>	<i>kg</i> <i>m</i>	inches	<i>mm</i>
		Iron Pipe	Schedule No.							
18 <i>450</i>	18 <i>457</i>	STD	10	10S	0.188	4.78	35.80	53.31	17.624	447.44
					0.250	6.35	47.44	70.57	17.500	444.30
					0.312	7.92	58.99	87.71	17.376	441.16
			20	40S	0.375	9.53	70.65	105.17	17.250	437.94
		XS	30		0.438	11.13	82.23	122.38	17.124	434.74
			40	80S	0.500	12.70	93.54	139.16	17.000	431.60
			60		0.562	14.27	104.76	155.81	16.876	428.46
			80		0.750	19.05	138.30	205.75	16.500	418.90
			100		0.938	23.83	171.08	254.57	16.124	409.34
			120		1.156	29.36	208.15	309.64	15.688	398.28
			140		1.375	34.93	244.37	363.58	15.250	387.14
			160		1.562	39.67	274.48	408.28	14.876	377.66
					1.781	45.24	308.79	459.39	14.438	366.52
20 <i>500</i>	20 <i>508</i>	STD	10	10S	0.218	5.54	46.10	68.61	19.564	496.92
					0.250	6.35	52.78	78.56	19.500	495.30
					0.375	9.53	78.67	117.15	19.250	488.94
		XS	20	40S	0.375	9.53	78.67	117.15	19.250	488.94
			30		0.500	12.70	104.23	155.13	19.000	482.60
			40		0.594	15.09	123.23	183.43	18.812	477.82
			60		0.812	20.62	166.56	247.84	18.376	466.76
			80		1.031	26.19	209.06	311.19	17.938	455.62
			100		1.281	32.54	256.34	381.55	17.438	442.92
			120		1.500	38.10	296.65	441.52	17.000	431.80
			140		1.750	44.45	341.41	508.15	16.500	419.10
			160		1.969	50.01	379.53	564.85	16.062	407.98
22 <i>550</i>	22 <i>559</i>	STD	10	10S	0.218	5.54	50.76	75.55	21.564	547.92
					0.250	6.35	58.13	86.55	21.500	546.30
					0.375	9.53	86.69	129.14	21.250	539.94
		XS	20	40S	0.375	9.53	86.69	129.14	21.250	539.94
			30		0.500	12.70	114.92	171.10	21.000	533.60
			60		0.875	22.23	197.60	294.27	20.250	514.54
			80		1.125	28.58	251.05	373.85	19.750	501.84
			100		1.375	34.93	303.16	451.45	19.250	489.14
			120		1.625	41.28	353.94	527.05	18.750	476.44
			140		1.875	47.63	403.38	600.67	18.250	463.74
			160		2.125	53.98	451.49	672.30	17.750	451.04
24 <i>600</i>	24 <i>610</i>	STD	10	10S	0.250	6.35	63.47	94.53	23.500	597.30
					0.375	9.53	94.71	141.12	23.250	590.94
					0.500	12.7	125.61	187.07	23.000	584.60
		XS	20	40S	0.562	14.27	140.81	209.65	22.876	581.46
			30		0.688	17.48	171.45	255.43	22.624	575.04
			40		0.969	24.61	238.57	355.28	22.062	560.78
			60		1.219	30.96	296.86	442.11	21.562	548.08
			80		1.531	38.89	367.74	547.74	20.938	532.22
			100		1.812	46.02	429.79	640.07	20.376	517.96
			120		2.062	52.37	483.57	720.19	19.876	505.26
			140		2.344	59.54	542.64	808.27	19.312	490.92
			160							

Pipe Dimensions and Weights (cont'd)

Weights are based on carbon steel pipe

Pipe Size	OD	Identification			Wall Thickness		Weight		Inside Diameter	
inches <i>mm</i>	inches <i>mm</i>	Steel		Stain- less Steel Schedule	inches	<i>mm</i>	lbm ft	kg m	inches	<i>mm</i>
		Iron Pipe	Schedule No.							
26 <i>650</i>	26 <i>660</i>	STD XS	10	40S 80S	0.312	7.92	85.68	127.36	25.376	644.16
			20		0.375	9.53	102.72	152.88	25.250	640.94
					0.500	12.70	136.30	202.74	25.000	634.60
28 <i>700</i>	28 <i>711</i>	STD	10	40S	0.312	7.92	92.35	137.32	27.376	695.16
			20		0.375	9.53	110.74	164.86	27.250	691.94
					0.500	12.70	146.99	218.71	27.000	685.60
					30	0.625	15.88	182.90	272.23	26.750
30 <i>750</i>	30 <i>762</i>	STD XS	10	10S	0.312	7.92	99.02	147.29	29.376	746.16
			20	40S	0.375	9.53	118.76	176.85	29.250	742.94
				80S	0.500	12.70	157.68	234.68	29.000	736.60
				30	0.625	15.88	196.26	292.2	28.750	730.24
32 <i>800</i>	32 <i>813</i>	STD XS	10	40S 80S	0.312	7.92	105.69	157.25	31.376	797.16
			20		0.375	9.53	126.78	188.83	31.250	793.94
					0.500	12.70	168.37	250.65	31.000	787.60
					0.625	15.88	209.62	312.17	30.750	781.24
					40	0.688	17.48	230.29	342.94	30.624
34 <i>850</i>	34 <i>864</i>	STD XS	10	40S 80S	0.312	7.92	112.36	167.21	33.376	848.16
			20		0.375	9.53	134.79	200.82	33.250	844.94
					0.500	12.70	179.06	266.63	33.000	838.60
					0.625	15.88	222.99	332.14	32.750	832.24
					40	0.688	17.48	245.00	364.92	32.624
36 <i>900</i>	36 <i>914</i>	STD XS	10	40S 80S	0.312	7.92	119.03	176.97	35.376	898.16
			20		0.375	9.53	142.81	212.57	35.250	894.94
					0.500	12.70	189.75	282.29	35.00	888.60
42 <i>1050</i>	42 <i>1067</i>		30		0.375	9.53	166.86	248.53	41.250	1047.94
			60		0.500	12.70	221.82	330.21	41.000	1041.60
48 <i>1200</i>	48 <i>1219</i>		30		0.375	9.53	190.92	284.25	47.250	1199.94
			60		0.500	12.70	253.89	377.81	47.000	1193.60

Tubing Sizes (U.S.)

Size (inches)	OD (inches)	Gauge (nominal inches)											
		24ga	22ga	20ga	18ga	16ga	14ga	12ga	11ga	9ga	7ga	1/4"	3/8"
		Inside Diameter (inches)											
		0.022	0.028	0.035	0.049	0.062	0.083	0.109	0.120	0.148	0.180	0.250	0.375
1/4	0.2500	0.206	0.194										
3/8	0.3750	0.331	0.319										
1/2	0.5000		0.444	0.430	0.402	0.376	0.334						
5/8	0.6250			0.555	0.527	0.501	0.459						
3/4	0.7500			0.680	0.652	0.626	0.584	0.532	0.510				
7/8	0.8750			0.805	0.777	0.751	0.709	0.657	0.635				
1	1.0000			0.930	0.902	0.876	0.834	0.782	0.760				
1.050	1.0500			0.980	0.952	0.926	0.884	0.832	0.810				
1-1/8	1.1250			1.055	1.027	1.001	0.959	0.907	0.885				
1-1/4	1.2500			1.180	1.152	1.126	1.084	1.032	1.010				
1-5/16	1.3125			1.243	1.215	1.189	1.147	1.095	1.073				
1-3/8	1.3750			1.305	1.277	1.251	1.209	1.157	1.135				
1-1/2	1.5000			1.430	1.402	1.376	1.334	1.282	1.260				
1-5/8	1.6250			1.555	1.527	1.501	1.459	1.407	1.385				
1.660	1.6600			1.590	1.562	1.536	1.494	1.442	1.420	1.364			
1-3/4	1.7500			1.680	1.652	1.626	1.584	1.532	1.510	1.454			
1-7/8	1.8750			1.805	1.777	1.751	1.709	1.657	1.635	1.579			
1.900	1.9000			1.830	1.802	1.776	1.734	1.682	1.660	1.604			
2	2.0000			1.930	1.902	1.876	1.834	1.782	1.760	1.704	1.640		
2-1/4	2.2500				2.152	2.126	2.084	2.032	2.010	1.954			
2-3/8	2.3750				2.277	2.251	2.209	2.157	2.135	2.079	2.015		
2-1/2	2.5000				2.402	2.376	2.334	2.282	2.260	2.204	2.140		
2-7/8	2.8750					2.751	2.709	2.657	2.635	2.579	2.515		
3	3.0000				2.902	2.876	2.834	2.782	2.760	2.704	2.640		
3-1/8	3.1250					3.001	2.959	2.907	2.885	2.829	2.765		
3-1/2	3.5000					3.376	3.334	3.282	3.260	3.204	3.140		
3-3/4	3.7500						3.584	3.532	3.510	3.454	3.390		
4	4.0000						3.834	3.782	3.760	3.704	3.640		
4-1/2	4.5000						4.334	4.282	4.260	4.204	4.140	4.000	
5	5.0000						4.834	4.782	4.760	4.704	4.640	4.500	
6-1/4	6.2500								6.010		5.890	5.750	5.500

Tubing Sizes (Metric)

Size	OD (mm)	Gauge (nominal mm)													
		24ga	22ga	20ga	18ga	16ga	14ga	12ga	11ga	9ga	7ga	1/4"	3/8"		
		Inside Diameter (mm)													
		0.600	0.700	0.900	1.300	1.600	2.100	2.800	3.100	3.800	4.600	6.400	9.600		
1/4"	6.4	5.2	5.0												
3/8"	9.5	8.3	8.1												
1/2"	12.7		11.3	10.9	10.1	9.5	8.5								
5/8"	15.9		14.1	13.3	12.7	11.7									
3/4"	19.1		17.3	16.5	15.9	14.9	13.5	12.9							
7/8"	22.2		20.4	19.6	19.0	18.0	16.6	16.0							
1"	25.4		23.6	22.8	22.2	21.2	19.8	19.2							
1.050"	26.7		24.9	24.1	23.5	22.5	21.1	20.5							
1-1/8"	28.6		26.8	26.0	25.4	24.4	23.0	22.4							
1-1/4"	31.8		30.0	29.2	28.6	27.6	26.2	25.6							
1-5/16"	33.4		31.6	30.8	30.2	29.2	27.8	27.2							
1-3/8"	35.0		33.2	32.4	31.8	30.8	29.4	28.8							
1-1/2"	38.1		36.3	35.5	34.9	33.9	32.5	31.9							
1-5/8"	41.3		39.5	38.7	38.1	37.1	35.7	35.1							
1.660"	42.2		40.4	39.6	39.0	38.0	36.6	36.0	34.6						
1-3/4"	44.5		42.7	41.9	41.3	40.3	38.9	38.3	36.9						
1-7/8"	47.7		45.9	45.1	44.5	43.5	42.1	41.5	40.1						
1.900"	48.3		46.5	45.7	45.1	44.1	42.7	42.1	40.7						
2"	50.8		49.0	48.2	47.6	46.6	45.2	44.6	43.2	41.6					
2-1/4"	57.2					54.6	54.0	53.0	51.6	51.0	49.6				
2-3/8"	60.4					57.8	57.2	56.2	54.8	54.2	52.8	51.2			
2-1/2"	63.5					60.9	60.3	59.3	57.9	57.3	55.9	54.3			
2-7/8"	73.1						69.9	68.9	67.5	66.9	65.5	63.9			
3"	76.2					73.6	73.0	72.0	70.6	70.0	68.6	67.0			
3-1/8"	79.4							76.2	75.2	73.8	73.2	71.8	70.2		
3-1/2"	88.9							85.7	84.7	83.3	82.7	81.3	79.7		
3-3/4"	95.3							91.1	89.7	89.1	87.7	86.1			
4"	101.6							97.4	96.0	95.4	94.0	92.4			
4-1/2"	114.3							110.1	108.7	108.1	106.7	105.1	101.5		
5"	127.0							122.8	121.4	120.8	119.4	117.8	114.2		
6-1/4"	158.8										152.6		149.6	146.0	139.6



7 MASS TRANSFER

7.1 Symbols and Definitions

Symbols

Symbol	Description	Units (U.S.)	Units (SI)
A	Area	ft ² or in ²	m ²
$\mathcal{A}$	Absorption factor	dimensionless	
a	Effective interfacial mass-transfer area per unit volume	$\frac{\text{ft}^2}{\text{ft}^3}$	$\frac{\text{m}^2}{\text{m}^3}$
B	Bottom product flow rate	$\frac{\text{lb mole}}{\text{hr}}$	$\frac{\text{mol}}{\text{s}}$
c	Concentration	$\frac{\text{lb mole}}{\text{ft}^3}$	$\frac{\text{mol}}{\text{m}^3}$
c_p	Heat capacity	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}} = \frac{\text{m}^2}{\text{s}^2 \cdot \text{K}}$
D	Distillate flow rate	$\frac{\text{lb mole}}{\text{hr}}$	$\frac{\text{mol}}{\text{s}}$
D_{AB}	Mass diffusivity (diffusion coefficient)	$\frac{\text{ft}^2}{\text{hr}}$	$\frac{\text{m}^2}{\text{s}}$
D, d	Diameter	ft or in.	m
E	Efficiency	dimensionless	
F	Feed flow	$\frac{\text{lb mole}}{\text{hr}}$	$\frac{\text{mol}}{\text{s}}$
f	Ratio of vapor-phase flow to feed flow (fraction vaporized)	dimensionless	
f	Darcy friction factor	dimensionless	
f	Fugacity of a pure component	$\frac{\text{lb f}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
$\hat{f}_i$	Fugacity of a component i in a mixture	$\frac{\text{lb f}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$

Symbols (cont'd)

Symbol	Description	Units (U.S.)	Units (SI)
G	Gas flow rate (stripper/absorber)	$\frac{\text{lb mole}}{\text{hr}}$	$\frac{\text{mol}}{\text{s}}$
G_S	Gas flow rate, solute-free basis	$\frac{\text{lb mole}}{\text{hr}}$	$\frac{\text{mol}}{\text{s}}$
g	Gravitational acceleration	$\frac{\text{ft}}{\text{sec}^2}$	$\frac{\text{m}}{\text{s}^2}$
$\hat{g}$	Molar Gibbs free energy	$\frac{\text{Btu}}{\text{lb mole}}$	$\frac{\text{J}}{\text{mol}}$
H	Henry's Law constant	$\frac{\text{lbf}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
ΔH	Heat input	$\frac{\text{Btu}}{\text{hr}}$	$W = \frac{\text{J}}{\text{s}} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3}$
h	Height	ft or in.	m
h	Head loss, pressure drop	ft or in.	m
h	Specific enthalpy	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}} = \frac{\text{m}^2}{\text{s}^2}$
$\hat{h}$	Molar specific enthalpy	$\frac{\text{Btu}}{\text{lb mole}}$	$\frac{\text{J}}{\text{mol}}$
Δh	Specific enthalpy change	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}} = \frac{\text{m}^2}{\text{s}^2}$
Δh_{vap}	Latent heat of vaporization	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}} = \frac{\text{m}^2}{\text{s}^2}$
HTU	Height of a transfer unit	ft or in.	m
j	Colburn Factor	dimensionless	
j_A	Molar flux of component A per area	$\frac{\text{lb mole}}{\text{ft}^2 \cdot \text{hr}}$	$\frac{\text{mol}}{\text{m}^2 \cdot \text{s}}$
K	Distribution coefficient for phase equilibrium	dimensionless	
L	Liquid flow (for a flash, in a column, stripper, or absorber)	$\frac{\text{lb mole}}{\text{hr}}$	$\frac{\text{mol}}{\text{s}}$
L_S	Liquid flow rate, solute-free basis	$\frac{\text{lb mole}}{\text{hr}}$	$\frac{\text{mol}}{\text{s}}$
l	Length, distance	ft or in.	m
m	Mass	lbm	kg
m	General phase equilibrium coefficient	dimensionless	
m	Slope of the operating line or slope of the equilibrium line	dimensionless	
MW	Molecular weight	$\frac{\text{lbm}}{\text{lb mole}}$	$\frac{\text{kg}}{\text{mol}}$
N	Number of stages	dimensionless	
n	Number of moles	lb mole	mol
$\dot{n}$	Molar flow per area, molar flux	$\frac{\text{lb mole}}{\text{ft}^2 \cdot \text{hr}}$	$\frac{\text{mol}}{\text{m}^2 \cdot \text{s}}$
NTU	Number of transfer units	dimensionless	
P	Pressure	$\frac{\text{lbf}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$

Symbols (cont'd)

Symbol	Description	Units (U.S.)	Units (SI)
P_c	Critical pressure	$\frac{\text{lbf}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
P_r	Reduced pressure	dimensionless	
P^*	Three-phase equilibrium pressure	$\frac{\text{lbf}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
p	Partial pressure	$\frac{\text{lbf}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
p_{sat}	Saturation pressure, or vapor pressure	$\frac{\text{lbf}}{\text{in}^2}$	$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
ϕ	Poynting correction factor	dimensionless	
q	Ratio of liquid-phase flow to feed flow	dimensionless	
$\dot{Q}$	Heat duty	$\frac{\text{Btu}}{\text{hr}}$	$\text{W} = \frac{\text{J}}{\text{s}} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3}$
q	Ratio of liquid-phase flow to feed flow (fraction not vaporized)	dimensionless	
R	Reflux ratio	dimensionless	
R	Universal gas constant	$\frac{\text{Btu}}{\text{lb mole} \cdot ^\circ\text{R}}$	$\frac{\text{J}}{\text{mol} \cdot \text{K}}$
S	Boil-up ratio	dimensionless	
$\mathcal{S}$	Stripping factor	dimensionless	
T	Temperature	$^\circ\text{R or } ^\circ\text{F}$	$\text{K or } ^\circ\text{C}$
T_c	Critical temperature	$^\circ\text{R or } ^\circ\text{F}$	$\text{K or } ^\circ\text{C}$
T_r	Reduced temperature	dimensionless	
u	Velocity	$\frac{\text{ft}}{\text{sec}}$	$\frac{\text{m}}{\text{s}}$
V	Volume	ft^3	m^3
V	Vapor flow (for a flash, in a column, stripper, or absorber)	$\frac{\text{lb mole}}{\text{hr}}$	$\frac{\text{mol}}{\text{s}}$
$\hat{v}$	Molar volume	$\frac{\text{ft}^3}{\text{lb mole}}$	$\frac{\text{m}^3}{\text{mol}}$
v	Specific volume	$\frac{\text{ft}^3}{\text{lbm}}$	$\frac{\text{m}^3}{\text{kg}}$
Δv	Specific volume change during phase change	$\frac{\text{ft}^3}{\text{lb mole}}$	$\frac{\text{m}^3}{\text{mol}}$
X	Mole ratio in liquid phase (solute-free basis)	dimensionless	
x	Mole fraction in liquid phase	dimensionless	
Y	Mole ratio in vapor phase (solute-free basis)	dimensionless	
y	Mole fraction in vapor phase	dimensionless	
Z	Compressibility factor	dimensionless	
z	Mole fraction in the feed	dimensionless	
z	Distance or length	ft or in.	m
α	Interfacial area per unit volume	$\frac{\text{ft}^2}{\text{ft}^3}$	$\frac{\text{m}^2}{\text{m}^3}$

Symbols (cont'd)

Symbol	Description	Units (U.S.)	Units (SI)
α_{ij}	Relative volatility for components i and j	dimensionless	
δ	Film thickness	ft or in.	m
γ	Activity coefficient	dimensionless	
γ	Surface tension, interfacial tension	$\frac{\text{lbf}}{\text{in.}}$	$\frac{\text{N}}{\text{m}} = \frac{\text{kg}}{\text{s}^2}$
ε	Void fraction	dimensionless	
μ	Dynamic viscosity	cP or $\frac{\text{lbm}}{\text{ft} \cdot \text{sec}}$	$\text{Pa} \cdot \text{s} = \frac{\text{kg}}{\text{m} \cdot \text{s}}$
ρ	Density	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{kg}}{\text{m}^3}$
ϕ_i	Fugacity coefficient i of a pure component in the vapor phase	dimensionless	
$\hat{\phi}_i$	Fugacity coefficient i of a component in a mixture in the vapor phase	dimensionless	
ϕ_d	Volume fraction of the dispersed phase (holdup)	dimensionless	

7.2 Fundamentals of Mass Transfer

7.2.1 Diffusion

Fick's Law of Diffusion: Molar Flux

$$j_A = -D_{AB} \frac{dc_A}{dz}$$

Mass transport due to diffusion and bulk flow:

$$\dot{n}_A = \dot{n} x_A + j_A = (\dot{n}_A + \dot{n}_B) x_A - c D_{AB} \frac{dx_A}{dz}$$

$$\dot{n}_B = \dot{n} x_B + j_B = (\dot{n}_A + \dot{n}_B) x_B - c D_{BA} \frac{dx_B}{dz}$$

where

$\dot{n}_A$ = molar flux of species A

$\dot{n}$ = bulk flow

Rules of Thumb for Diffusion Coefficients at 25°C

	$D_{AB} \left(\frac{\text{ft}^2}{\text{sec}} \right)$	$D_{AB} \left(\frac{\text{m}^2}{\text{s}} \right)$
In Gases		
Air	$0.43 \times 10^{-4} - 2.4 \times 10^{-4}$	$0.4 \times 10^{-5} - 2.2 \times 10^{-5}$
Hydrogen	$1.8 \times 10^{-4} - 8.1 \times 10^{-4}$	$1.7 \times 10^{-5} - 7.5 \times 10^{-5}$
Carbon dioxide	$0.32 \times 10^{-4} - 1.7 \times 10^{-4}$	$0.3 \times 10^{-5} - 1.6 \times 10^{-5}$
In Liquids		
Gases in water	$0.75 \times 10^{-8} - 2.2 \times 10^{-8}$	$0.7 \times 10^{-9} - 2.0 \times 10^{-9}$
Acids in water	$1.3 \times 10^{-8} - 3.2 \times 10^{-8}$	$1.2 \times 10^{-9} - 3.0 \times 10^{-9}$
Organics in water	$0.43 \times 10^{-8} - 1.5 \times 10^{-8}$	$0.4 \times 10^{-9} - 1.5 \times 10^{-9}$
Organic solvents	$1.6 \times 10^{-8} - 3.2 \times 10^{-8}$	$1.5 \times 10^{-9} - 3.0 \times 10^{-9}$

Diffusion Coefficient (Pressure and Temperature Dependence)

For dilute, binary gas systems, changes in the diffusion coefficient can be predicted at any temperature and at any pressure below 25 atm by:

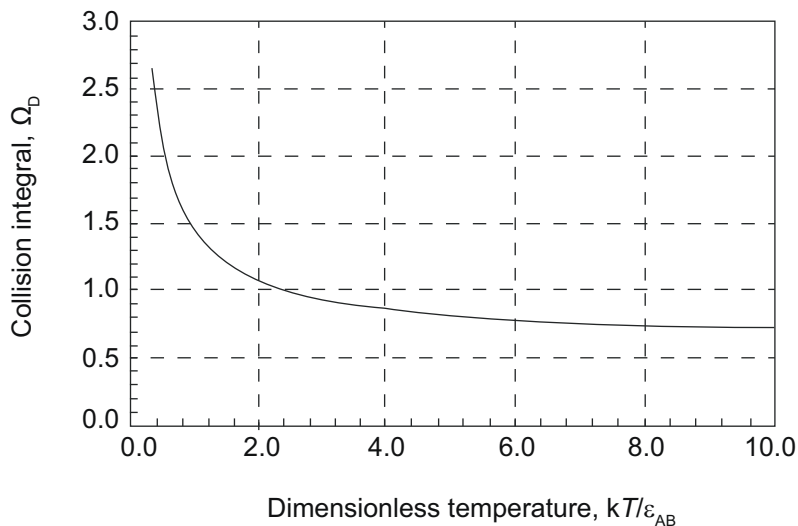
$$D_{AB}(T_2, P_2) = D_{AB}(T_1, P_1) \left(\frac{P_1}{P_2} \right) \left(\frac{T_2}{T_1} \right)^{\frac{3}{2}} \frac{\Omega_D(T_1)}{\Omega_D(T_2)}$$

where

$D_{AB}(T, P)$ = diffusion coefficient as a function of pressure and temperature

$\Omega_D(T)$ = the "collision integral" for molecular diffusion, which is a dimensionless function of temperature and of the intermolecular potential field for one molecule of A and one molecule of B.

Collision Integral for Diffusion as a Function of Dimensionless Temperature



Source: *Fundamentals of Momentum, Heat, and Mass Transfer*, James R. Welty, Gregory L. Rorrer, and David G. Foster. Copyright © 2015 John Wiley & Sons, Inc. Reproduced with permission of John Wiley & Sons, Inc.

where

k = Boltzmann constant

ϵ_{AB} = Leonard-Jones force constant

Integrated Fick's Law

Steady-state equimolar counterdiffusion of two components (No bulk flow, $D_{AB} = D_{BA}$, ideal gas):

$$\dot{n}_A = \frac{D_{AB}}{\delta} (c_A - c_{A,i}) = \frac{c D_{AB}}{\delta} (x_A - x_{A,i})$$

For an ideal gas:

$$\dot{n}_A = \frac{D_{AB}}{\delta R T} (p_A - p_{A,i}) = \frac{D_{AB} P}{\delta R T} (y_A - y_{A,i})$$

where

i = conditions at the interface

δ = film thickness

Steady-state diffusion of A through a stagnant film ($\dot{n}_B = 0$)

$$\dot{n}_A = \frac{c D_{AB}}{\delta} \ln \left(\frac{1 - x_{A,i}}{1 - x_{A,b}} \right)$$

where

$x_{A,i}$ = concentration of A at the interface

x_A = concentration of A at distance z from the interface

Concentration profile:

$$\ln \left(\frac{1 - x_A}{1 - x_{A,i}} \right) = \frac{z}{\delta} \ln \left(\frac{1 - x_{A,b}}{1 - x_{A,i}} \right)$$

where

$x_{A,b}$ = concentration of A in the bulk fluid

z = distance from the interface

For an ideal gas:

$$\dot{n}_A = \frac{D_{AB} P}{\delta R T} \ln \left(\frac{1 - y_{A,i}}{1 - y_{A,b}} \right) = \frac{D_{AB} P}{\delta R T} \frac{y_{A,i} - y_{A,b}}{y_{lm}}$$

$$y_{lm} = \frac{(1 - y_{A,b}) - (1 - y_{A,i})}{\ln \left(\frac{1 - y_{A,b}}{1 - y_{A,i}} \right)}, \quad p_{lm} = \frac{(P - p_{A,b}) - (P - p_{A,i})}{\ln \left(\frac{P - p_{A,b}}{P - p_{A,i}} \right)}$$

where

y_{lm} = logarithmic mean of the mole fractions in the gas phase and at the interface

p_{lm} = logarithmic mean of the partial pressures in the gas phase and at the interface

For diffusion of one component through a multicomponent mixture, the equation above with an effective diffusion coefficient can be used:

$$D_{A,mix} = \frac{1 - y_A}{\sum_{j \neq A} \frac{y_j}{D_{Aj}}}$$

7.2.2 Mass-Transfer Coefficients

Definitions of the Mass-Transfer Coefficient

System	Mole Fraction	Concentration	Pressure
Equimolar counter-diffusion, liquid	$\dot{n}_A = k_x \Delta x_A$ $k_x = \frac{c D_{AB}}{\delta}$	$\dot{n}_A = k_c \Delta c_A$ $k_c = \frac{D_{AB}}{\delta}$	
Equimolar counter-diffusion, ideal gas	$\dot{n}_A = k_y \Delta y_A$ $k_y = \frac{D_{AB} P}{\delta R T}$	$\dot{n}_A = k_c \Delta c_A$ $k_c = \frac{D_{AB}}{\delta}$	$\dot{n}_A = k_G \Delta p_A$ $k_G = \frac{D_{AB}}{\delta R T}$
Diffusion through a stagnant film, liquid	$\dot{n}_A = k_{x'} \Delta x_A$ $k_{x'} = \frac{c D_{AB}}{\delta c_{lm}}$	$\dot{n}_A = k_{c'} \Delta c_A$ $k_{c'} = \frac{D_{AB}}{\delta c_{lm}}$	
Diffusion through a stagnant film, ideal gas	$\dot{n}_A = k_{y'} \Delta y_A$ $k_{y'} = \frac{D_{AB} P}{\delta R T y_{lm}}$	$\dot{n}_A = k_c \Delta c_A$ $k_c = \frac{D_{AB}}{\delta c_{lm}}$	$\dot{n}_A = k_{G'} \Delta p_A$ $k_{G'} = \frac{D_{AB}}{\delta R T p_{lm}}$
Diffusion through a stagnant film, ideal gas, mass-basis	$\dot{m}_A = k_{y'} \Delta y_A$ $k_{y'} = \frac{D_{AB} P M W_A}{\delta R T y_{lm}}$		

where $c_{lm} = \frac{c_{Ao} - c_A}{\ln \frac{c_{Ao} - c_{Ai}}{c_A - c_{Ai}}}$

k_c = mass-transfer coefficient for liquid (concentration basis)

k_G = mass-transfer coefficient for gas (pressure basis)

k_x = mass-transfer coefficient for liquid (mole fraction basis)

k_y = mass-transfer coefficient for gas (mole fraction basis)

$k_{c'}$ = mass-transfer coefficient for liquid (concentration basis), corrected for inert component

$k_{G'}$ = mass-transfer coefficient for gas (concentration basis), corrected for inert component

$k_{x'}$ = mass-transfer coefficient for liquid (mole fraction basis), corrected for inert component

$k_{y'}$ = mass-transfer coefficient for gas (mole fraction basis), corrected for inert component

7.2.3 Convective Mass Transfer

Reynolds analogy between momentum, heat, and mass transfer with Colburn correction:

$$j_M = \frac{k_G}{G'} \left(\frac{\mu}{\rho D_{AB}} \right)^{2/3}$$

$$j_H = \frac{h}{c_p G_M} \left(\frac{c_p \mu}{k} \right)^{2/3}$$

For flow through straight tubes and across plane surfaces:

$$j_M = j_H = \frac{f}{8}$$

For turbulent flow around cylinders:

$$j_M = j_H \leq \frac{f}{8}$$

where

f = Darcy friction factor

G_M = mass flux in $\frac{\text{lbm}}{\text{ft}^2 \cdot \text{hr}}$ or $\frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$

G' = molar flux

h = heat-transfer coefficient in $\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$ or $\frac{\text{W}}{\text{m}^2 \cdot \text{K}}$

j_H = Colburn heat-transfer factor

j_M = Colburn mass-transfer factor

k = thermal conductivity in $\frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$ or $\frac{\text{W}}{\text{m} \cdot \text{K}}$

k_G = gas-phase mass-transfer coefficient

Other correlations for the mass-transfer coefficient:

Mass Transfer¹ for Simple Situations

Fluid Motion	Range of Conditions	Equation
Inside circular pipes	$Re = 4000-60,000$ $Sc = 0.6-3000$	$j_M = 0.023 Re^{-0.17}$ $Sh = 0.023 Re^{0.83} Sc^{1/3}$
	$Re = 10,000-400,000$ $Sc > 100$	$j_M = 0.0149 Re^{-0.12}$ $Sh = 0.0149 Re^{0.88} Sc^{1/3}$
Unconfined flow parallel to flat plates ²	Transfer begins at leading edge $Re_x < 50,000$	$j_M = 0.664 Re_x^{-0.5}$
	$Re_x = 5 \times 10^5 - 3 \times 10^7$ $Pr = 0.7-380$	$Nu = 0.037 Re_x^{0.8} Pr_0^{0.43} \left(\frac{Pr_0}{Pr_i} \right)^{0.25}$
	$Re_x = 2 \times 10^4 - 5 \times 10^5$ $Pr = 0.7-380$	Between above equation and $Nu = 0.0027 Re_x Pr_0^{0.43} \left(\frac{Pr_0}{Pr_i} \right)^{0.25}$
Confined gas flow parallel to a flat plate in a duct	$Re_e = 2600-22,000$	$j_M = 0.11 Re_e^{-0.29}$
Liquid film in wetted-wall tower, transfer between liquid and gas	$\frac{4\Gamma}{\mu} = 0-1200$ ripples suppressed	See note 4.
	$\frac{4\Gamma}{\mu} = 1300-8300$	$Sh = (1.76 \times 10^{-5}) \left(\frac{4\Gamma}{\mu} \right)^{1.506} Sc^{0.5}$
Perpendicular to single cylinders	$Re = 400-25,000$ $Sc = 0.6-2.6$	$\frac{k_G P}{G'} Sc^{0.56} = 0.281 (Re')^{0.4}$
	$Re' = 0.1-10^5$ $Pr = 0.7-1500$	$Nu = [0.35 + 0.34(Re')^{0.5} + 0.15(Re')^{0.58}] Pr^{0.3}$
Past single spheres	$Re'' Sc^{0.5} = 1.8-600,000$ $Sc = 0.6-3200$	$Sh = Sh_0 + 0.347 (Re'' Sc^{0.5})^{0.62}$ $Sh_0 = \begin{cases} 2.0 + 0.569 (Gr_M Sc)^{0.250} & Gr_M Sc < 10^8 \\ 2.0 + 0.0254 (Gr_M Sc)^{0.333} Sc^{0.244} & Gr_M Sc > 10^8 \end{cases}$
Through fixed beds of pellets ³	$Re'' = 90-4000$ $Sc = 0.6$	$j_M = j_H = \frac{2.06}{\epsilon} (Re'')^{-0.575}$
	$Re'' = 5000-10,300$ $Sc = 0.6$	$j_M = 0.95 j_H = \frac{20.4}{\epsilon} (Re'')^{-0.815}$
	$Re'' = 0.0016-55$ $Sc = 168-70,600$	$j_M = \frac{1.09}{\epsilon} (Re'')^{-2/3}$
	$Re'' = 5-1500$ $Sc = 168-70,600$	$j_M = \frac{0.250}{\epsilon} (Re'')^{-0.31}$

1. Average mass-transfer coefficients throughout, for constant solute concentrations at the phase surface. Generally, fluid properties are evaluated at the average conditions between the phase surface and the bulk fluid. The heat-mass-transfer analogy is valid throughout.
2. Mass-transfer data for this case scatter badly but are reasonably well represented by setting $j_M = j_H$.

3. For fixed beds, the relation between ε and d_p is $a = \frac{6(1-\varepsilon)}{d_p}$, where a is the specific solid surface, surface per volume of bed. For mixed sizes:

$$d_p = \frac{\sum_{i=1}^n n_i d_{pi}^3}{\sum_{i=1}^n n_i d_{pi}^2}$$

4. For small rates of flow or long contact times:

$$\frac{k_{L,av} \delta}{D_{AB}} = Sh_{av} \approx 3.41$$

For large Reynold numbers of short contact times:

$$k_{L,av} = \left(\frac{6D_{AB}\Gamma}{\pi \rho \delta l} \right)^{\frac{1}{2}}$$

$$Sh_{av} = \left(\frac{3}{2\pi} \frac{\delta}{l} Re Sc \right)^{\frac{1}{2}}$$

Total absorption rate from the average k_L :

$$N_{A,av} = \frac{\bar{u}_y \delta}{l} (\bar{c}_{A,l} - c_{A0}) = k_{L,av} (c_{A,i} - \bar{c}_A)_M$$

$$(c_{A,i} - \bar{c}_A)_M = \frac{(c_{A,i} - c_{A0}) - (c_{A,i} - \bar{c}_{A,l})}{\ln \left[\frac{(c_{A,i} - c_{A0})}{(c_{A,i} - \bar{c}_{A,l})} \right]}$$

where

a = specific surface of a fixed bed of pellets, pellet surface/volume of bed

$c_{A,i}$ = concentration of A at the interface

c_{A0} = concentration of A at the approach, or initial, value

$\bar{c}_A$ = bulk-average concentration of A

$\bar{c}_{A,l}$ = bulk-average concentration of A across length l

D_{AB} = molecular diffusivity of A in B

d_c = diameter of a cylinder

d_e = equivalent diameter of a noncircular duct = 4 (cross-sectional area)/perimeter

d_p = diameter of a sphere; for a nonspherical particle, diameter of a sphere of the same surface as the particle

Gr_M = Grashof number for mass transfer $\frac{g l^3 \Delta \rho}{\rho} \left(\frac{\rho}{\mu} \right)^2$

k = mass-transfer coefficient

$k_{L,av}$ = average mass-transfer coefficient across length l

l = length

$N_{A,av}$ = average mass-transfer flux of A at, and relative to, a phase boundary

n_i = a number, dimensionless

Nu = Nusselt number $\frac{h\delta}{k}$

Pr = Prandtl number $\frac{c_p \mu}{k}$

Pr_0 = Prandtl number at the approach, or initial, value

Pr_i = Prandtl number at the interface

Re = Reynolds number $\frac{dG}{\mu}$ or $\frac{lG}{\mu}$

Re' = Reynolds number for flow outside a cylinder $\frac{d_c G}{\mu}$

Re'' = Reynolds number for flow past a sphere $\frac{d_p G}{\mu}$

Re_e = Reynolds number for flow in a noncircular duct $\frac{d_e G}{\mu}$

Re_x = Reynolds number with x as the length dimension $\frac{xG}{\mu}$

Sc = Schmidt number $\frac{\mu}{\rho D_{AB}}$

Sh = Sherwood number $\frac{kl}{D_{AB}}$

Sh_0 = Sherwood number at the approach, or initial, value

Sh_i = Sherwood number at the interface

$\bar{u}_y$ = bulk average velocity in the y direction (parallel to the direction of flow)

Γ = mass flow rate per unit width

δ = thickness of a layer

ε = void fraction

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7.2.4 Mass Transfer Between Phases for Dilute Systems

$$\dot{n}_A = k_L(x - x_i) = k_G(y_i - y) = k'_L(c - c_i) = k'_G(p_i - p)$$

$$k_L = k'_L \bar{\rho}_L \quad \text{and} \quad k_G = k'_G P$$

$$\frac{y_i - y}{x - x_i} = \frac{k_L}{k_G} = \frac{k'_L \bar{\rho}_L}{k'_G P} = \frac{L' HTU_G}{G' HTU_L}$$

where

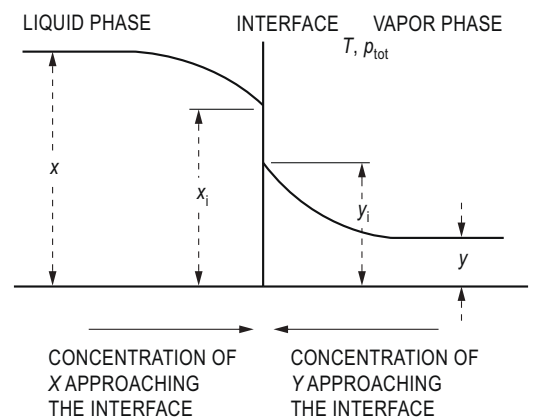
G' = molar flux (gas phase)

HTU_L = height of a transfer unit based on liquid-phase resistance

HTU_G = height of a transfer unit based on vapor-phase resistance

i = subscript meaning concentration at interface

k_L = liquid-phase mass-transfer coefficient (mole fraction basis)



- k'_L = liquid-phase mass-transfer coefficient (concentration basis)
 k_G = gas-phase mass-transfer coefficient (mole fraction basis)
 k'_G = gas-phase mass-transfer coefficient (partial pressure basis)
 L' = molar flux (liquid phase)
 p_i = partial pressure
 $\bar{\rho}_L$ = average molar density of liquid phase

In most types of separation equipment, the interfacial area for mass transfer cannot be accurately determined, and transfer coefficients based on volume of the device are used:

$$\frac{1}{K_G a} = \frac{1}{k_G a} + \frac{m}{k_L a} \quad \text{and} \quad \frac{1}{K_L a} = \frac{1}{m k_G a} + \frac{1}{k_L a}$$

where

a = effective interfacial mass-transfer area per unit volume, in $\frac{\text{ft}^2}{\text{ft}^3}$ or $\frac{\text{m}^2}{\text{m}^3}$

K_G = overall gas-phase mass-transfer coefficient

K_L = overall liquid-phase mass-transfer coefficient

m = slope of equilibrium line

Overall Mass-Transfer Coefficients K_L and K_G for Dilute Systems

$$\dot{n}_A = K_L (x - x^{\text{eq}}) = K_G (y^{\text{eq}} - y)$$

where

x^{eq} = liquid mole fraction in equilibrium with vapor phase

y^{eq} = vapor mole fraction in equilibrium with liquid phase

Overall Mass-Transfer Coefficients for Dilute Systems

	Gas Phase	Liquid Phase
Equilibrium: $y = m \cdot x$	$\frac{1}{K_G} = \frac{1}{k_G} + \frac{m}{k_L}$	$\frac{1}{K_L} = \frac{1}{m k_G} + \frac{1}{k_L}$
Use for:	High solubility, low m ; gas-phase resistance is controlling	Low solubility, high m ; liquid-phase resistance is controlling

7.2.5 Mass Transfer Between Phases for Concentrated Systems

$$\dot{n}_A = \frac{\hat{k}_L (x - x_i)}{x_{\text{BM}}} = \frac{\hat{k}_G (y_i - y)}{y_{\text{BM}}} = \frac{\hat{K}_L (x - x^{\text{eq}})}{x_{\text{BM}}^{\text{eq}}} = \frac{\hat{K}_G (y^{\text{eq}} - y)}{y_{\text{BM}}^{\text{eq}}}$$

$$x_{\text{BM}} = \frac{(1-x) - (1-x_i)}{\ln \frac{(1-x)}{(1-x_i)}} \quad x_{\text{BM}}^{\text{eq}} = \frac{(1-x) - (1-x^{\text{eq}})}{\ln \frac{(1-x)}{(1-x^{\text{eq}})}}$$

$$y_{\text{BM}} = \frac{(1-y) - (1-y_i)}{\ln \frac{(1-y)}{(1-y_i)}} \quad y_{\text{BM}}^{\text{eq}} = \frac{(1-y) - (1-y^{\text{eq}})}{\ln \frac{(1-y)}{(1-y^{\text{eq}})}}$$

$$\hat{k}_G = k_G y_{BM} = k'_G P y_{BM}$$

$$\hat{k}_L = k_L x_{BM} = k'_L \bar{\rho}_L x_{BM}$$

$$\frac{y_i - y}{x - x_i} = \frac{k_L}{k_G} = \frac{\hat{k}_L y_{BM}}{\hat{k}_G x_{BM}} = \frac{L_M HTU_G y_{BM}}{G_M HTU_L x_{BM}}$$

where

$\hat{k}_G$ = gas-phase mass-transfer coefficient for concentrated systems

$\hat{K}_G$ = overall gas-phase mass-transfer coefficient for concentrated systems

$\hat{k}_L$ = liquid-phase mass-transfer coefficient for concentrated systems

$\hat{K}_L$ = overall liquid-phase mass-transfer coefficient for concentrated systems

x_{BM} = logarithmic-mean solvent concentration between bulk and interface

y_{BM} = logarithmic-mean gas concentration between bulk and interface

L' = molar flux (liquid phase)

G' = molar flux (gas phase)

HTU_L = height of a transfer unit based on liquid-phase resistance

HTU_G = height of a transfer unit based on vapor-phase resistance

$\bar{\rho}_L$ = average molar density of liquid phase

Overall Mass-Transfer Coefficients $\hat{K}_L$ and $\hat{K}_G$ for Concentrated Systems

$$\frac{1}{\hat{K}_G} = \frac{y_{BM}}{y_{BM}^{eq} \hat{k}_G} + \frac{x_{BM}}{y_{BM}^{eq} \hat{k}_L} \frac{(y^{eq} - y_i)}{(x - x_i)}$$

$$\frac{1}{\hat{K}_L} = \frac{x_{BM}}{x_{BM}^{eq} \hat{k}_L} + \frac{y_{BM}}{x_{BM}^{eq} \hat{k}_G} \frac{(x_i - x^{eq})}{(y_i - y)}$$

7.2.6 Height of a Transfer Unit

$$HTU_G = \frac{G'}{k_G a y_{BM}} = \frac{G'}{\hat{k}_G a}$$

$$HTU_L = \frac{L'}{k_L a x_{BM}} = \frac{L'}{\hat{k}_L a}$$

$$HTU_{OG} = \frac{G'}{K_G a y_{BM}^{eq}} = \frac{G'}{\hat{K}_G a} = \frac{y_{BM}}{y_{BM}^{eq}} HTU_G + \frac{m G'}{L'} \frac{x_{BM}}{y_{BM}^{eq}} HTU_L$$

$$HTU_{OL} = \frac{L'}{K_L a x_{BM}^{eq}} = \frac{L'}{\hat{K}_L a} = \frac{x_{BM}}{x_{BM}^{eq}} HTU_L + \frac{L'}{m G'} \frac{y_{BM}}{x_{BM}^{eq}} HTU_G$$

where

HTU_G = height of a transfer unit based on vapor-phase resistance

HTU_{OG} = height of an overall vapor-phase mass-transfer unit

HTU_L = height of a transfer unit based on liquid-phase resistance

HTU_{OL} = height of an overall liquid-phase mass-transfer unit

Height Equivalent to One Theoretical Plate (HETP)

If equilibrium line and operating line are parallel $\left(\frac{mG'}{L'} = 1\right)$, then:

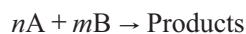
$$HETP = HTU$$

If equilibrium line and operating line are straight, but not parallel, then:

$$\frac{HTU_{OG}}{HETP} = \frac{\frac{mG'}{L'} - 1}{\ln\left(\frac{mG'}{L'}\right)}$$

7.2.7 Mass Transfer with Reaction

Consider a reaction between a dissolving gas A and a liquid-phase reactant B, with q moles of B reacting per mole of A, so that:



$$q = \frac{m}{n}$$

where

q = number of moles of B reacting per mole of A

C_{AL} and C_{BL} = molar concentrations of A and B, respectively, in the liquid

The rate of reaction of A, J_L , is then given by

$$J_L = k_{nm} C_{AL}^n C_{BL}^m$$

where k_{nm} = reaction velocity constant, in $\left(\frac{\text{m}^3}{\text{mol}}\right)^{m+n-1}$

J_L has units, moles/sec/unit volume of liquid. Alternatively,

$$J = k_{nm} C_{AL}^n C_{BL}^m \varepsilon_L$$

J_L is the rate of reaction and has units of $\frac{\text{mol}}{\text{s} \cdot \text{m}^3}$, n and m are the orders of reaction in A and B, and ε_L is the liquid hold-up fraction. A "reaction time" t_R can be defined as

$$t_R = \frac{(n+1)}{2k_{nm} C_{AL}^{(n-1)} C_{BL}^m}$$

The mass transfer of A in the liquid is given by

$$J = k_L a (C_{AL}^* - C_{AL})$$

where

J = reaction rate in moles/sec/unit volume of reactor

C_{AL}^* = dissolved gas concentration in liquid bulk in $\frac{\text{mol}}{\text{m}^3}$

k_L = interphase mass-transfer coefficient in $\frac{\text{m}}{\text{s}}$

a = gas-liquid interphase surface area/unit dispersion volume in $\frac{1}{\text{m}}$

J is the rate of reaction and has units of $\frac{\text{mol}}{\text{s} \cdot \text{m}^3}$. A mass-transfer "diffusion time," t_D , can be defined as

$$t_D = \frac{D_{AL}}{k_L^2}$$

where D_{AL} = diffusivity of A in the liquid

If a fast reaction is occurring near the interface within the "diffusion film," it will enhance the mass-transfer rate, and the equation for the mass transfer of A into liquid, above, becomes

$$J = k_L^* a (C_{AL}^* - C_{AL})$$

$$k_L^* = \left[\frac{2D_{AL} k_{nm} (C_{AL}^* - C_{AL})^{(n-1)} C_{BL}^m}{n+1} \right]^{\frac{1}{2}}$$

where k_L^* = enhanced liquid-film mass-transfer coefficient in $\frac{\text{m}}{\text{s}}$

Various Gas-Liquid Reaction Regimes and Parameters of Importance

Regime	Conditions	Important Variables	Concentration Profiles
I Kinetic control Slow reaction	$\sqrt{\frac{t_D}{t_R}} < 0.02$	Rate $\propto \epsilon_L$ $\propto k_{nm}$ $\propto (C_{AL}^*)^n$ $\propto (C_{BL}^*)^m$ Independent of a (if a is adequate) Independent of k_L	
II Diffusion control Moderately fast reaction in bulk of liquid, $C_{AL} \approx 0$	$0.02 < \sqrt{\frac{t_D}{t_R}} < 2$ Design so that $\frac{\epsilon_L}{a} > 100 \frac{D_{AL}}{k_L}$	Rate $\propto a$ $\propto k_L$ $\propto C_{AL}^*$ Independent of k_{nm} Independent of ϵ_L (if ϵ_L is adequate)	
III Fast reaction Reaction in film, $C_{AL} \approx 0$ (pseudo first order in A')	$2 < \sqrt{\frac{t_D}{t_R}} < \frac{C_{BL}}{q C_{AL}^*}$ $C_{BL} \gg C_{AL}^*$	Rate $\propto a$ $\propto \sqrt{k_{nm}}$ $\propto (C_{AL}^*)^{\frac{n+1}{2}}$ Independent of k_L Independent of ϵ_L	
IV Very fast reaction General case of III	$2 < \sqrt{\frac{t_D}{t_R}}$ $C_{BL} \sim C_{AL}^*$	Rate $\propto a$ depends on $k_L, k_{nm}, C_{AL}^*, C_{BL}$ Independent of ϵ_L	
V Instantaneous reaction Reaction at interface; controlled by transfer of B to interface from bulk, $J \propto k_L a$	$\sqrt{\frac{t_D}{t_R}} \gg \frac{C_{BL}}{q C_{AL}^*}$	Rate $\propto a$ $\propto k_L$ Independent of C_{AL}^* Independent of k_{nm} Independent of ϵ_L	

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7.3 Vapor-Liquid Separations

7.3.1 Batch Distillation

Rayleigh Equation

$$\int_{n_0}^{n_f} \frac{dn}{n} = \ln \frac{n_f}{n_0} = \int_{x_0}^{x_f} \frac{dx}{y-x}$$

where

n_f = moles in still at end of run

n_0 = initial moles in still

x_f = mole fraction in liquid phase at end of run

x_0 = initial mole fraction in liquid phase in still

Relative Volatility Equation

$$\alpha_{AB} = \frac{\left(\frac{y}{x}\right)}{\left(\frac{1-y}{1-x}\right)}$$

where

α_{AB} = relative volatility

y = mole fraction of light component in vapor phase

Rearranging:

$$y = \frac{\alpha_{AB} x}{1 + (\alpha_{AB} - 1)x}$$

Therefore,

$$\ln \frac{n_A}{n_{0A}} = \alpha_{AB} \ln \frac{n_B}{n_{0B}}$$

where

n_A = moles of liquid "A" left in the still at any time

n_B = moles of liquid "B" left in the still at any time

0 = time zero

α_{AB} = relative volatility

Operating Line for Batch Distillation With Reflux

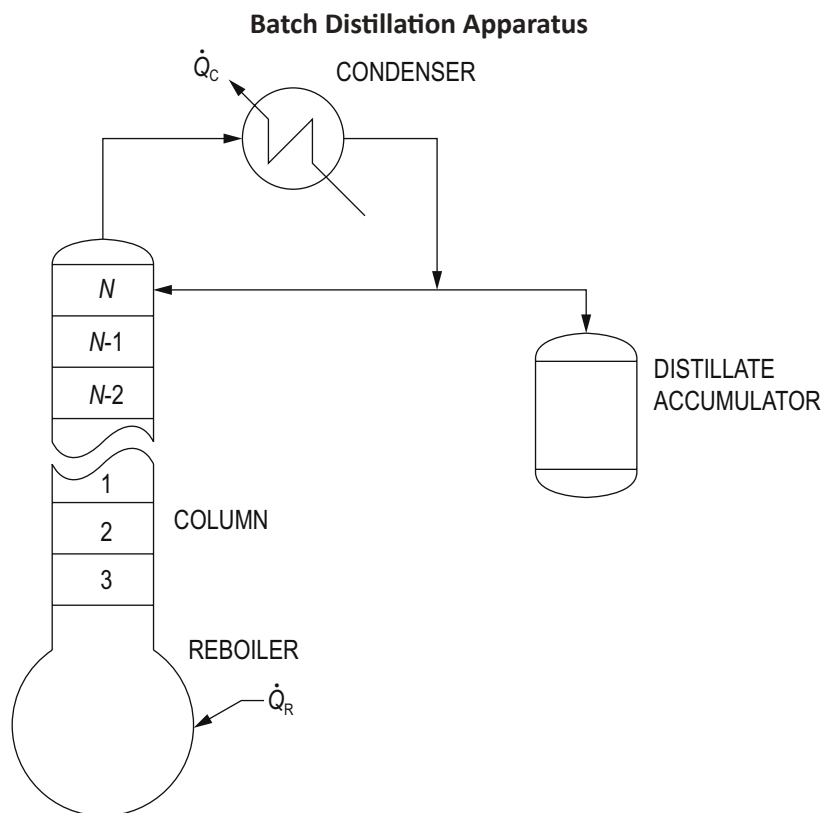
$$y_{n+1} = \frac{R_D}{R_D + 1} x_n + \frac{x_D}{R_D + 1}$$

where

R_D = reflux ratio based on the distillate rate

x = liquid composition

Batch Distillation Apparatus



7.3.2 Continuous Distillation

7.3.2.1 Theoretical Stage

An ideal theoretical stage has the following characteristics:

1. It operates in steady state and has a liquid product and a vapor product.
2. All vapor and liquid entering the stage are intimately contacted and perfectly mixed.
3. Total vapor leaving the stage is in equilibrium with total liquid leaving the stage.

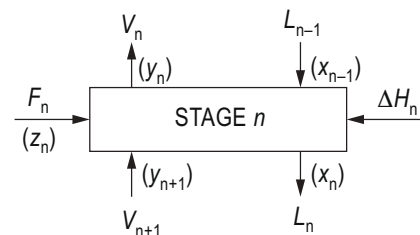
For a single binary distillation stage, the following balances and equilibrium relationships apply.

Overall mass balance:

$$F_n + V_{n+1} + L_{n-1} = V_n + L_n$$

Component mass balance:

$$z_n F_n + y_{n+1} V_{n+1} + x_{n-1} L_{n-1} = y_n V_n + x_n L_n$$



Energy balance:

$$\hat{h}_{f,n} F_n + \hat{h}_{v,n+1} V_{n+1} + \hat{h}_{L,n-1} L_{n-1} + \Delta H_n = \hat{h}_{v,n} V_n + \hat{h}_{L,n} L_n$$

where

$\hat{h}$ = molar specific enthalpy

F_n = feed flow to stage n

V_n = vapor flow leaving stage n

L_n = liquid flow leaving stage n

ΔH_n = heat input to stage n

7.3.2.2 Constant Molal Overflow

When the molar heats of vaporization of the components are nearly equal, the molar flow rates of the vapor and liquid are nearly constant in each section of the column.

In the rectifying section, the following assumptions then apply:

$$L = L_0 = L_1 = L_n \quad \text{and} \quad V = V_1 = V_n$$

And in the stripping section, the following assumptions then apply:

$$L' = L_N = L_m \quad \text{and} \quad V' = V_N = V_m$$

where

L = liquid flow in the rectifying section

V = vapor flow in the rectifying section

L' = liquid flow in the stripping section

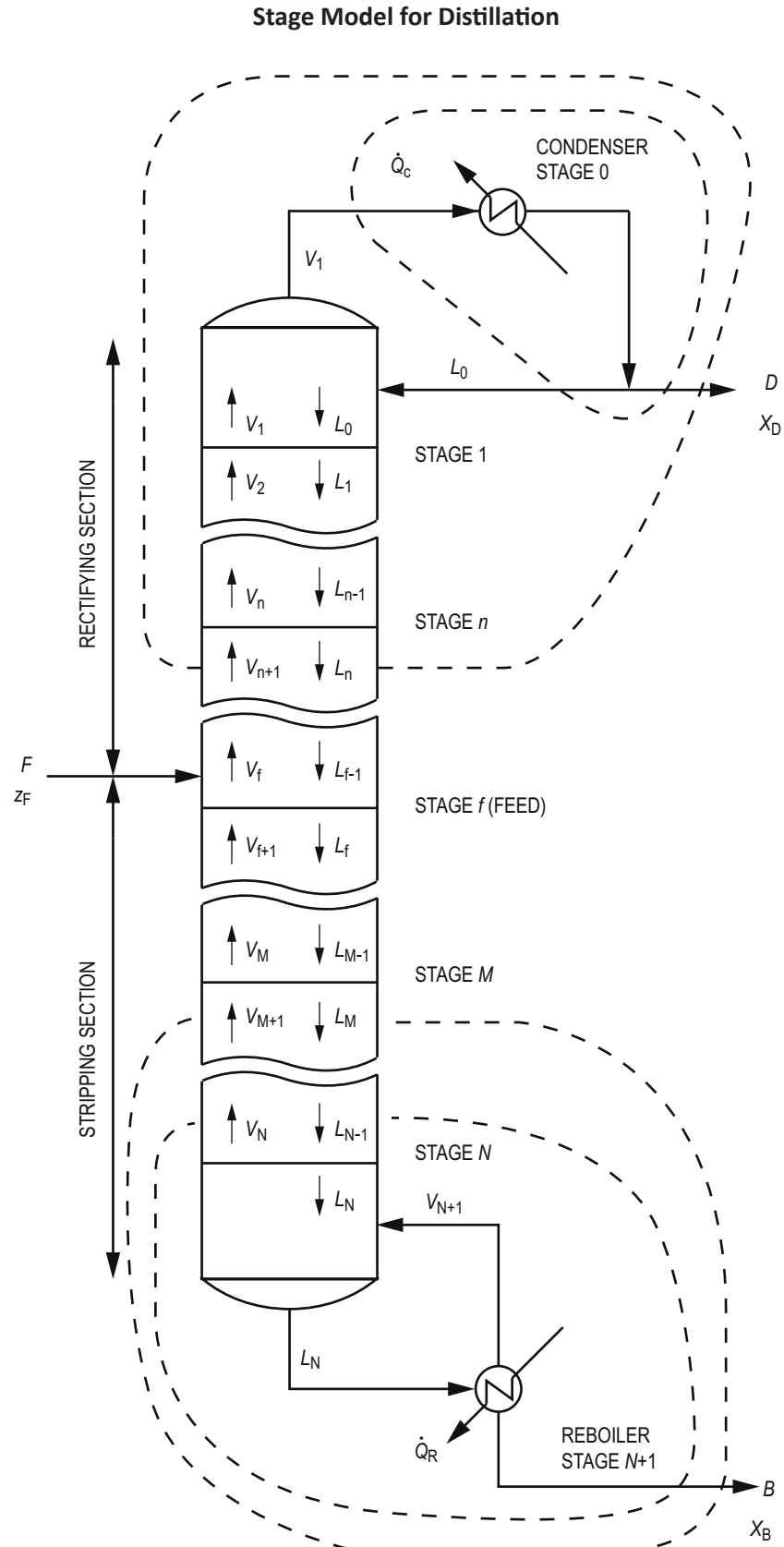
V' = vapor flow in the stripping section

N = total number of stages

m = stage in stripping section

n = stage in rectifying section

7.3.2.3 Column Material Balance



Overall mass balance:

$$F = D + B$$

Component mass balance:

$$z_F F = x_D D + x_B B$$

Ratios:

$$\frac{D}{F} = \frac{z_F - x_B}{x_D - x_B}$$

$$\frac{B}{F} = \frac{x_D - z_F}{x_D - x_B}$$

For the rectifying section, the following balances apply:

$$D = V_1 - L_0 = V_{n+1} - L_n$$

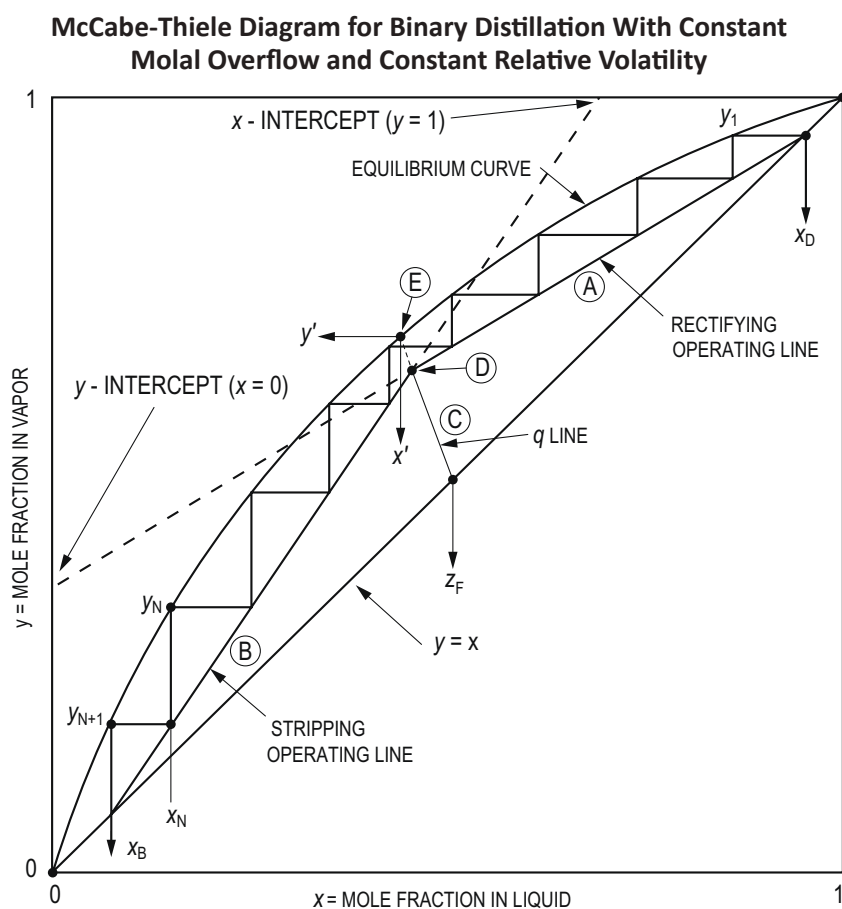
$$x_D D = y_1 V_1 - x_0 L_0 = y_{n+1} V_{n+1} - x_n L_n$$

For the stripping section, the following balances apply:

$$B = L_{N-1} - V_N = L_m - V_{m+1}$$

$$x_B B = x_{N-1} L_{N-1} - y_N V_N = x_m L_m - y_{m+1} V_{m+1}$$

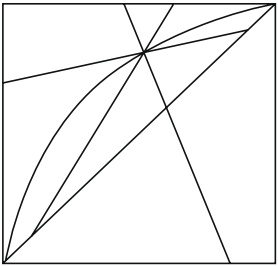
7.3.2.4 Graphical Solution for Binary Distillation (McCabe-Thiele Diagram)



Equations for the McCabe-Thiele Diagram

Name	Equations
Equilibrium Line	$y = \frac{\alpha x}{1 + x(\alpha - 1)}$
Operating Line for the Rectifying Section Ⓐ	$y_{n+1} = \frac{L}{V} x_n + \frac{x_D D}{V} = \frac{L}{L+D} x_n + \frac{x_D D}{L+D} = \frac{R}{R+1} x_n + \frac{x_D}{R+1} = \frac{L}{V} x_n + \left(1 - \frac{L}{V}\right) x_D$ Slope: $\frac{L}{V} = \frac{R}{R+1}$ y-Intercept ($x = 0$): $y_{x=0} = \frac{x_D}{R+1} = \frac{x_D D}{L+D}$ Reflux Ratio: $R = \frac{L}{D} = \frac{V-D}{D} = \frac{x_D}{y_{x=0}} - 1$
Operating Line for the Stripping Section Ⓑ	$y_{m+1} = \frac{L'}{V'} x_m - \frac{x_B B}{V'} = \frac{L'}{L'-B} x_m - \frac{x_B B}{L'-B} = \frac{S+1}{S} x_m - \frac{x_B}{S} = \frac{\frac{L'}{B}}{\frac{L'}{B}-1} x_m - \frac{x_B}{\frac{L'}{B}-1}$ $\frac{L'}{B} = (1-f) + (R+1-f) \frac{x_F - x_B}{x_D - x_F}$ Slope: $\frac{L'}{V'} = \frac{S+1}{S}$ x-Intercept ($y = 1$): $x_{y=1} = \frac{x_B + \frac{L'}{B} - 1}{\frac{L'}{B}}$ Boil-up Ratio: $S = \frac{V'}{B} = \frac{L'}{B} - 1 = \frac{x_{y=1} - x_B}{1 - x_{y=1}}$
Feed Line Ⓒ	$y = \frac{f-1}{f} x + \frac{z_F}{f} = \frac{q}{q-1} x - \frac{z_F}{q-1}$ Feed Condition: q = mole fraction liquid in feed = $\frac{\text{molar enthalpy to convert feed to saturated vapor}}{\text{molar enthalpy of vaporization}}$ f = mole fraction vapor in feed $q + f = 1$ Slope: $\frac{f-1}{f} = \frac{q}{q-1}$ Intercept: For $z_F \geq (1-f)$ $x_{y=0} = \frac{z_F}{1-f} = \frac{z_F}{q}$ For $z_F \leq (1-f)$ $y_{x=1} = \frac{z_F + f - 1}{f} = \frac{q - z_F}{q-1}$
Intersection of Feed Line/ Operating Lines Ⓓ	$x_I = \left(\frac{z_F}{f} - \frac{x_D}{R+1} \right) \frac{f(R+1)}{1+R-f}$ $y_I = \frac{z_F}{f} + \left(\frac{z_F}{f} - \frac{x_D}{R+1} \right) \frac{(f-1)(R+1)}{1+R-f}$

Equations for the McCabe-Thiele Diagram (cont'd)

Name	Equations
Intersection of Feed Line/ Equilibrium Line Ⓔ	For constant α : $x' = -\frac{1}{2} \left[\frac{1}{\alpha - 1} + \frac{z_F}{f - 1} - \frac{\alpha f}{(\alpha - 1)(f - 1)} \right] + \sqrt{\frac{1}{4} \left[\frac{1}{\alpha - 1} + \frac{z_F}{f - 1} - \frac{\alpha f}{(\alpha - 1)(f - 1)} \right]^2 - \frac{z_F}{(\alpha - 1)(f - 1)}}$ $y' = x' \left(\frac{f - 1}{f} \right) + \frac{z_F}{f}$
Operating Line for Total Reflux	$y = x$
Operating Line for Minimum Reflux	$R_{\min} = \frac{x_D - y'}{y' - x'}$ 

Circled A, B, C, D, and E in table above refer to the previous graph, "Binary Distillation With Constant Molal Overflow."

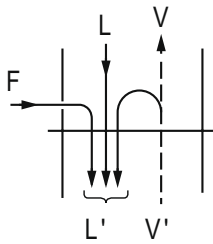
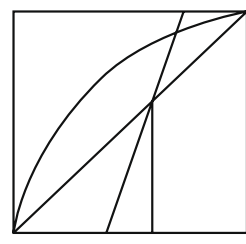
7.3.2.5 Feed Conditions

The feed condition is defined by

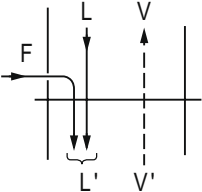
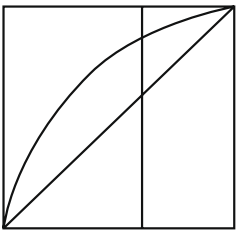
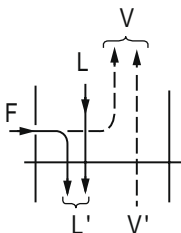
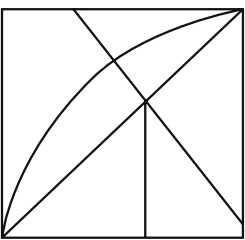
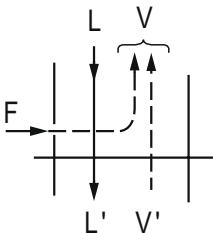
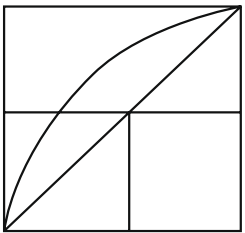
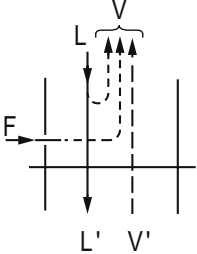
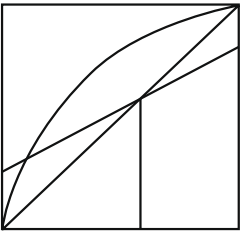
$$L' = L + qF = L + (1 - f)F$$

$$V = V' + (1 - q)F = V' + fF$$

Feed Conditions

Feed Condition	Values for f and q	Flows at Feed Location	Feed Line in McCabe-Thiele
Subcooled Liquid	$f < 0$ $f = -\frac{c_{pL}(T_b - T_F)}{\Delta h_{\text{vap}}}$ $q > 1$ $q = 1 + \frac{c_{pL}(T_b - T_F)}{\Delta h_{\text{vap}}}$		

Feed Conditions (cont'd)

Feed Condition	Values for f and q	Flows at Feed Location	Feed Line in McCabe-Thiele
Bubble Point (Saturated Liquid)	$f = 0$ $q = 1$		
Partially Vaporized	$0 < f < 1$ $0 < q < 1$		
Dew Point (Saturated Vapor)	$f = 1$ $q = 0$		
Superheated Vapor	$f > 1$ $f = 1 + \frac{c_{pV}(T_F - T_d)}{\Delta h_{vap}}$ $q < 0$ $q = -\frac{c_{pV}(T_F - T_d)}{\Delta h_{vap}}$		

where

c_{pL} = heat capacity of the liquid

c_{pV} = heat capacity of the vapor

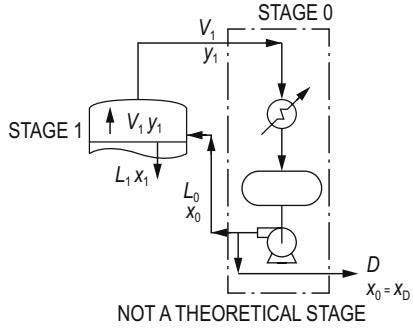
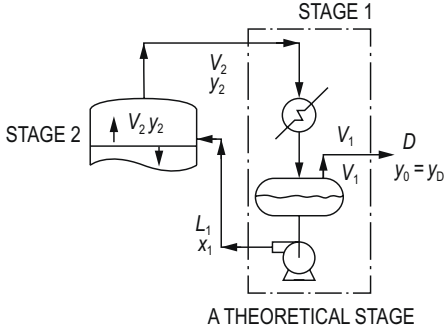
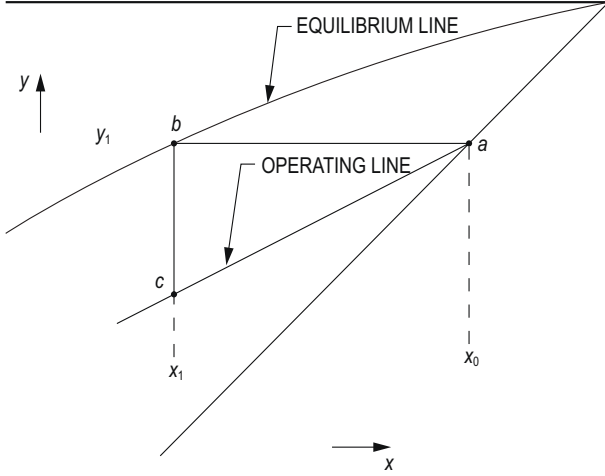
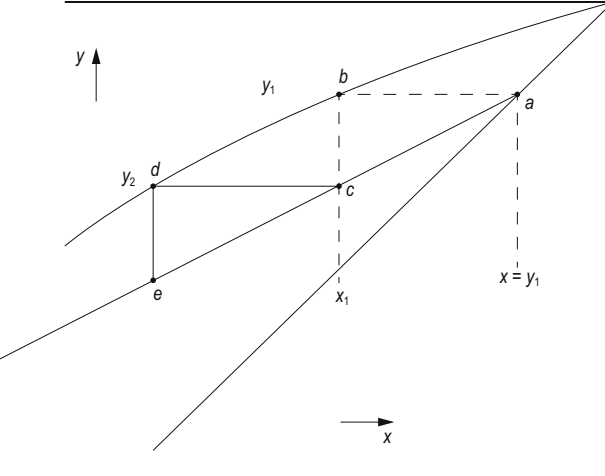
T_F = temperature of the feed

T_b = bubble point temperature of the liquid

T_d = dew point temperature of the vapor

7.3.2.6 Condensers

Types of Condensers

Total Condenser	Partial Condenser
A total condenser does not represent a theoretical stage.	A partial condenser represents a theoretical stage.
 STAGE 0 NOT A THEORETICAL STAGE	 STAGE 1 A THEORETICAL STAGE
 The triangle indicated by <i>abc</i> represents the top stage of the distillation column.	 The triangle indicated by <i>cde</i> represents the top stage of the distillation column, and the triangle indicated by <i>abc</i> represents the partial condenser.
Heat Duty: $\dot{Q}_{TC} = V_1 \Delta h_{\text{vap}} = D(R + 1) \Delta h_{\text{vap}}$	Heat Duty: $\dot{Q}_{PC} = L_1 \Delta h_{\text{vap}} = D R \Delta h_{\text{vap}}$

For subcooled reflux:

If the reflux is subcooled, a portion of the vapor entering the top stage of the column will condense, providing heat to increase the liquid temperature to the bubble point. The additional amount of liquid that is condensed inside the column is determined by:

$$\Delta L = \frac{L_{\text{ER}} c_{\text{pR}} (T_1 - T_{\text{R}})}{\Delta h_{\text{vap}}}$$

Effective reflux ratio (also called internal reflux ratio) for the stages in the column:

$$\frac{L}{D} = \frac{L_{\text{ER}} + \Delta L}{D} = \frac{L_{\text{ER}} \left[1 + c_{\text{pR}} \frac{(T_1 - T_{\text{R}})}{\Delta h_{\text{vap}}} \right]}{D}$$

The temperature of the top stage in the column, T_1 , may be estimated as equal to the bubble point of the external reflux.

where

T_1 = temperature of top stage

T_R = temperature of the reflux

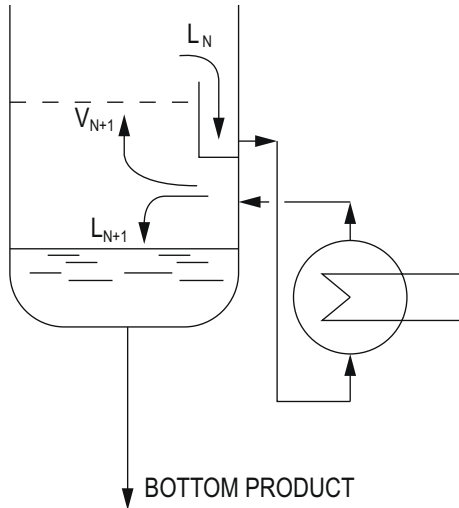
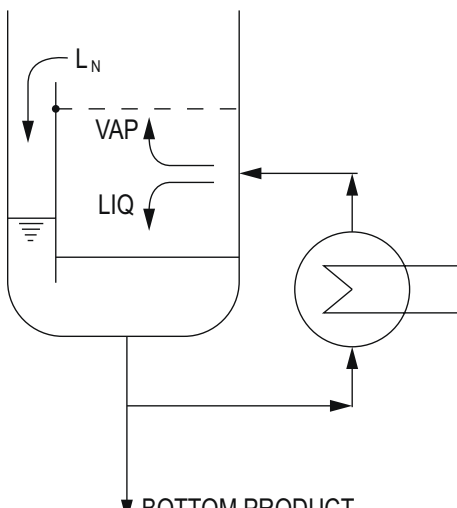
L_{ER} = external reflux ($L_{ER} = RD$)

ΔL = rate of liquid condensed on top stage of the column

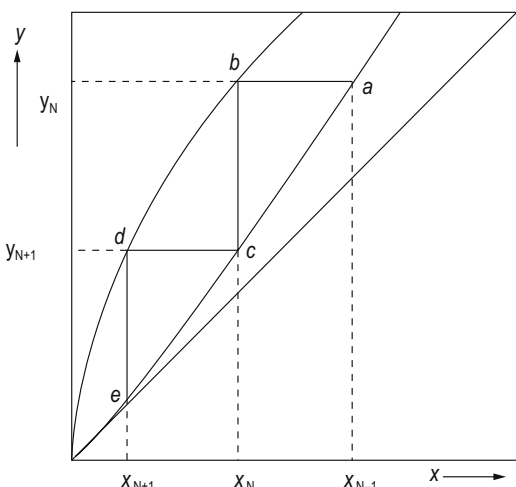
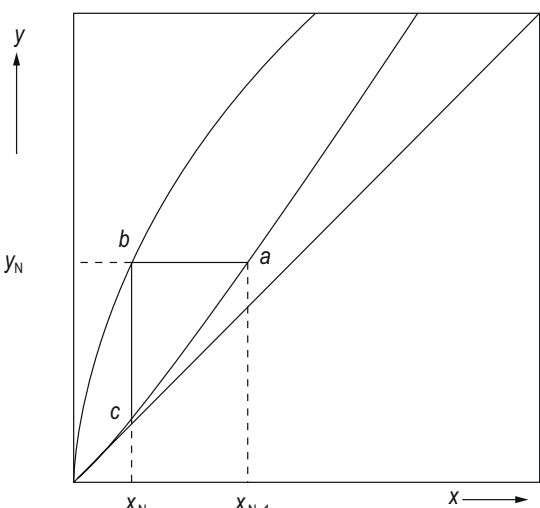
c_{pR} = heat capacity of the reflux

7.3.2.7 Reboilers

Types of Reboilers

Reboiler Without Mixing	Reboiler With Mixing
If the vapor effluent from the reboiler is in equilibrium with the bottom product, then the reboiler represents a theoretical stage. Other examples: kettle reboiler, internal heating coil.	If liquid effluent from the reboiler mixes with the liquid from the bottom stage of the column, the reboiler does not represent a theoretical stage.
	

Types of Reboilers

Reboiler Without Mixing	Reboiler With Mixing
 The triangle indicated by <i>abc</i> represents the bottom stage of the distillation column and the triangle indicated by <i>cde</i> represents the reboiler. Heat Duty: $\dot{Q}_R = V_{N+1} \Delta h_{\text{vap}} = S B \Delta h_{\text{vap}}$	 The triangle indicated by <i>abc</i> represents the bottom stage of the distillation column. Heat Duty: $\dot{Q}_R = V_R \Delta h_{\text{vap}} = S B \Delta h_{\text{vap}}$
Heat Duty: $\dot{Q}_R = B \left[(R + 1 - f) \frac{x_F - x_B}{x_D - x_F} - f \right] \Delta h_{\text{vap}}$	

7.3.2.8 Minimum Reflux

Underwood Method With No Distributed Nonkey Components

The Underwood method assumes constant relative volatilities and constant molal overflows, and it requires a trial-and-error solution.

First, by trial-and-error, find a value for ϕ that is between the relative volatilities of the light key and heavy key components. The relative volatilities are based on a characteristic temperature for the column, such as the bubble-point temperature of the distillate or the flashed feed temperature at the column pressure. The heavy key is the reference component j for the relative volatilities of each component i .

$$1 - q = \sum \frac{\alpha_{ij} z_{Fi}}{\alpha_{ij} - \phi} = \sum f_i$$

Second, calculate the value of $R_{\min}$ from:

$$R_{\min} + 1 = \sum \frac{\alpha_{ij} x_{Di}}{\alpha_{ij} - \phi}$$

where

z_{Fi} = mole fraction of component i in the feed

q = moles of feed to stripping section per mole of feed

α_{ij} = relative volatility between components i and j

ϕ = adjustable parameter, which has no physical significance

f_i = fraction of component i in the feed that is vaporized

x_{Di} = mole fraction of component i in the distillate

7.3.2.9 Minimum Theoretical Stages

The *Fenske equation* applies when the relative volatility is constant across the column. If the relative volatility varies across the column, a geometric mean of the range of values for the relative volatility may be used as an approximation. For example:

$$\alpha_{ij} = (\alpha_{\text{top}} \alpha_{\text{bot}})^{1/2}$$

or

$$\alpha_{ij} = (\alpha_{\text{top}} \alpha_{\text{mid}} \alpha_{\text{bot}})^{1/3}$$

For a binary separation, the Fenske equation for the number of stages (including any theoretical stages represented by the condenser and reboiler) at total reflux is

$$N_{\min} = \frac{\ln \left[\frac{x_D}{1-x_D} \frac{1-x_B}{x_B} \right]}{\ln \alpha}$$

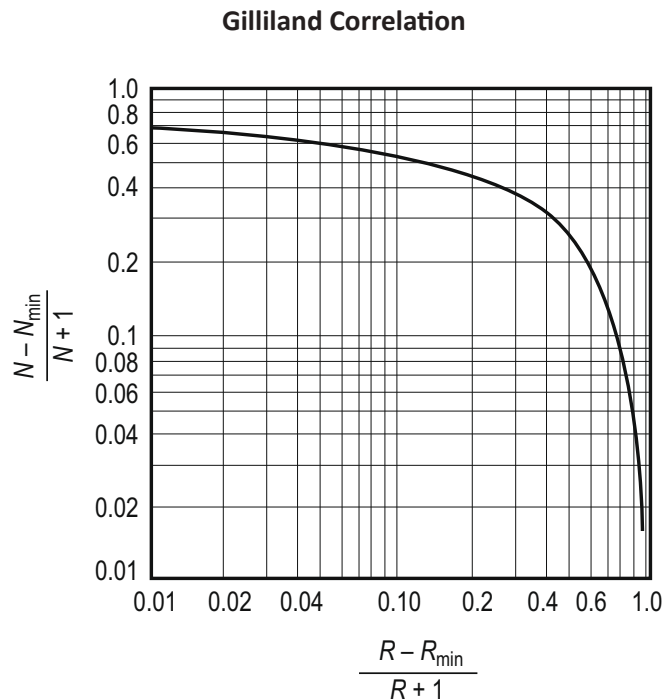
For a multicomponent separation with the light key indicated by i and the heavy key indicated by j , the Fenske equation is

$$N_{\min} = \frac{\ln \left[\frac{x_{Di}}{x_{Dj}} \frac{x_{Bj}}{x_{Bi}} \right]}{\ln \alpha_{ij}}$$

where $N_{\min}$ = minimum number of stages, including any theoretical stages represented by the condenser and reboiler

7.3.2.10 Shortcut Estimates for Number of Theoretical Stages

Estimated Number of Theoretical Stages: Gilliland Correlation



Source: McCabe, Warren L., Julian C. Smith, and Peter Harriott, *Unit Operations of Chemical Engineering*, 5th ed., New York: McGraw-Hill, 1993.

Estimated Number of Theoretical Stages: Underwood Correlation

The Underwood correlation can be used for constant volatility and partial reflux.

Underwood Correlation

Rectifying Section (Top)	Stripping Section (Bottom)
$0 \leq K_1 \leq 1, \quad K_2 > 1$	$K_1 < 0, \quad 0 \leq K_2 \leq 1$
$\frac{V}{L} = \frac{R+1}{R}$	$\frac{V}{L} = \frac{(R+1+f) \frac{x_F - x_B}{x_D - x_F} - f}{(1-f) + (R+1-f) \frac{x_F - x_B}{x_D - x_F}}$
$b = \frac{x_D}{R+1}$	$b = -\frac{x_B}{(R+1-f) \frac{x_F - x_B}{x_D - x_F} - f}$
$K_{1,2} = -\frac{1}{2} \left(b \frac{V}{L} - \frac{1 - \alpha_{12} \frac{V}{L}}{\alpha_{12} - 1} \right) \pm \sqrt{\frac{1}{4} \left(b \frac{V}{L} - \frac{1 - \alpha_{12} \frac{V}{L}}{\alpha_{12} - 1} \right)^2 - \frac{b \frac{V}{L}}{\alpha_{12} - 1}}$	
Intersection of feed line with operating lines: $x_1 = \left(\frac{x_F}{f} - \frac{x_D}{R+1} \right) \frac{f(R+1)}{1+R-f}$	
$N_R = \frac{\ln \frac{(x_D - K_1)(K_2 - x_I)}{(x_I - K_1)(K_2 - x_D)}}{\ln \left[\frac{\alpha_{12} \frac{V}{L}}{(1 + (\alpha_{12} - 1)K_1)^2} \right]}$	$N_S = \frac{\ln \frac{(x_I - K_1)(K_2 - x_B)}{(x_B - K_1)(K_2 - x_I)}}{\ln \left[\frac{\alpha_{12} \frac{V}{L}}{(1 + (\alpha_{12} - 1)K_1)^2} \right]}$

where

b = intercept of the operating line with the vertical axis

N_R = number of stages in the rectifying section

N_S = number of stages in the stripping section

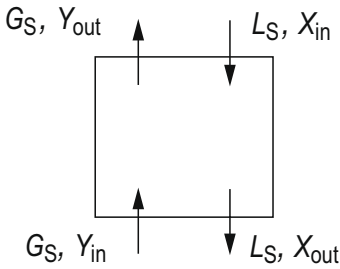
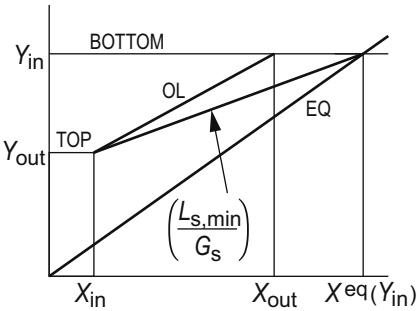
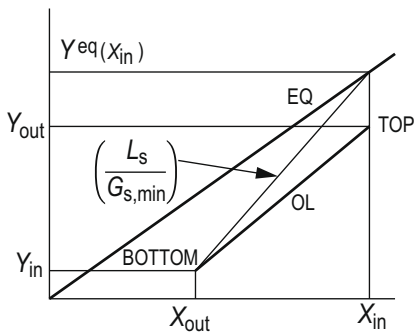
7.3.3 Absorption and Stripping

For dilute solutions ($x_{\text{solvent}} \approx 1$), use solute-free basis for the concentrations (X, Y) and the flow rates (G_S, L_S):

$$Y_A = \frac{y_A}{1 - y_A} = \frac{p_A}{P_{\text{tot}} - p_A} \quad X_A = \frac{x_A}{1 - x_A}$$

$$G_S = G(1 - y_A) = \frac{G}{(1 + Y_A)} \quad L_S = L(1 - x_A) = \frac{L}{(1 + X_A)}$$

Absorption and Stripping

Absorber		Stripper
Feed Y_{in} For fresh solvent: $X_{in} = 0$		Feed X_{in} For fresh stripping gas: $Y_{in} = 0$
Material Balance $Y_{out} = Y_{in} - \frac{L_S}{G_S} (X_{out} - X_{in}) \quad \text{and} \quad X_{out} = X_{in} - \frac{G_S}{L_S} (Y_{out} - Y_{in})$		
Equilibrium Line (EQ): $y = m x$ $Y_{EQ} = \frac{m X}{1 + X(1 - m)} \quad \text{or} \quad X_{EQ} = \frac{Y}{m + Y(m - 1)}$		
Operating Line (OL) $Y = \frac{L_S}{G_S} X + Y_{in} - \frac{L_S}{G_S} X_{out}$	Operating Line (OL) $Y = \frac{L_S}{G_S} X + Y_{out} - \frac{L_S}{G_S} X_{in}$	
Minimum Flow $L_{S,min} = \frac{G_S (Y_{in} - Y_{out})}{\frac{Y_{in}}{m + Y_{in}(m - 1)} - X_{in}}$	Minimum Flow $G_{S,min} = \frac{L_S (X_{in} - X_{out})}{\frac{m X_{in}}{m + X_{in}(m - 1)} - Y_{in}}$	
Diagram 	Diagram 	
Absorption Factors:	Equilibrium Equations:	Stripping Factors:
$A = \frac{L}{mG}$	General: $y = m x$	$S = \frac{mG}{L}$
$A = \frac{L}{KG}$	Vapor/Liquid: $y = K x$	$S = \frac{KG}{L}$
$A = \frac{P_{tot} L}{H G}$	Henry's Law: $y = \frac{H}{P_{tot}} x$	$S = \frac{H G}{P_{tot} L}$
$A = \frac{P_{tot} L}{p^{sat} G}$	Raoult's Law: $y = \frac{p^{sat}}{P_{tot}} x$	$S = \frac{p^{sat} G}{P_{tot} L}$

Absorption and Stripping (cont'd)

Absorber	Stripper
Efficiency $E = \frac{\left(\frac{L_s}{G_s}\right)_{\min}}{\left(\frac{L_s}{G_s}\right)_{\text{act}}} = \frac{X_{\text{out}} - X_{\text{in}}}{X^{\text{eq}}(Y_{\text{in}}) - X_{\text{in}}}$	Efficiency $E = \frac{\left(\frac{G_s}{L_s}\right)_{\min}}{\left(\frac{G_s}{L_s}\right)_{\text{act}}} = \frac{Y_{\text{out}} - Y_{\text{in}}}{Y^{\text{eq}}(X_{\text{in}}) - Y_{\text{in}}}$
Theoretical Stages $N_A = \frac{\ln \left[(1 - A^{-1}) \frac{[Y_{\text{in}} - Y^{\text{eq}}(X_{\text{in}})]}{[Y_{\text{out}} - Y^{\text{eq}}(X_{\text{in}})]} + A^{-1} \right]}{\ln(A)}$ $N_A = \frac{Y_{\text{out}} - Y_{\text{in}}}{Y^{\text{eq}}(X_{\text{in}}) - Y_{\text{out}}} \quad \text{for } A = 1$	Theoretical Stages $N_S = \frac{\ln \left[(1 - S^{-1}) \frac{[X_{\text{in}} - X^{\text{eq}}(X_{\text{in}})]}{[X_{\text{out}} - X^{\text{eq}}(X_{\text{in}})]} + S^{-1} \right]}{\ln(S)}$ $N_S = \frac{X_{\text{out}} - X_{\text{in}}}{X^{\text{eq}}(X_{\text{in}}) - X_{\text{out}}} \quad \text{for } A = 1$
NTU (Number of Transfer Units) $NTU_{\text{OY}} = \frac{Y_{\text{in}} - Y_{\text{out}}}{\Delta Y_{\text{lm}}}$ $\Delta Y_{\text{lm}} = \frac{[Y_{\text{in}} - Y^{\text{eq}}(X_{\text{out}})] - [Y_{\text{out}} - Y^{\text{eq}}(X_{\text{in}})]}{\ln \left[\frac{Y_{\text{in}} - Y^{\text{eq}}(X_{\text{out}})}{Y_{\text{out}} - Y^{\text{eq}}(X_{\text{in}})} \right]}$	NTU (Number of Transfer Units) $NTU_{\text{OX}} = \frac{X_{\text{in}} - X_{\text{out}}}{\Delta X_{\text{lm}}}$ $\Delta X_{\text{lm}} = \frac{[X_{\text{in}} - X^{\text{eq}}(Y_{\text{out}})] - [X_{\text{out}} - X^{\text{eq}}(Y_{\text{in}})]}{\ln \left[\frac{X_{\text{in}} - X^{\text{eq}}(Y_{\text{out}})}{X_{\text{out}} - X^{\text{eq}}(Y_{\text{in}})} \right]}$

where

ΔY_{lm} = log-mean concentration difference in the vapor phase (solute-free basis)

ΔX_{lm} = log-mean concentration difference in the liquid phase (solute-free basis)

NTU_{OY} = overall number of transfer units based on the gas phase

NTU_{OX} = overall number of transfer units based on the liquid phase

7.4 Design of Columns

7.4.1 Trayed Columns

7.4.1.1 Primary Tray Design Parameters

- Number of passes
- Tray spacing
- Tray type
- Outlet weir type and height
- Downcomer type and area
- Clearance under downcomer
- Hole size, valve size, or bubble cap size and style
- Fractional hole area for sieve and valve trays
- Tray pressure drop

- Tray efficiency
- Tray capacity
- Tray hydraulics (flooding)

Tray sizing calculations are performed at points where the column loading is expected to be the highest and lowest in each section. Typically, these are

- The top tray
- Above every feed, product draw-off, and point of heat addition or removal
- Below every feed, product draw-off, and point of heat addition or removal
- The bottom tray
- At any point in the column where the calculated vapor or liquid loading peaks

Starting Dimensions for Cross-Flow Sieve Trays

Dimension (units)	Vacuum	Atmospheric	Pressure
Tray spacing (in.)	24	24	24
Downcomer area (% column)	5	10	15
Active area (% column)	90	80	70
Hole area (% active)	12	10	8
Weir height (in.)	1	2	2
Hole diameter (in.)	0.25	0.25	0.25
Downcomer clearance (in.)	0.5	1.0	1.5

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Tray Selection

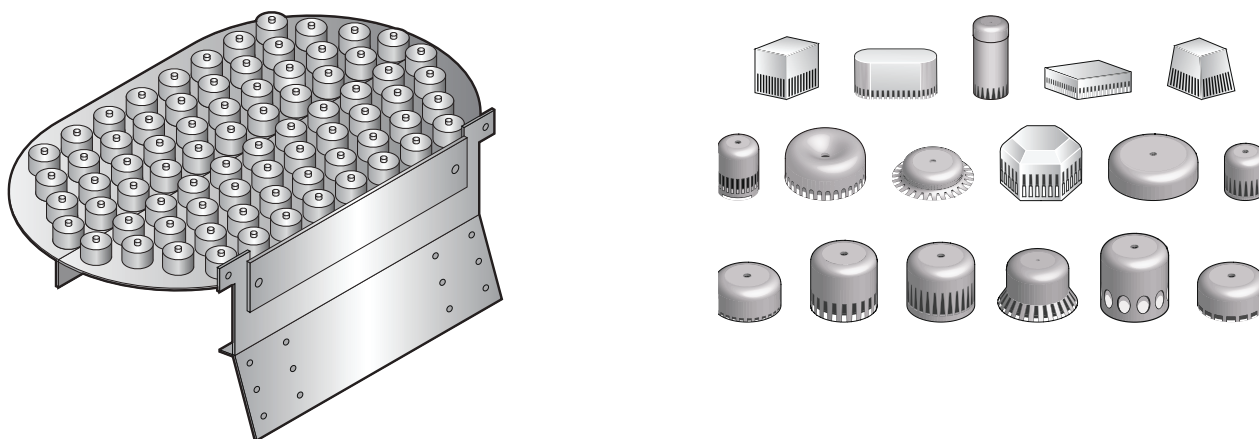
Criteria for Selecting a Distillation Column Device

Criterion	Details
Vapor-Handling Capacity	Entrainment flooding. At incipient flooding, the minimum column diameter is fixed.
Liquid-Handling Capacity	Fixes the size of downcomers. Downcomer backup can lead to flooding.
Mass-Transfer Efficiency	Sets required height for a given number of theoretical stages. Efficiency can be a function of column diameter.
Flexibility	Of concern when the column must be operated under a wide range of feed rates or when future capacity needs must be considered in the initial design.
Pressure Drop	Low pressure drop is critical for vacuum columns, especially when a low bottoms temperature must be maintained.
Cost	Consider total cost of the system, including auxiliary equipment; a more expensive device may lead to lower operating costs.
Design Limitations	Device should be proven commercially. Also, the user needs to understand how the device was designed (if by a vendor).
Special Concerns	Fouling, corrosion, ease of installation or removal, potential foaming problems, adequate residence time for reactions, special heat-transfer needs.

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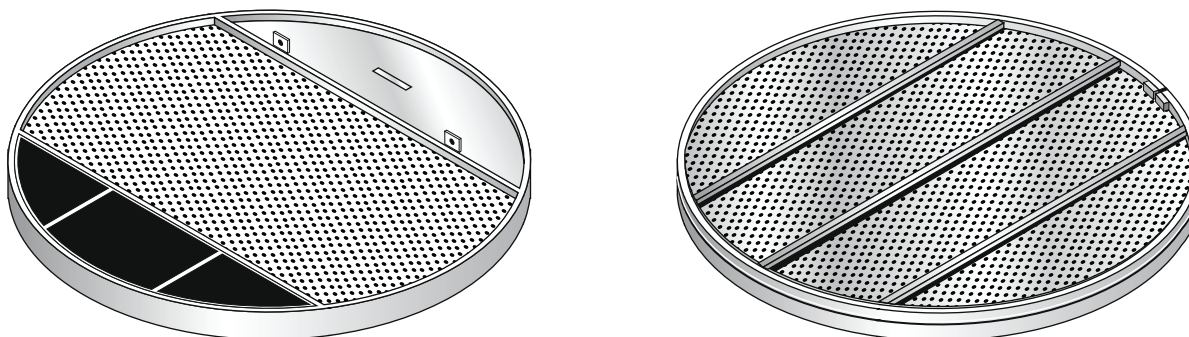
7.4.1.2 Common Types of Distillation Trays

Bubble Cap Tray (left) and Various Caps (right)



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Sieve Tray (left) and Dual-Flow Tray (right)



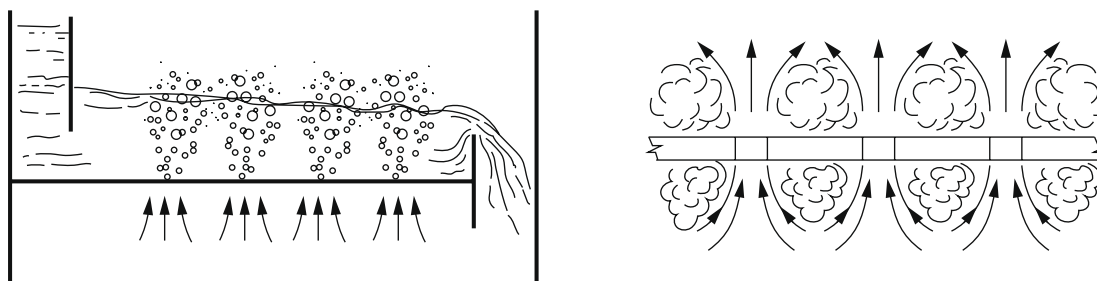
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Bubble Cap Trays



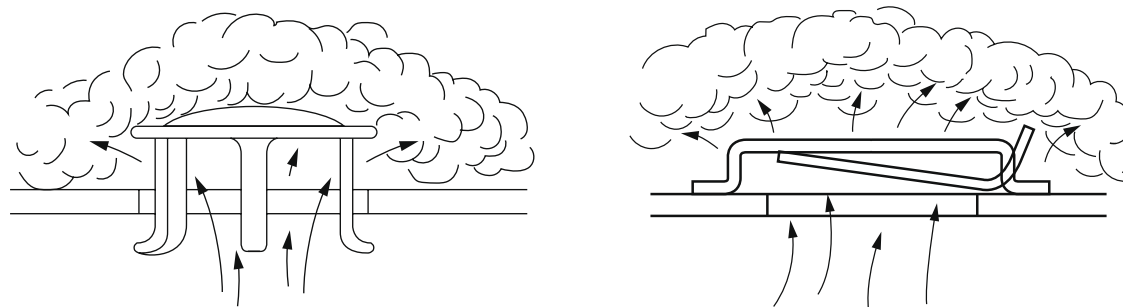
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Sieve Trays



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Valve Trays



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7.4.1.3 Comparison of Common Types of Distillation Trays

Comparison of the Common Tray Types

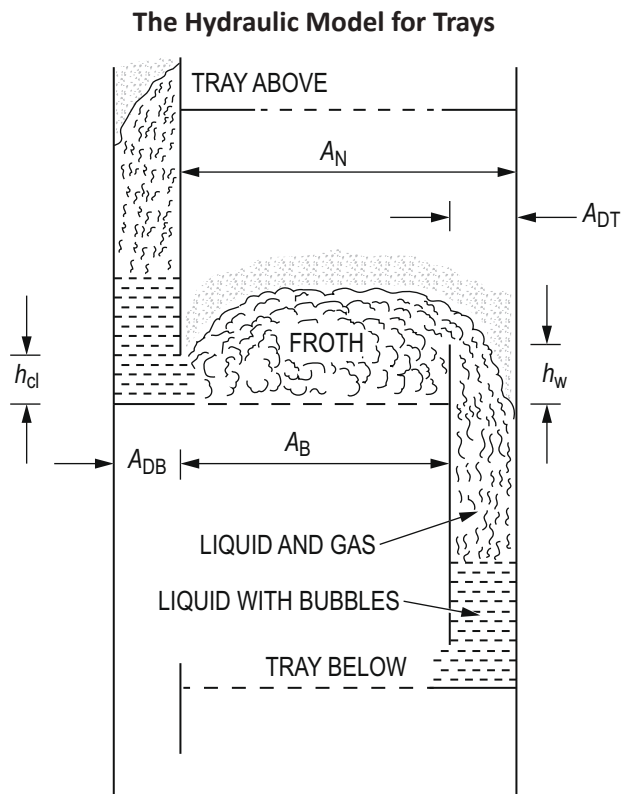
Feature	Sieve Trays	Valve Trays	Bubble Cap Trays	Dual-Flow Trays
Capacity	High	High to very high	Moderately high	Very high
Efficiency	High	High	Moderately high	Lower than other types
Turndown	About 2:1; not generally suitable for operation under variable loads	About 4–5:1; some special designs achieve (or claim) 10:1 or more	Excellent; better than valve trays; good at extremely low liquid rates	Low; even lower than sieve trays; unsuitable for variable load operation
Entrainment	Moderate	Moderate	High; about 3 times higher than sieve trays	Low to moderate
Pressure Drop	Moderate	Moderate; early designs somewhat higher; recent designs same as sieve trays	High	Low to moderate

Comparison of the Common Tray Types (cont'd)

Feature	Sieve Trays	Valve Trays	Bubble Cap Trays	Dual-Flow Trays
Cost	Low	About 20 percent higher than sieve trays	High; about 2–3 times the cost of sieve trays	Low
Maintenance	Low	Low to moderate	Relatively high	Low
Fouling Tendency	Low	Low to moderate	High; tends to collect solids	Extremely low; suitable where fouling is extensive and for slurry handling
Effects of Corrosion	Low	Low to moderate	High	Very low
Availability of Design Information	Well-known	Proprietary, but information readily available	Well-known	Some information available
Other				Instability sometimes occurs in large diameter (> 8 ft) columns
Main Applications	Most columns when turn-down is not critical	Most columns, services where turndown is important	Extremely low-flow conditions; where leakage must be minimized	Capacity revamps where efficiency and turndown can be sacrificed; highly fouling and corrosive services

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7.4.1.4 Hydraulic Model for Trays



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Tray Area Definitions

Tray Area	Symbol	Definition
Total tower cross-sectional area	A_T	The inside cross-section area of the empty tower without downcomers or trays
Net area	A_N	Total cross-section area minus the area at top of the downcomer; also referred to as <i>free area</i> ; represents smallest area available for vapor flow in the intertray spacing
Bubbling area	A_B	Total tower cross-section area minus total downcomer area, downcomer seal area, and any other nonperforated regions; also referred to as the active area (A_a); represents the area available to vapor flow near the tray floor
Hole area	A_h	Total area of perforations on the tray; smallest area available for vapor passage
Slot area	A_s	Total vertical curtain area for all valves through which vapor passes in a horizontal direction as it leaves the valves, based on the narrowest opening of the valves; smallest area available for vapor flow on a valve tray
Open slot area	A_{s0}	Slot area when all valves are fully opened
Fractional hole area	A_f	Ratio of hole area to bubbling area (in sieve trays) or slot area to bubbling area (in valve trays)
Downcomer top area	A_{DT}	Area at top of downcomer
Downcomer bottom area	A_{DB}	Area at bottom of downcomer

7.4.1.5 Definitions of Vapor Load

Several different parameters are used for characterization of the vapor load.

The *vapor load* (V_{load}), in $\frac{\text{ft}^3}{\text{sec}}$ or $\frac{\text{m}^3}{\text{s}}$, is

$$V_{load} = CFS \sqrt{\frac{\rho_G}{\rho_L - \rho_G}}$$

where

$$CFS = \text{vapor flow rate at conditions, in } \frac{\text{ft}^3}{\text{sec}} \text{ or } \frac{\text{m}^3}{\text{s}}$$

ρ_L, ρ_G = densities of the liquid and gas phases, respectively

The *F-factor* for gas loading, in $\frac{\text{ft}}{\text{sec}} \left(\frac{\text{lbm}}{\text{ft}^3} \right)^{0.5}$ or $\frac{\text{m}}{\text{s}} \left(\frac{\text{kg}}{\text{m}^3} \right)^{0.5}$, is

$$F = u \sqrt{\rho_G}$$

where u = superficial linear gas velocity

The *C-factor* for gas loading, in $\frac{\text{ft}}{\text{sec}}$ or $\frac{\text{m}}{\text{s}}$, is

$$C = u \sqrt{\frac{\rho_G}{\rho_L - \rho_G}}$$

In practice, the F-factor and the C-factor may be based on bubbling area A_B , net area A_N , or some other area, depending on the source of data and correlations. Care must be taken to use the correct area basis, depending on the source.

These terms are related as follows:

$$C = \frac{V_{load}}{A} = \frac{F}{\sqrt{\rho_L - \rho_G}}$$

7.4.1.6 Definitions of Liquid Load

The tray liquid load Q_L , in $\frac{\text{gpm}}{\text{in.}}$ or $\frac{\text{m}^3}{\text{hr} \cdot \text{m}}$, is

$$Q_L = \frac{\dot{V}_L}{L_W}$$

where

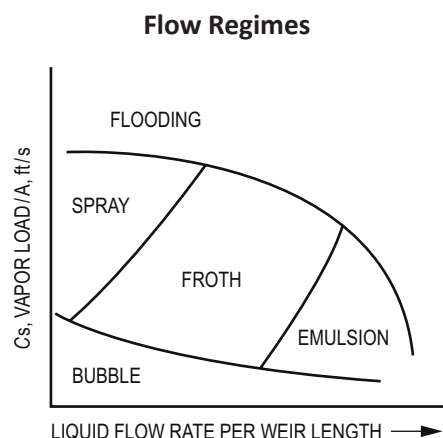
$\dot{V}_L$ = liquid volumetric flow rate, in $\frac{\text{gal}}{\text{min}}$ or $\frac{\text{m}^3}{\text{s}}$

L_W = outlet weir length, in inches or meters

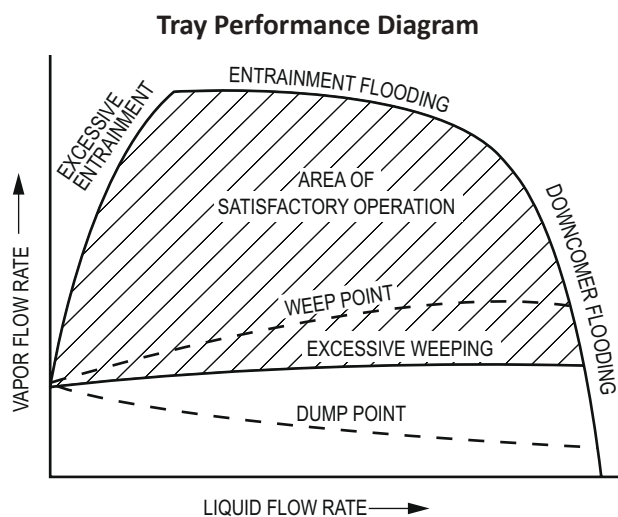
The downcomer liquid load Q_D , in $\frac{\text{gpm}}{\text{ft}^2}$ or $\frac{\text{ft}}{\text{sec}}$ or $\frac{\text{m}}{\text{s}}$, is

$$Q_D = \frac{\dot{V}_L}{A_{DT}}$$

7.4.1.7 Flow Regimes on Trays



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7.4.1.8 Column Flooding

Effect of Design Parameters on Flooding

Design Parameters That Lower Flooding Point	Spray Entrainment Flooding	Froth Entrainment Flooding	Downcomer Backup Flooding	Downcomer Choke Flooding
Low bubbling area	X	X	X	
Low fractional hole area (< 8%)	X	X	X	
Low tray spacing	X	X	X	
High weirs (> 4 in)		X	X	
Small weir length		X	X	
Small clearance under downcomer			X	
Small downcomer top area				X

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7.4.1.9 Entrainment Flooding

The correlations for entrainment given below are based on C-factors, specifically the Souders and Brown constant

C_{SB} at the entrainment flood point, in $\frac{\text{ft}}{\text{sec}}$ or $\frac{\text{m}}{\text{s}}$.

$$C_{SB, \text{flood}} = u_{S, \text{flood}} \sqrt{\frac{\rho_G}{\rho_L - \rho_G}}$$

where $u_{S, \text{flood}}$ = superficial gas velocity at the entrainment flood point

Fair's Entrainment Flooding Correlation

C_{SB} at the entrainment flood point, in $\frac{\text{ft}}{\text{sec}}$ or $\frac{\text{m}}{\text{s}}$

$$C_{SB, \text{flood}} = u_{N, \text{flood}} \left(\frac{20}{\gamma} \right)^{0.2} \left(\frac{\rho_G}{\rho_L - \rho_G} \right)^{0.5}$$

where $u_{N, \text{flood}}$ = superficial gas velocity at the entrainment flood point based on the net area A_N

γ = surface tension, in dyne/cm

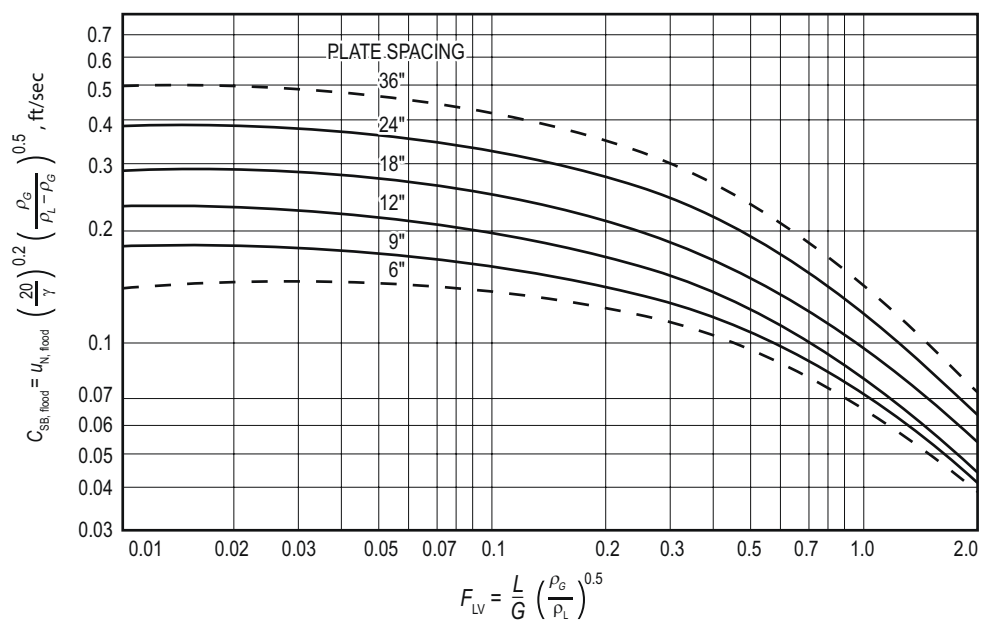
$C_{SB, \text{flood}}$ and $u_{N, \text{flood}}$ are based on the net area A_N . The correlation is applicable to sieve trays, valve trays, and bubble cap trays.

These restrictions apply:

1. System is nonfoaming or low-foaming.
2. Weir height is less than 15 percent of tray spacing.
3. Sieve-tray perforations are 13 mm (1/2 in.) or less in diameter.
4. Ratio of slot (bubble cap), perforation (sieve), or full valve opening (valve plate) area A_h to active area A_a is 0.1 or greater. Otherwise the value of $u_{N, \text{flood}}$ should be corrected using the following table:

$\frac{A_h}{A_a}$	$u_{N,flood}$ Correction Factor
0.10	1.00
0.08	0.90
0.06	0.80

Fair's Entrainment Flooding Correlation



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7.4.1.10 Downcomer Backup Flooding

The downcomer backup is determined by a pressure balance for the downcomer:

$$h_{dc} = h_t + h_w + h_{ow} + h_{hg} + h_{da}$$

where

h_{dc} = height of clear liquid in downcomer, in inches liquid or mm liquid

h_t = total tray pressure drop, in inches liquid or mm liquid

h_w = height of weir at tray outlet, in inches liquid or mm liquid

h_{ow} = height of liquid crest over weir, in inches liquid or mm liquid

h_{hg} = liquid hydraulic gradient across tray, in inches liquid or mm liquid

h_{da} = head loss due to liquid flow under downcomer apron, in inches liquid or mm liquid

The height of aerated liquid in the downcomer is determined by:

$$h'_{dc} = \frac{h_{dc}}{\phi_{dc}}$$

where

h'_{dc} = height of aerated liquid froth in downcomer, in inches froth or mm froth

ϕ_{dc} = relative froth density (froth density to liquid density)

To prevent downcomer backup flooding, the following criterion must be met:

$$h'_{dc} < S + h_w$$

where

S = tray spacing, in inches or millimeters

Downcomer Choke Flooding

Glitsch Correlation

The maximum clear liquid velocity at the downcomer entrance to avoid downcomer choke flooding is the lowest of the three following correlations:

$$(Q_{D,max})_1 = 250 SF$$

$$(Q_{D,max})_2 = 41 \sqrt{\rho_L - \rho_G} SF$$

$$(Q_{D,max})_3 = 7.5 \sqrt{S(\rho_L - \rho_G)} SF$$

where

S = tray spacing, in inches or millimeters

SF = derating factor

$Q_{D,max}$ = maximum downcomer liquid load, in $\frac{\text{gpm}}{\text{ft}^2}$ or $\frac{\text{ft}}{\text{sec}}$ or $\frac{\text{m}}{\text{s}}$

Generalized Criteria for Maximum Downcomer Velocity

Maximum Downcomer Velocities

Foaming Tendency	Example	Clear Liquid Velocity in Downcomer, $\frac{\text{ft}}{\text{sec}}$		
		18-in. Spacing	24-in. Spacing	30-in. Spacing
Low	Low-pressure (< 100 psia) light hydrocarbons, stabilizers, air-water simulators	0.4–0.5	0.5–0.6	0.6–0.7
Medium	Oil systems, crude oil distillation, absorbers, midpressure (100–300 psia) hydrocarbons	0.3–0.4	0.4–0.5	0.5–0.6
High	Amines, glycerine, glycols, high-pressure (> 300 psia) light hydrocarbons	0.2–0.25	0.2–0.25	0.2–0.3

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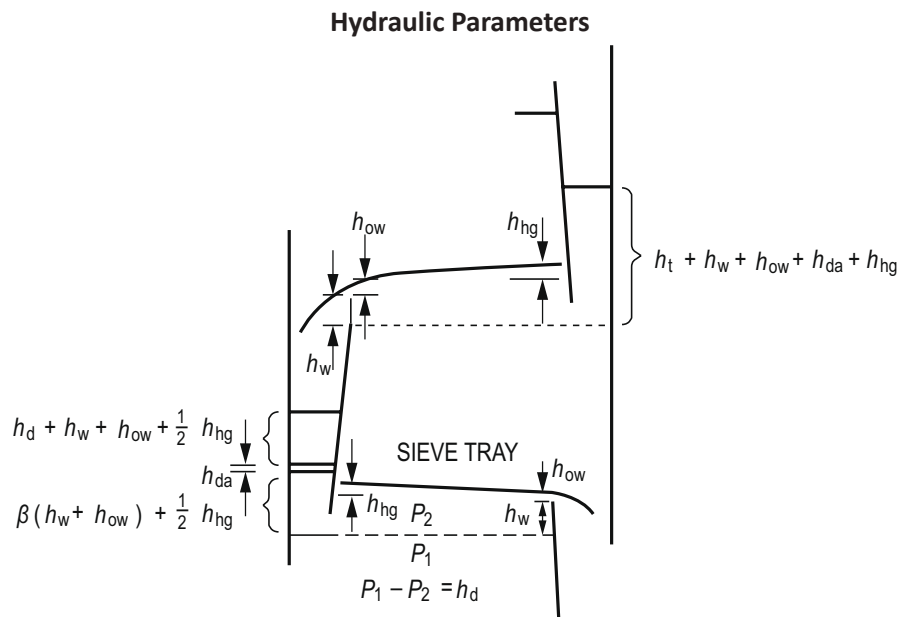
System Factors

Capacity Discount Factors for Foaming Systems

System Type	Examples	Factor
Nonfoaming		1.00
Fluorine systems	Freon, BF_3	0.90
Moderate foaming	Oil absorbers, amine, and glycol regenerators	0.85
Heavy foaming	Amine and glycol absorbers	0.73
Severe foaming	MEK units	0.60
Foam-stable	Caustic regenerators	0.30

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7.4.1.11 Tray Hydraulic Parameters



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where

h_d = dry tray pressure drop, in inches liquid or mm liquid

h_{da} = head loss due to liquid flow under downcomer apron, in inches liquid or mm liquid

h_{hg} = liquid hydraulic gradient across tray, in inches liquid or mm liquid

h_{ow} = height of liquid crest over weir, in inches liquid or mm liquid

h_t = total tray pressure drop, in inches liquid or mm liquid

h_w = height of weir at tray outlet, in inches liquid or mm liquid

β = tray aeration factor in pressure drop equation, dimensionless

7.4.1.12 Tray Pressure Drop

The total pressure drop across a tray, h_t :

$$h_t = h_d + h_l$$

where h_l = pressure drop through the aerated liquid on the tray, in inches liquid or mm liquid

7.4.1.13 Efficiency

The point efficiency is the ratio of the change of composition at a point to the change that would occur on a theoretical stage:

$$E_{OG} = \left(\frac{y_n - y_{n-1}}{y_n^{eq} - y_{n-1}} \right)_{\text{point}}$$

The Murphree tray efficiency applies to an entire tray instead of to a single point on a tray:

$$E_{MV} = \left(\frac{y_n - y_{n-1}}{y_n^{eq} - y_{n-1}} \right)_{\text{tray}}$$

Overall column efficiency:

$$E_{OC} = \frac{N_t}{N_a}$$

The overall column efficiency is related to the Murphree efficiency by:

$$E_{OC} = \frac{\ln[1 + E_{MV}(\lambda - 1)]}{\ln \lambda} \quad \text{with } \lambda = m \frac{V}{L}$$

where

E_{OC} = overall column efficiency

E_{OG} = point efficiency for a tray

E_{MV} = Murphree tray efficiency

N_t = number of theoretical stages in a column

N_a = number of actual stages in a column

y_n^{eq} = vapor mole fraction in equilibrium with the liquid

λ = ratio of slope of equilibrium curve to operating line

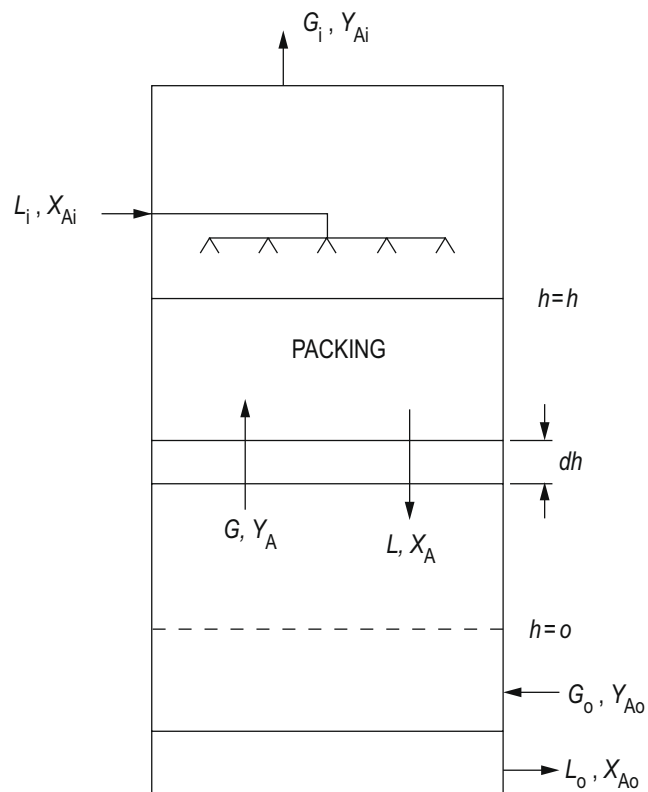
7.4.2 Packed Columns

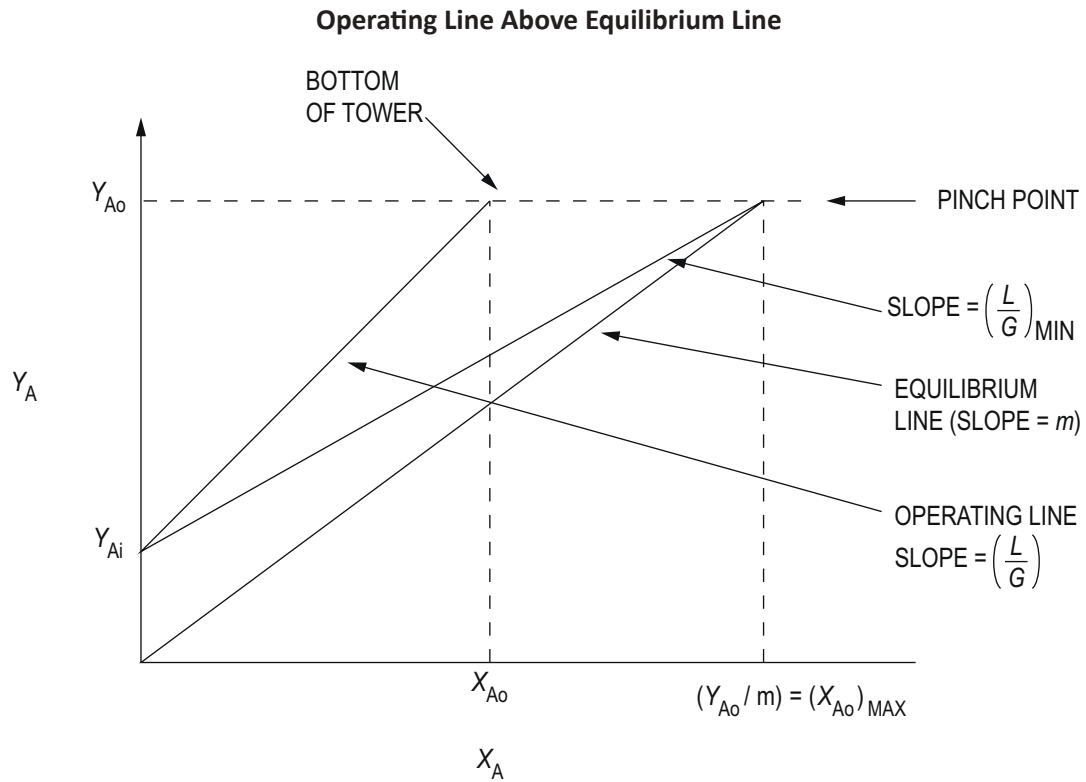
7.4.2.1 Primary Packing Design Parameters

- Type of tower separation
- Packing height
- Packing type and packing factors
- Tower pressure drop
- Flooding velocity calculation

7.4.2.2 Absorption and Stripping

Gas Absorption With Countercurrent Flow





where

G = mass velocity of gas phase

L = mass velocity of liquid phase

X_A = mass ratio A in liquid phase

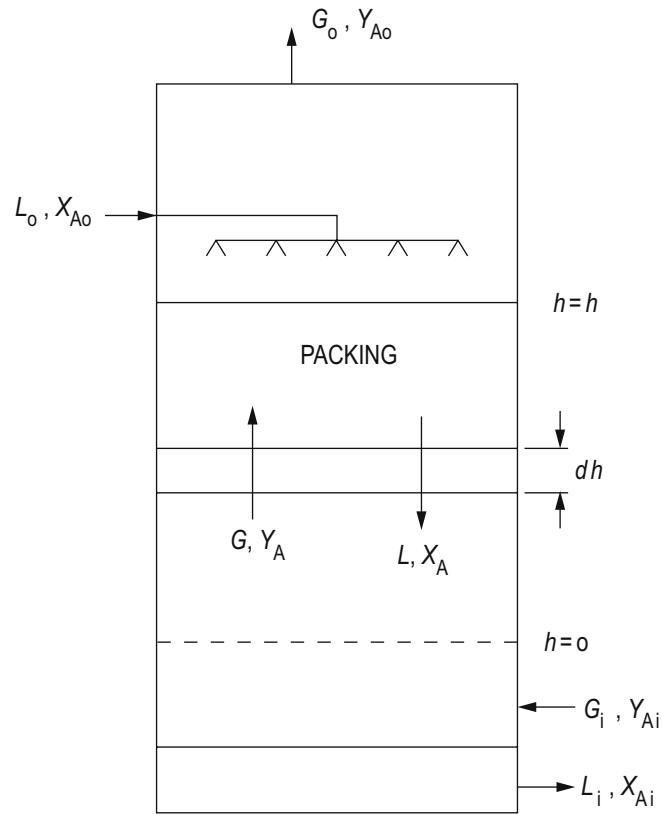
Y_A = mass ratio A in gas phase

h = height of packing

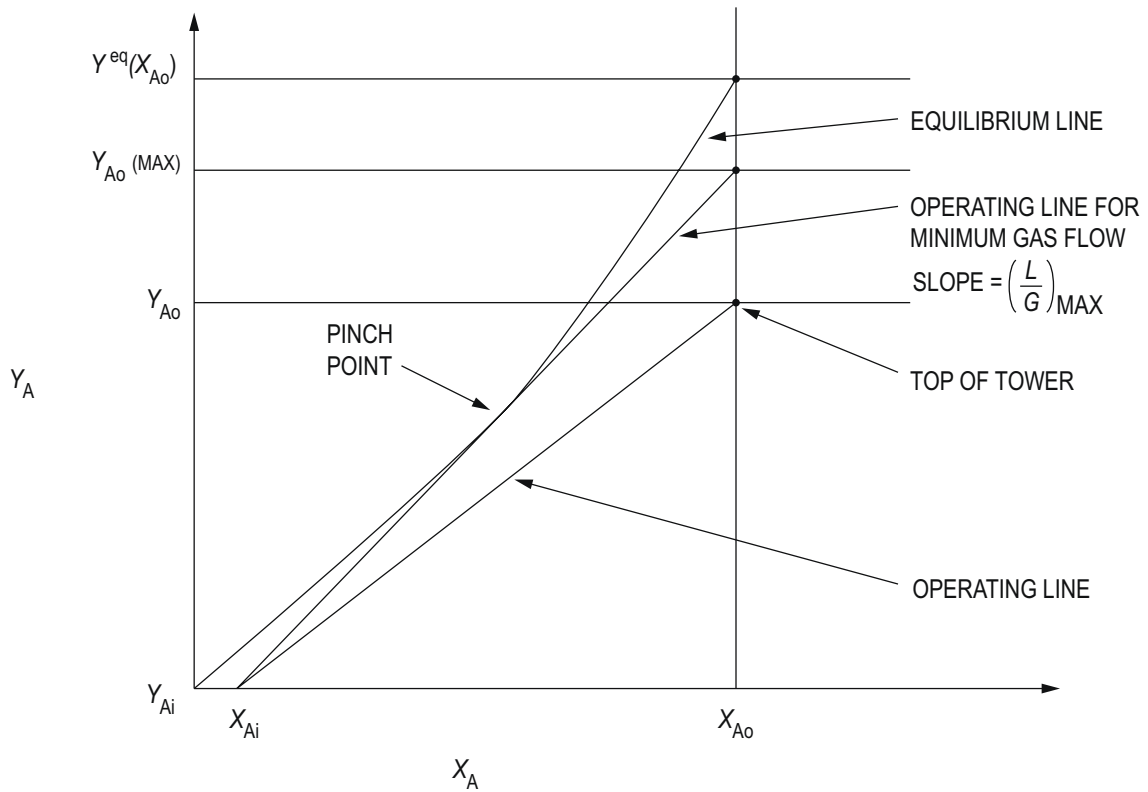
i = dilute end

o = rich end

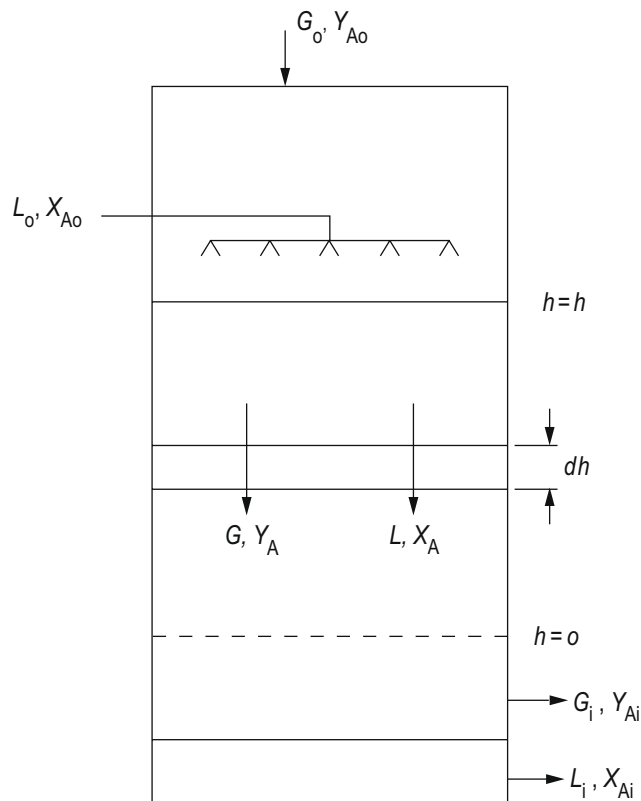
Desorption or Stripping With Countercurrent Flow



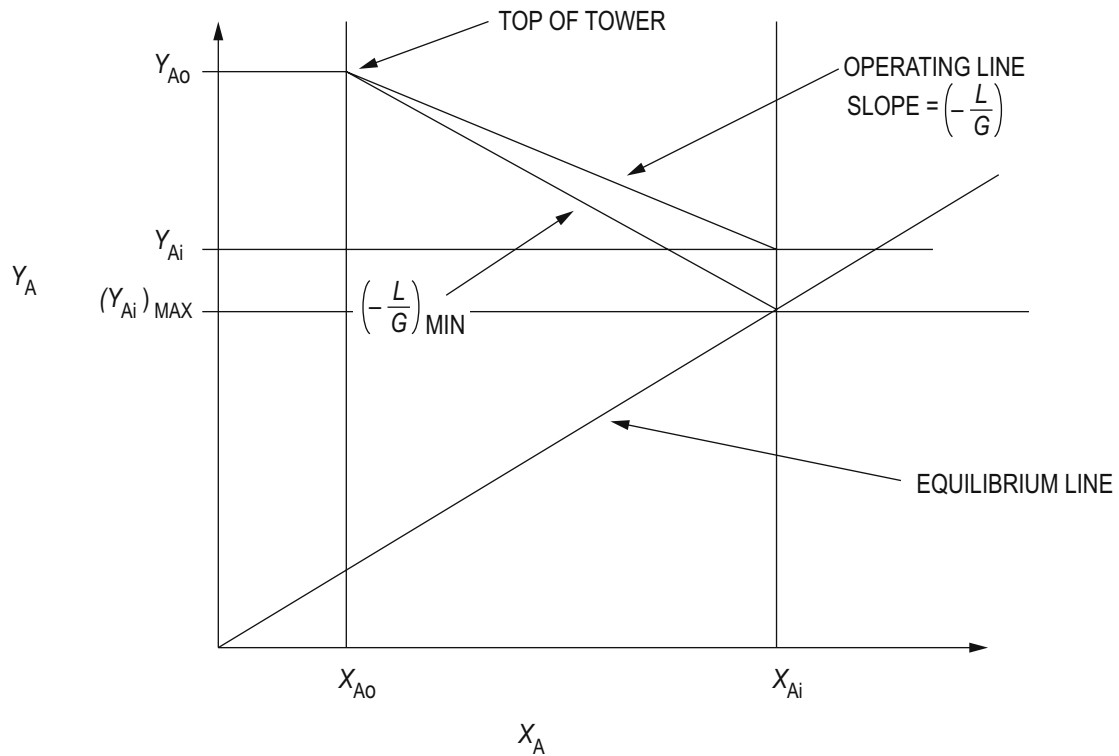
Operating Line Below Equilibrium Line



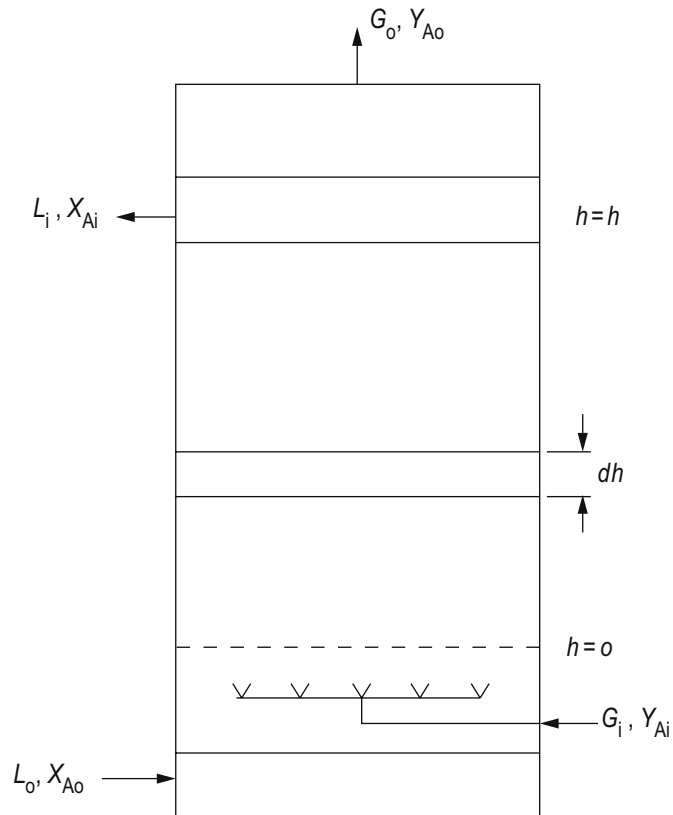
Gas Absorption With Concurrent Flow

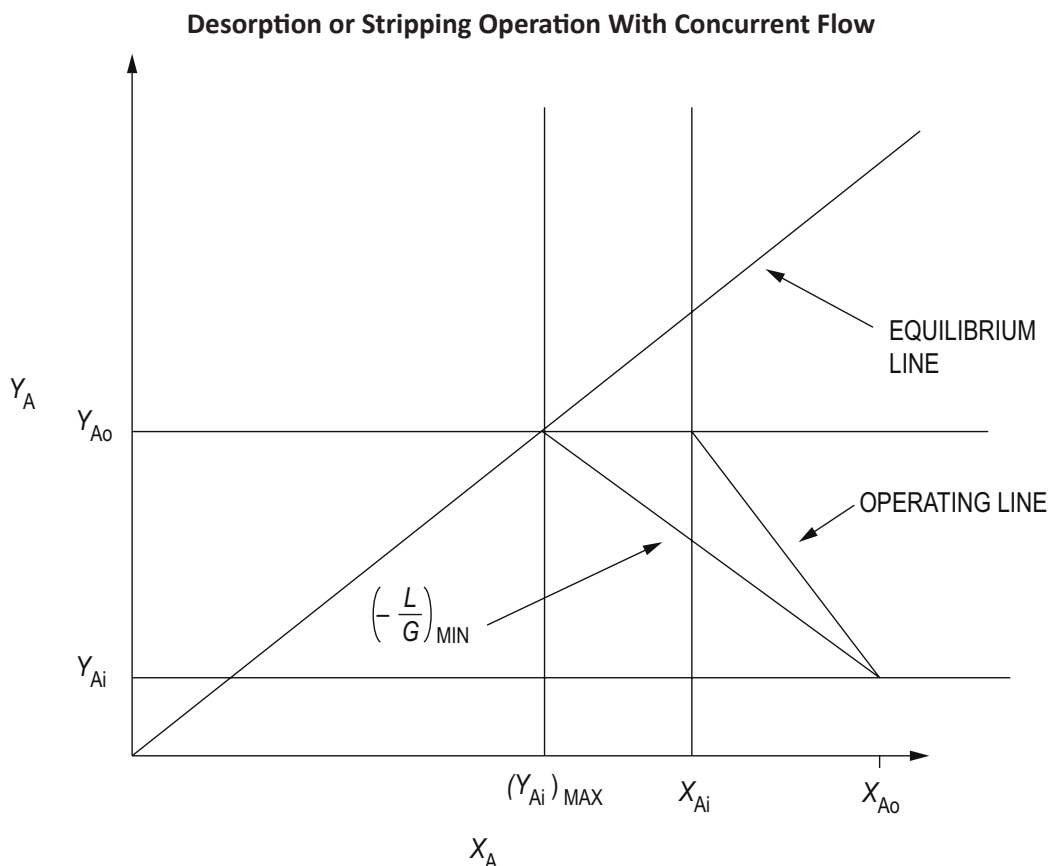


Absorption Operation With Concurrent Flow



Desorption or Stripping With Concurrent Flow





7.4.2.3 Mass-Transfer Coefficients

$$N_A = k_y (y_A - y_{As})$$

$$N_A = k_x (x_{As} - x_A)$$

$$N_A = K_x (x_A^* - x_A)$$

$$N_A = K_y (y_A - y_A^*)$$

where

N_A = molar flux of A

k_x, k_y = individual mass-transfer coefficients

K_x, K_y = overall mass-transfer coefficients

x_{As}, y_{As} = solute mole fraction at interface in liquid and gas phase, respectively

x_A^* = mole fraction of solute in the liquid phase at equilibrium

y_A^* = mole fraction of solute in the gas phase at equilibrium

$$(N_A)_{AVG} A_i = (k_y)_{AVG} (y_A - y_{As}) A_i$$

where A_i = total interfacial area

$$A_i = a A h$$

where

a = interfacial area per unit volume, in ft^2

A = cross-sectional area, in ft^2

h = height of packing, in ft

7.4.2.4 Packing Design

Operating Line

The equation for the operating line is

$$G_o y_{Ao} - G y_A = L_o x_{Ao} - L x_A \quad \text{or} \quad y_A = \frac{L}{G} x_A + \frac{1}{G} (G y_{Ao} - L_o x_{Ao})$$

Packing Height of Transfer Unit

The height of packing is

$$h = H_G \int_{y_{Ai}}^{y_{Ao}} \frac{(1 - y_A)_{lm} dy_A}{(1 - y_A)(y_A - y_{As})} \quad \text{and} \quad h = n_G H_G$$

where

n_G = number of gas-phase transfer units

lm = log mean

The number of gas-phase transfer units is

$$n_G = \int_{y_{Ai}}^{y_{Ao}} \frac{(1 - y_A)_{lm} dy_A}{(1 - y_A)(y_A - y_{As})}$$

where $(1 - y_A)_{lm} = \frac{(1 - y_A) + (1 - y_{As})}{2}$ as an approximation

For dilute solutions, assume L , G , and slope m are constant.

$$H_{OG} = H_G + \frac{mG}{L} H_L = \frac{mG}{L} H_{OL}$$

where H_{OG} and H_{OL} = height of overall transfer units in gas and liquid phases, respectively

$\frac{L}{mG} = \mathcal{A}$ = absorption factor, which ranges from 1.0 to 1.4

$\frac{1}{\mathcal{A}} = \mathcal{S}$ = stripping factor

$$m = \frac{H'}{P}$$

where

P = absolute pressure

H' = Henry's constant

$$H_{OG} = \frac{H_{OL}}{\mathcal{A}}$$

$$n_{OG} = \frac{y_{Ao} - y_{Ai}}{(y_A - y_A^*)_{lm}} \quad \text{and} \quad n_{OL} = \frac{x_{Ao} - x_{Ai}}{(x_A^* - x_A)_{lm}}$$

where $(y_A - y_A^*)_{lm} = \frac{(y_A - y_A^*)_o - (y_A - y_A^*)_i}{\ln \left[\frac{(y_A - y_A^*)_o}{(y_A - y_A^*)_i} \right]}$

and similarly for $(x_A^* - x_A)_{lm}$

For dilute solutions:

$$n_{OL} = \frac{x_{Ao} - x_{Ai}}{(x_A - x_A^*)_{lm}}$$

Packing HETP

For gas absorption: $h = n_{OG} H_{OG}$

For gas stripping: $h = n_{OL} H_{OL}$

Also, $h = N_{TP} HETP$

where

N_{TP} = number of theoretical plates

$HETP$ = height of an equivalent theoretical plate

$$N_{TP} = \frac{\ln \left[\left(1 - \frac{HG}{pL} \right) \left(\frac{y_{Ao} - \frac{Hx_{Ai}}{p}}{y_{Ai} - \frac{Hx_{Ao}}{p}} \right) + \frac{HG}{pL} \right]}{\ln \frac{pL}{HG}}$$

$$\mathcal{A} = \frac{L}{mG} \quad m = \frac{H}{p} \quad \mathcal{A} = \frac{pL}{HG}$$

$$\mathcal{S} = \frac{1}{\mathcal{A}} = \frac{HG}{pL}$$

$$N_{TP} = \frac{\ln \left[\left(1 - \frac{1}{\mathcal{S}} \right) \left(\frac{x_{Ao} - \frac{y_{Ai}}{m}}{x_{Ai} - \frac{y_{Ao}}{m}} \right) + \frac{1}{\mathcal{S}} \right]}{\ln(\mathcal{S})}$$

where

$\mathcal{A}$ = absorption factor

$\mathcal{S}$ = stripping factor

Note: For absorption and stripping, calculations for tower height are the same, although the operating line slope will differ.

Height of Packing

$$h = NTU_{OG} HTU_{OG} = NTU_{OL} HTU_{OL} = N_T HETP$$

where

$HETP$ = height equivalent of a theoretical stage

HTU_{OG} = height of an overall vapor-phase mass-transfer unit

HTU_{OL} = height of an overall liquid-phase mass-transfer unit

NTU_{OG} = overall number of transfer units based on gas phase

NTU_{OL} = overall number of transfer units based on liquid phase

N_T = number of theoretical stages

h = height of packing

Height of Overall Transfer Unit

$$H_{OG} = \frac{G_s}{K_y a (1 - y_A)_{lm}}$$

where $(1 - y_A)_{lm} = \frac{(1 - y_A^*) - (1 - y_A)}{\ln \left(\frac{1 - y_A^*}{1 - y_A} \right)}$

$$H_{OL} = \frac{L_s}{K_x a (1 - x_A)_{lm}}$$

where $(1 - x_A)_{lm} = \frac{(1 - x_A) - (1 - x_A^*)}{\ln \left(\frac{1 - x_A}{1 - x_A^*} \right)}$

Number of Gas-Phase Transfer Units

$$n_{OG} = \int_{y_{A1}}^{y_{A0}} \frac{(1 - y_A)_{lm}}{(1 - y_A)(y_A - y_A^*)} dy_A$$

Using the log-mean average:

$$n_{OG} = 0.5 \ln \frac{1 - y_{Ai}}{1 - y_{Ao}} + \int_{y_{Ai}}^{y_{Ao}} \frac{dy_A}{(y_A - y_A^*)}$$

In dilute solutions:

$$n_{OG} = \frac{y_{Ao} - y_{Ai}}{(y_A - y_A^*)_{lm}}$$

where $(y_A - y_A^*)_{lm} = \frac{(y_A - y_A^*)_{bottom} - (y_A - y_A^*)_{top}}{\ln \frac{(y_A - y_A^*)_{bottom}}{(y_A - y_A^*)_{top}}}$

Number of Liquid-Phase Transfer Units

$$n_{OL} = \int_{x_{Ai}}^{x_{Ao}} \frac{(1 - x_A)_{lm}}{(1 - x_A)(x_A^* - x_A)} dx_A$$

$$n_{OL} = 0.5 \ln \frac{1 - x_{Ao}}{1 - x_{Ai}} + \int_{x_{Ai}}^{x_{Ao}} \frac{dx_A}{x_A^* - x_A}$$

Absorption With Reaction

Dissolved solute reacts with solvent in liquid phase if irreversible reaction:

$$n_{OG} = \ln \frac{y_{Ao}}{y_{Ai}}$$

7.4.2.5 Correlations for Mass-Transfer Coefficients

For insoluble gases that do not react chemically with the liquid:

$$H_x = \frac{1}{\alpha} \left(\frac{G_x}{\mu_x} \right)^\eta \left(\frac{\mu_x}{\rho_x D_{vx}} \right)^{0.5}$$

where

H_x = individual liquid-phase HTU

G_x = mass velocity of liquid

μ_x = viscosity of liquid

D_{vx} = diffusivity of liquid

ρ_x = liquid density

α and η = constants given in the table below

Values of α and η in Equations¹ for Various Packing Materials at 77°F

Packing Type	Packing Size (in.)	α	η
Rings	2	80	0.22
	1.5	90	0.22
	1	100	0.22
	0.5	280	0.35
	0.375	550	0.46
Saddles	1.5	160	0.28
	1	170	0.28
	0.5	150	0.28
Tile	3	110	0.28

1. All quantities in equations must be expressed in fps units if these values of α are used.

Source: McCabe, W.L., and J.C. Smith, *Unit Operations of Chemical Engineering*, 3rd ed., New York: McGraw-Hill, 1976.

The temperature effect of liquids on the HTU can be evaluated as:

$$H_x = H_{x0} e^{-0.013(T - T_0)}$$

where

$$H_x = HTU \text{ at } T \text{ } ^\circ\text{F}$$

$$H_{x0} = HTU \text{ at } T_0 \text{ } ^\circ\text{F}$$

$$T = \text{final temperature in } ^\circ\text{F}$$

$$T_0 = \text{initial temperature in } ^\circ\text{F}$$

Henry's Law: $y^* = m x$

$$x^* = \frac{y}{m}$$

where m = Henry's Law Constant/Total Pressure

7.4.2.6 Packing Factors

Selection of packing is based primarily on packing factors and avoidance of flooding.

Packing Factors (ft^{-1})

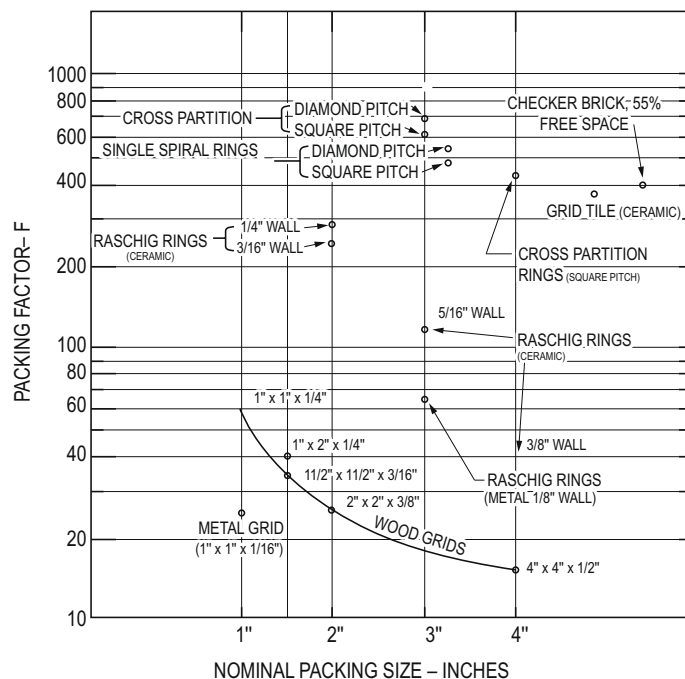
(WET AND DUMP PACKED)

TYPE OF PACKING	MAT'L.	NOMINAL PACKING SIZE (INCHES)										
		¼	⅜	½	⅝	¾	1	1 ¼	1 ½	2	3	3 ½
SUPER INTALOX	CERAMIC	—	—	—	—	—	60	—	—	30	—	—
SUPER INTALOX	PLASTIC	—	—	—	—	—	33	—	—	21	16	—
INTALOX SADDLES	CERAMIC	725	330	200	—	145	98	—	52	40	22	—
HY-PAK RINGS	METAL	—	—	—	—	—	42	—	—	18	15	—
PALL RINGS	PLASTIC	—	—	—	97	—	52	—	40	25	—	16
PALL RINGS	METAL	—	—	—	70	—	48	—	28	20	—	16
BERL SADDLES	CERAMIC	× 900	—	× 240	—	170	110	—	65	× 45	—	—
		⊖		⊖		□	□		□			
RASCHIG RINGS	CERAMIC	× 1600	× 1000	580	380	255	155	× 125	95	65	× 37	—
		⊖	⊖	⊕	⊕	⊕	⊕	⊗	⊗	●	●	
RASCHIG RINGS 1/32" WALL	METAL	× 700	× 390	× 300	170	155	× 115	—	—	—	—	—
RASCHIG RINGS 1/16" WALL	METAL	—	—	410	290	220	137	× 110	83	57	× 32	—

× EXTRAPOLATED ⊕ 1/8" WALL × × F ≈ 0/ε³ OBTAINED IN 16" AND 30" I.D. TOWER
 ○ 1/32" WALL ⊗ 3/16" WALL □ DATA BY LEVA
 ⊖ 1/16" WALL ● 1/4" WALL
 ⊕ 3/32" WALL ● 3/8" WALL

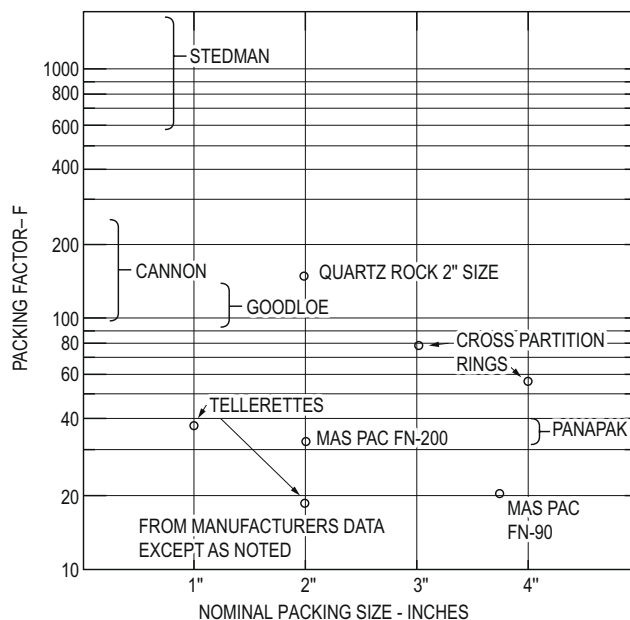
Source: Eckert, J.S., E.H. Foote, R.R. Nemunaitis, and L.H. Rollison, Akron, OH: Norton Chemical Process Products Division, 1972 (revised 2001).

Packing Factors: Stacked Packings & Grids



Source: Eckert, J.S., E.H. Foote, R.R. Nemunaitis, and L.H. Rollison, Akron, OH: Norton Chemical Process Products Division, 1972 (revised 2001).

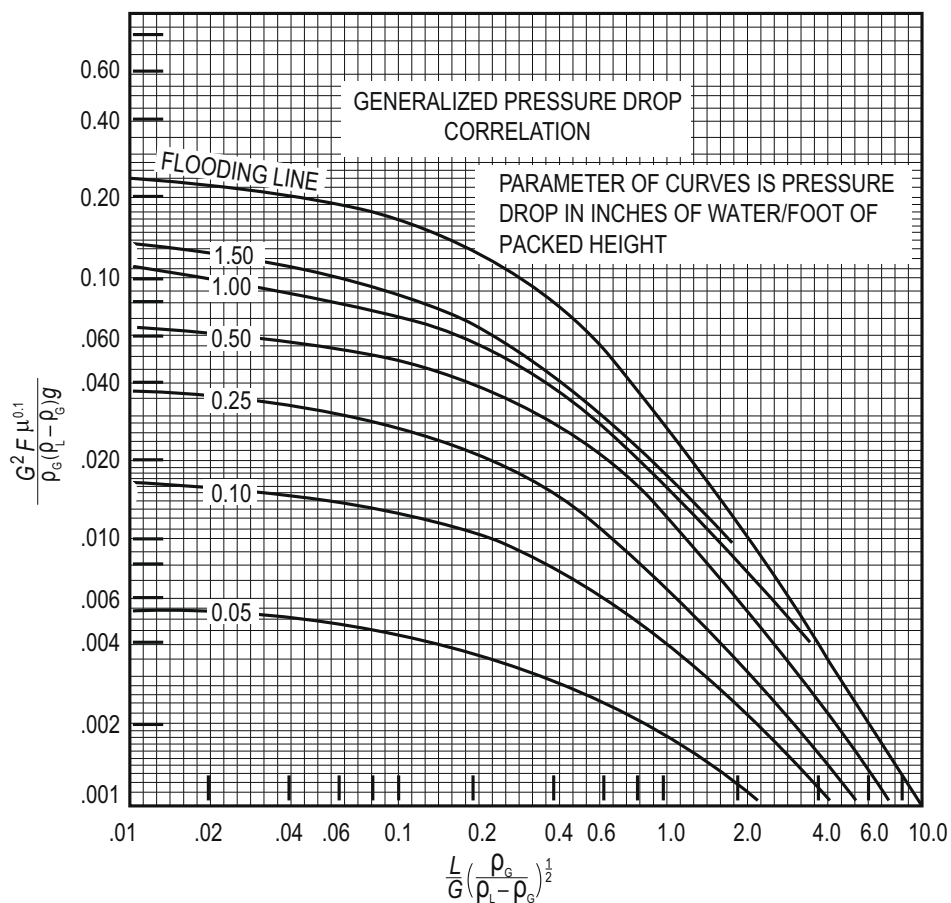
Packing Factors: Screen Packing & Random Dumped Packing



Source: Eckert, J.S., E.H. Foote, R.R. Nemunaitis, and L.H. Rollison, Akron, OH: Norton Chemical Process Products Division, 1972 (revised 2001).

7.4.2.7 Flooding and Pressure Drop

Generalized Pressure Drop Correlation



L = LIQUID RATE, $\frac{\text{lbm}}{\text{sec-ft}^2}$

G = GAS RATE, $\frac{\text{lbm}}{\text{sec-ft}^2}$

ρ_L = LIQUID DENSITY, $\frac{\text{lbm}}{\text{ft}^3}$

ρ_g = GAS DENSITY, $\frac{\text{lbm}}{\text{ft}^3}$

F = PACKING FACTOR

μ = VISCOSITY OF LIQUID, cP

g = GRAVITATIONAL ACCELERATION

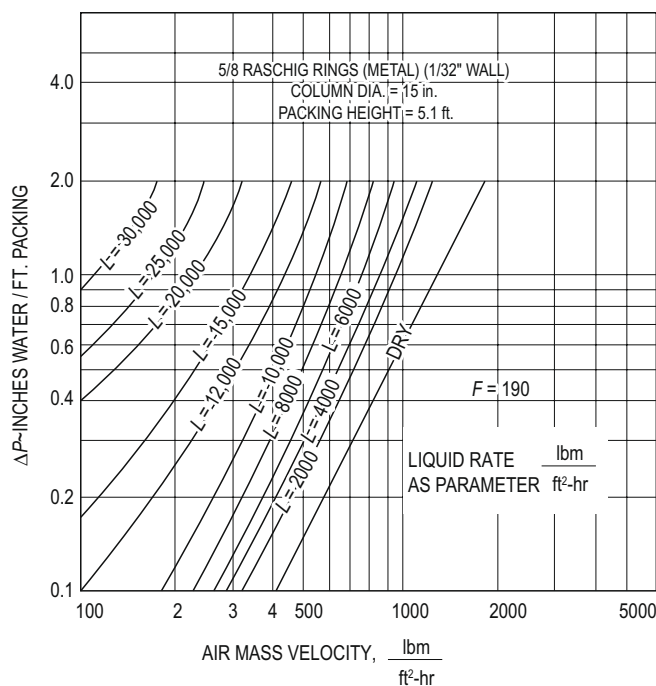
Source: Eckert, J.S., E.H. Foote, R.R. Nemunaitis, and L.H. Rollison, Akron, OH: Norton Chemical Process Products Division, 1972 (revised 2001).

Determination of column diameter D :

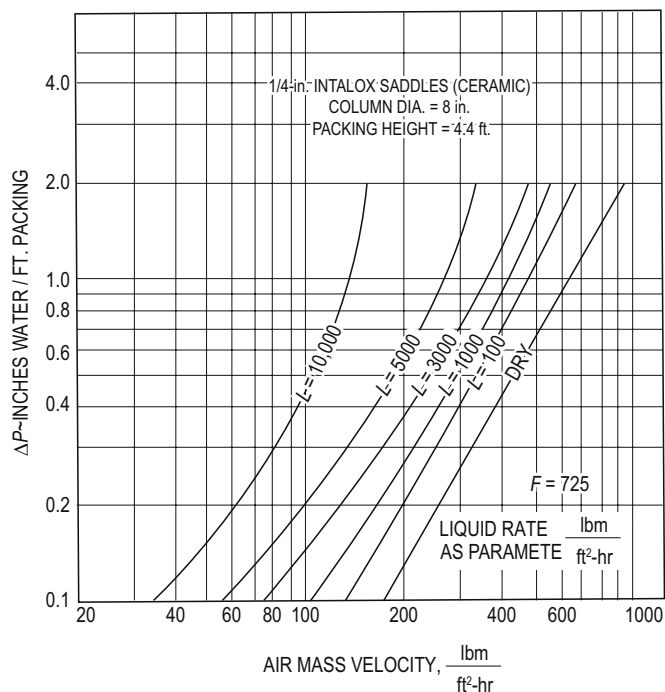
$$D = \sqrt{\left(\frac{4}{\pi}\right) \left(\frac{G_A}{G}\right)} \quad \text{where} \quad G_A = \text{actual gas flow rate of the packed column}$$

Chapter 7: Mass Transfer

Pressure Drop Versus Gas Rate

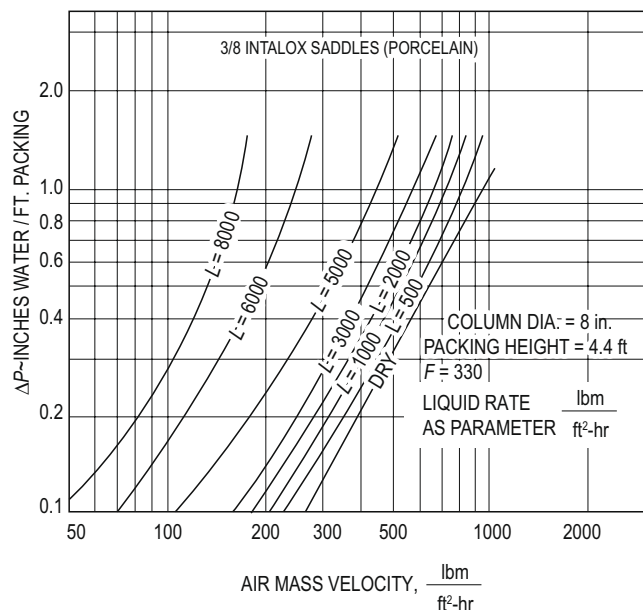


Source: Eckert, J.S., E.H. Foote, R.R. Nemunaitis, and L.H. Rollison, Akron, OH: Norton Chemical Process Products Division, 1972 (revised 2001).

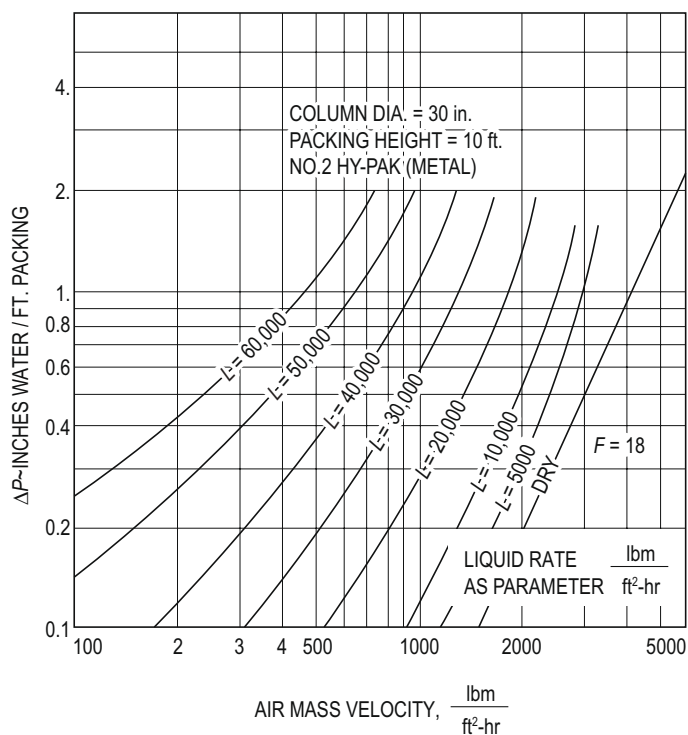


Source: Eckert, J.S., E.H. Foote, R.R. Nemunaitis, and L.H. Rollison, Akron, OH: Norton Chemical Process Products Division, 1972 (revised 2001).

Pressure Drop Versus Gas Rate (cont'd)

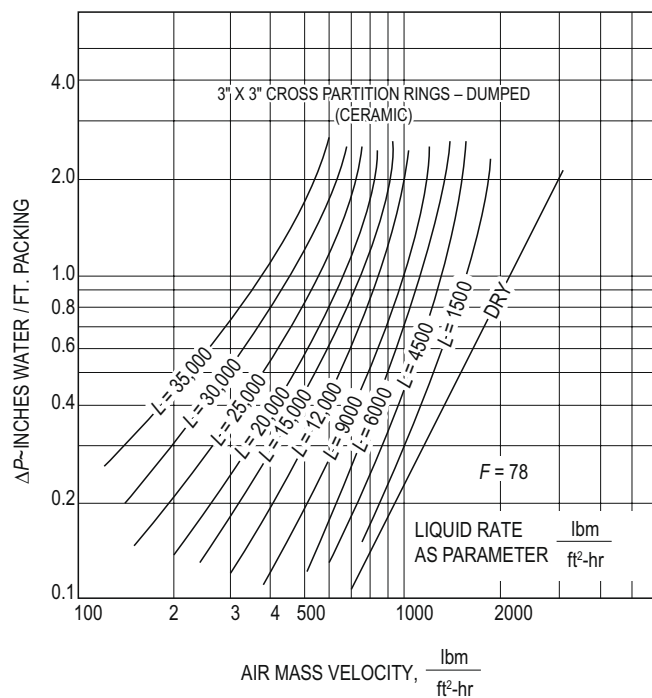


Source: Eckert, J.S., E.H. Foote, R.R. Nemunaitis, and L.H. Rollison, Akron, OH: Norton Chemical Process Products Division, 1972 (revised 2001).

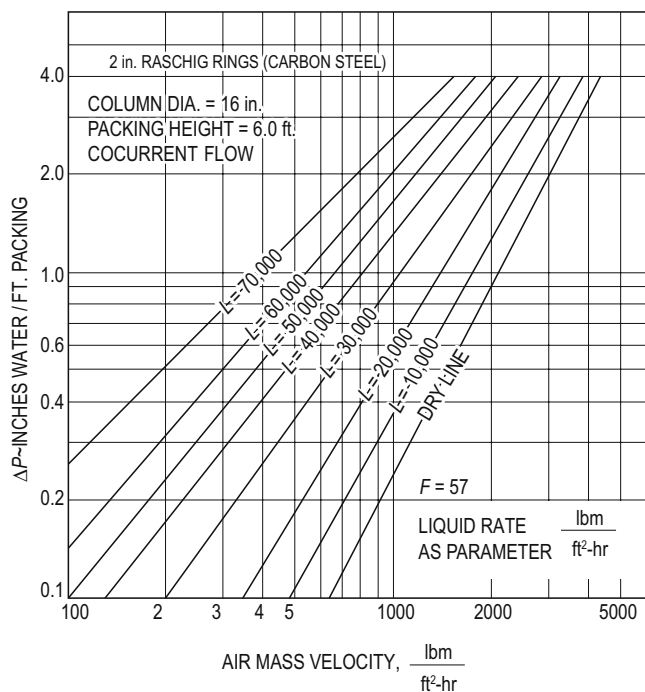


Source: Eckert, J.S., E.H. Foote, R.R. Nemunaitis, and L.H. Rollison, Akron, OH: Norton Chemical Process Products Division, 1972 (revised 2001).

Pressure Drop Versus Gas Rate (cont'd)

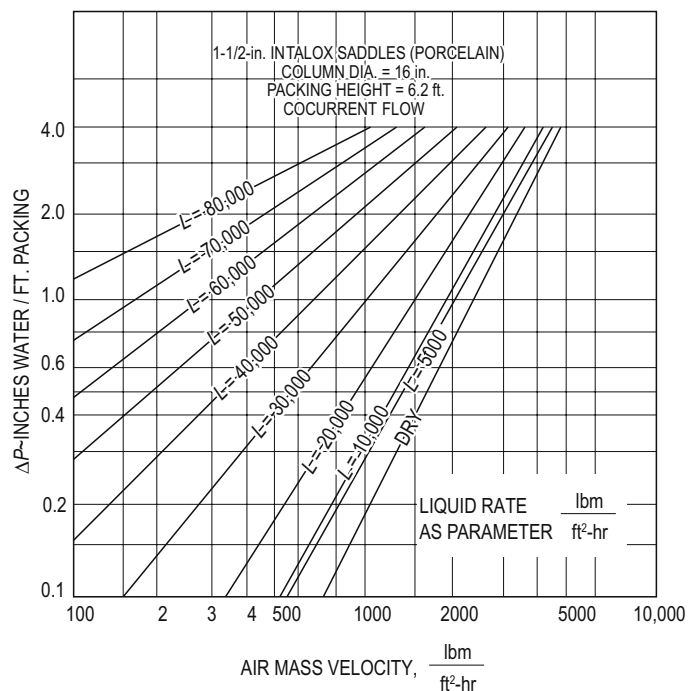


Source: Eckert, J.S., E.H. Foote, R.R. Nemunaitis, and L.H. Rollison, Akron, OH: Norton Chemical Process Products Division, 1972 (revised 2001).

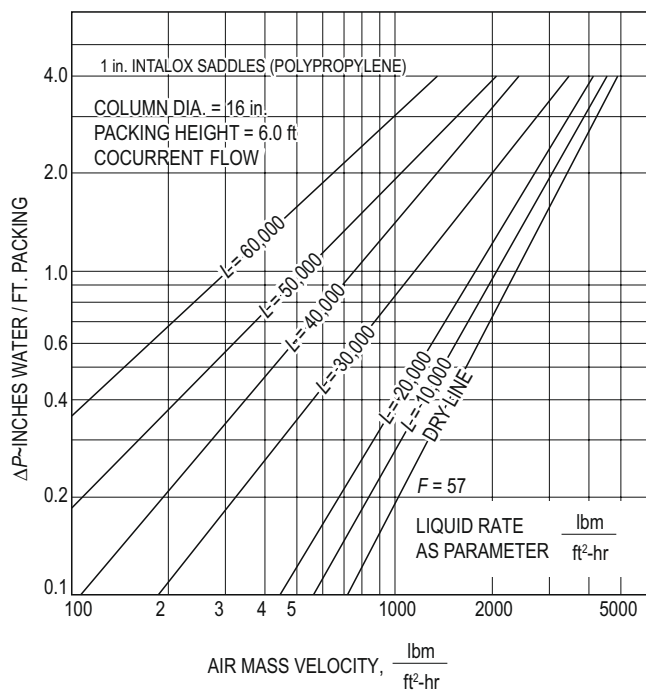


Source: Eckert, J.S., E.H. Foote, R.R. Nemunaitis, and L.H. Rollison, Akron, OH: Norton Chemical Process Products Division, 1972 (revised 2001).

Pressure Drop Versus Gas Rate (cont'd)

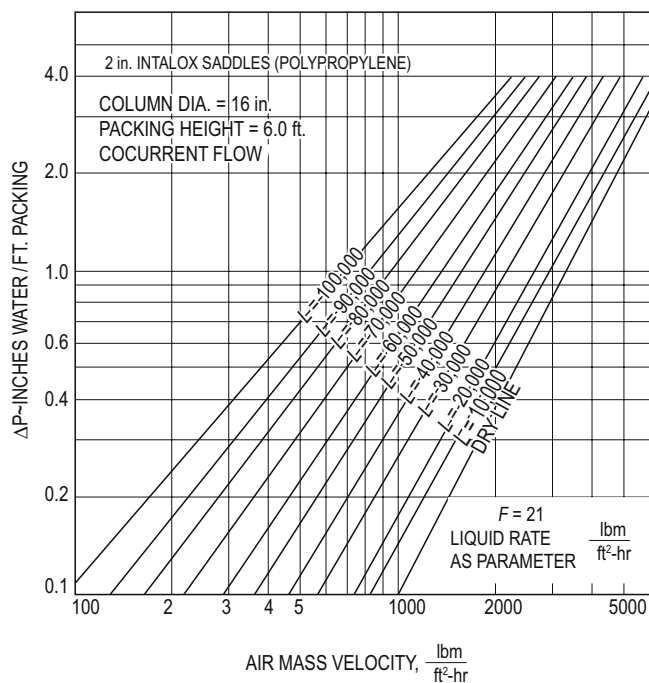


Source: Eckert, Foote, Nemunaitis, and Rollison, Akron, OH: Norton Chemical Process Products Division, 1972 (revised 2001).

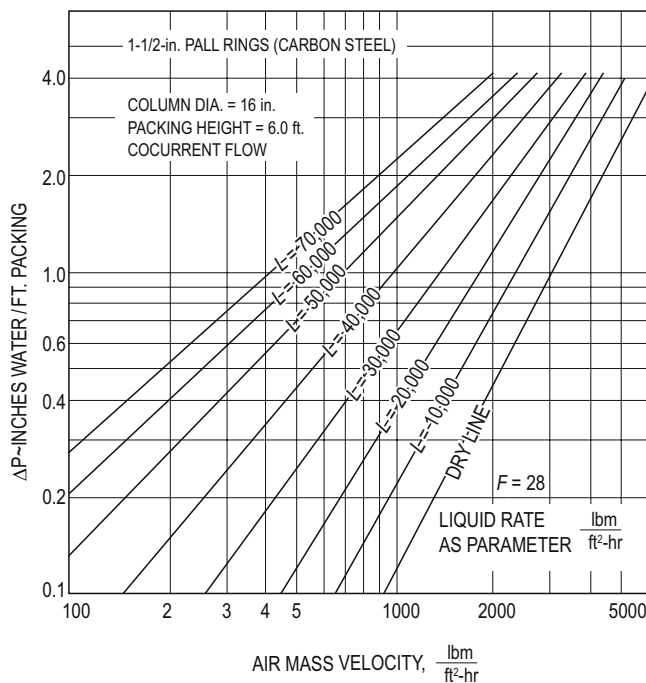


Source: Eckert, Foote, Nemunaitis, and Rollison, Akron, OH: Norton Chemical Process Products Division, 1972 (revised 2001).

Pressure Drop Versus Gas Rate (cont'd)

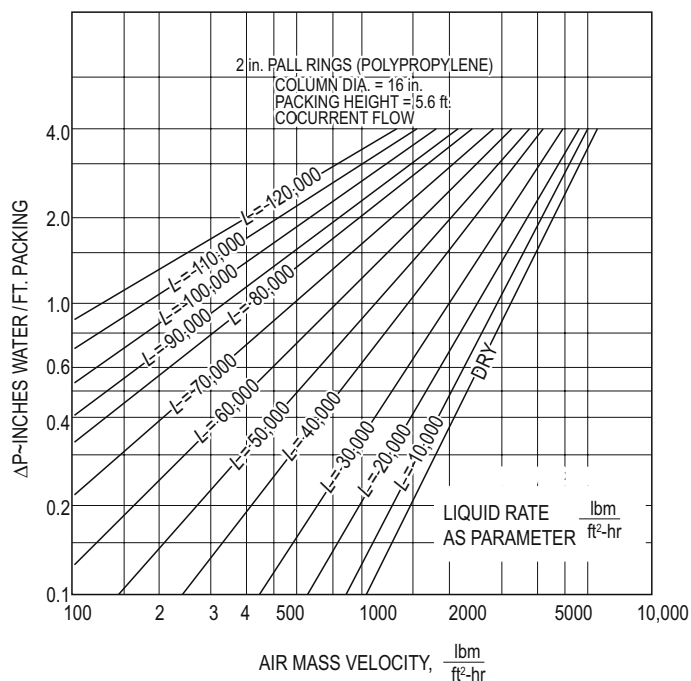


Source: Eckert, Foote, Nemunaitis, and Rollison, Akron, OH: Norton Chemical Process Products Division, 1972 (revised 2001).



Source: Eckert, J.S., E.H. Foote, R.R. Nemunaitis, and L.H. Rollison, Akron, OH: Norton Chemical Process Products Division, 1972 (revised 2001).

Pressure Drop Versus Gas Rate (cont'd)



Source: Eckert, J.S., E.H. Foote, R.R. Nemunaitis, and L.H. Rollison, Akron, OH: Norton Chemical Process Products Division, 1972 (revised 2001).

7.5 Liquid-Liquid Extraction

7.5.1 Fundamentals of Liquid-Liquid Extraction

7.5.1.1 Partition Ratio

The *equilibrium partition ratio* in mole fraction units is

$$K_i^o = \frac{y_i}{x_i} = \frac{\gamma_i^{\text{raffinate}}}{\gamma_i^{\text{extract}}}$$

where

y_i = mole fraction of solute i in the extract phase

x_i = mole fraction of solute i in the raffinate phase

γ_i = activity coefficient of solute i in the indicated phase

The equilibrium partition ratio in mass ratio units K_i' is

$$K_i' = \frac{Y_i'}{X_i'} = \frac{\left(\frac{m_{\text{solute}}^{\text{extract}}}{m_{\text{extraction solvent}}} \right)}{\left(\frac{m_{\text{solute}}^{\text{raffinate}}}{m_{\text{raffinate solvent}}} \right)}$$

where

Y_i' = ratio of mass solute i to mass extract solvent in extract phase

X_i' = ratio of mass solute i to mass raffinate (feed) solvent in raffinate phase

m = mass flow rate, in $\frac{\text{lbm}}{\text{hr}}$ or $\frac{\text{kg}}{\text{s}}$

The advantage of using the solute-free basis is that the feed solvent and extraction solvent flows do not change during the extraction.

7.5.1.2 Extraction Factor

On a McCabe-Thiele type of diagram, $\mathcal{E}$ is the slope of the equilibrium line divided by the slope of the operating line $\frac{F}{S}$.

$$\mathcal{E}_i = m_i \frac{S}{F}$$

where

$\mathcal{E}_i$ = extraction factor

m_i = local slope of the equilibrium line

S = mass flow rate of the solvent phase, in $\frac{\text{lbm}}{\text{hr}}$ or $\frac{\text{kg}}{\text{s}}$

F = mass flow rate of the feed phase, in $\frac{\text{lbm}}{\text{hr}}$ or $\frac{\text{kg}}{\text{s}}$

For dilute systems with straight equilibrium lines, the slope of the equilibrium line is equal to the partition ratio:

$$m_i = K_i'$$

7.5.1.3 Separation Factor

The separation factor indicates the relative enrichment of a given component in the extract phase after one theoretical stage of extraction.

$$\alpha_{ij}' = \frac{\left(\frac{Y_i'}{Y_j'}\right)_{\text{extract}}}{\left(\frac{X_i'}{X_j'}\right)_{\text{raffinate}}} = \frac{\left(\frac{Y_i'}{X_i'}\right)}{\left(\frac{Y_j'}{X_j'}\right)} = \frac{K_i'}{K_j'}$$

where α_{ij}' = separation factor for solute i with respect to solute j (mass ratio basis)

7.5.1.4 Interfacial Mass Transfer

$$\dot{n} = k_y(y_{\text{int}} - y) \quad \dot{n} = k_{oy}(y^* - y)$$

$$\dot{n} = k_x(x_{\text{int}} - x) \quad \dot{n} = k_{ox}(x - x^*)$$

where

$\dot{n}$ = molar flow per area

x_{int} = mole fraction of solute i in the raffinate phase at the interface

x^* = mole fraction of solute i in the raffinate phase in equilibrium with the extract phase

y_{int} = mole fraction solute i in the extract phase at the interface

y^* = mole fraction of solute i in the extract phase in equilibrium with the raffinate phase

$$NTU_G = \int_{y_s}^{y_e} \frac{(1-y)_{\text{lm}} dy}{(1-y)(y_{\text{int}} - y)}$$

For dilute solutions:

$$NTU_{OL} = \frac{x_f - x_r}{(x - x^*)_{lm}}$$

where

x_f = mole fraction of solute i in the feed

x_r = mole fraction of solute i in the raffinate

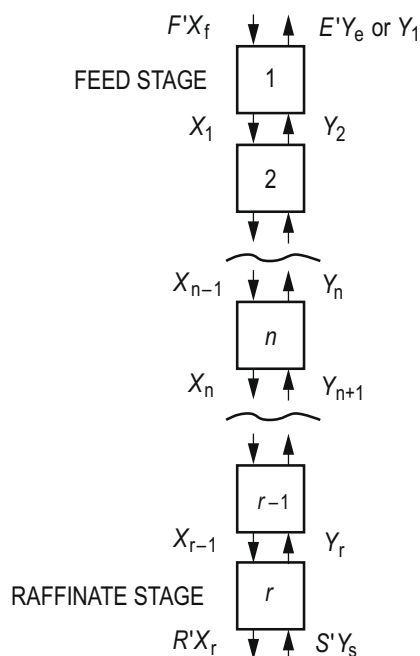
NTU_G = number of transfer units based on gas phase

NTU_{OL} = number of transfer units based on liquid phase

$(\quad)_{lm}$ = log mean

7.5.2 Theoretical (Equilibrium) Stage Calculations

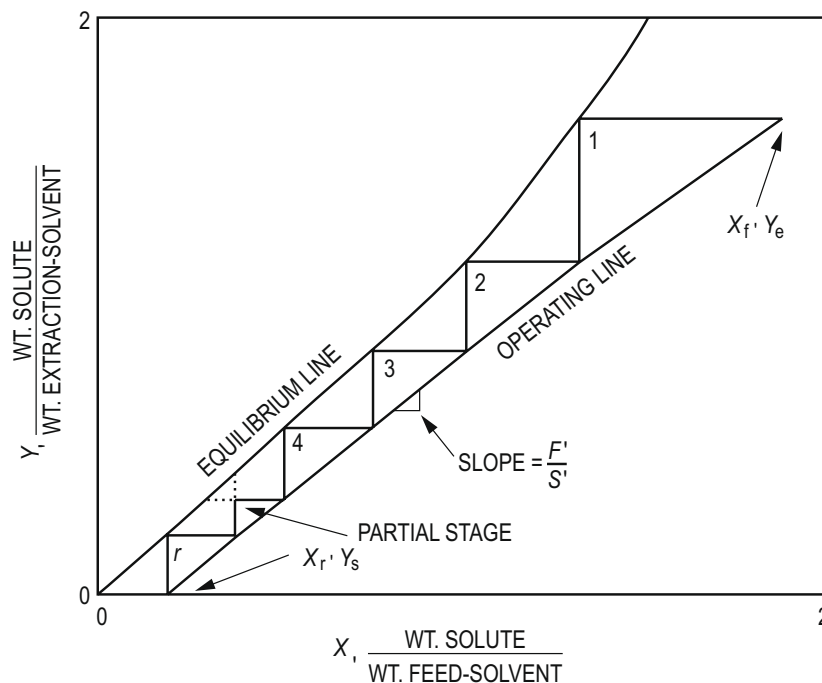
Countercurrent Extraction Cascade



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7.5.2.1 McCabe-Thiele Method

McCabe-Thiele Graphical Stage Calculation Using Bancroft Coordinates



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For immiscible feed and extraction solvents, the operating line for the feed end (stage 1 to stage n) is

$$Y'_{n+1} = \frac{F'}{S'} X'_n + \frac{E' Y'_e - F' X'_f}{S'}$$

where

X'_f = mass ratio of solute in feed

Y'_e = mass ratio of solute in extract

E' = mass flow rate of extraction solvent only

F' = mass flow rate of feed solvent only

S' = mass flow rate of extraction solvent only

For immiscible feed and extraction solvents, the operating line for the raffinate end (stage n to stage r) is

$$Y'_n = \frac{F'}{S'} X'_{n-1} + \frac{S' Y'_s - R' X'_r}{S'}$$

where

X'_r = mass ratio of solute in raffinate

Y'_s = mass ratio of solute in solvent

R' = mass flow rate of raffinate solvent only

The overall material balance is

$$Y_e' = \frac{F' X_f' + S' Y_s' - R' X_r'}{E'}$$

7.5.2.2 Kremser-Souders-Brown (KSB) Theoretical Stage Equation

For straight equilibrium and operating lines, the number of theoretical stages N is approximated by:

$$N = \frac{\ln \left[\left(\frac{X_f' - Y_s'/m'}{X_r' - Y_s'/m'} \right) \left(1 - \frac{1}{\mathcal{E}} \right) + \frac{1}{\mathcal{E}} \right]}{\ln \mathcal{E}} \quad \text{for } \mathcal{E} = m' \frac{S'}{F'}, \mathcal{E} \neq 1$$

where

N = number of theoretical stages

m' = local slope of equilibrium line in mass ratio units

S' = mass flow rate of the solvent only (solute-free basis), in $\frac{\text{lbm}}{\text{hr}}$ or $\frac{\text{kg}}{\text{s}}$

F' = mass flow rate of the feed solvent (solute-free basis), in $\frac{\text{lbm}}{\text{hr}}$ or $\frac{\text{kg}}{\text{s}}$

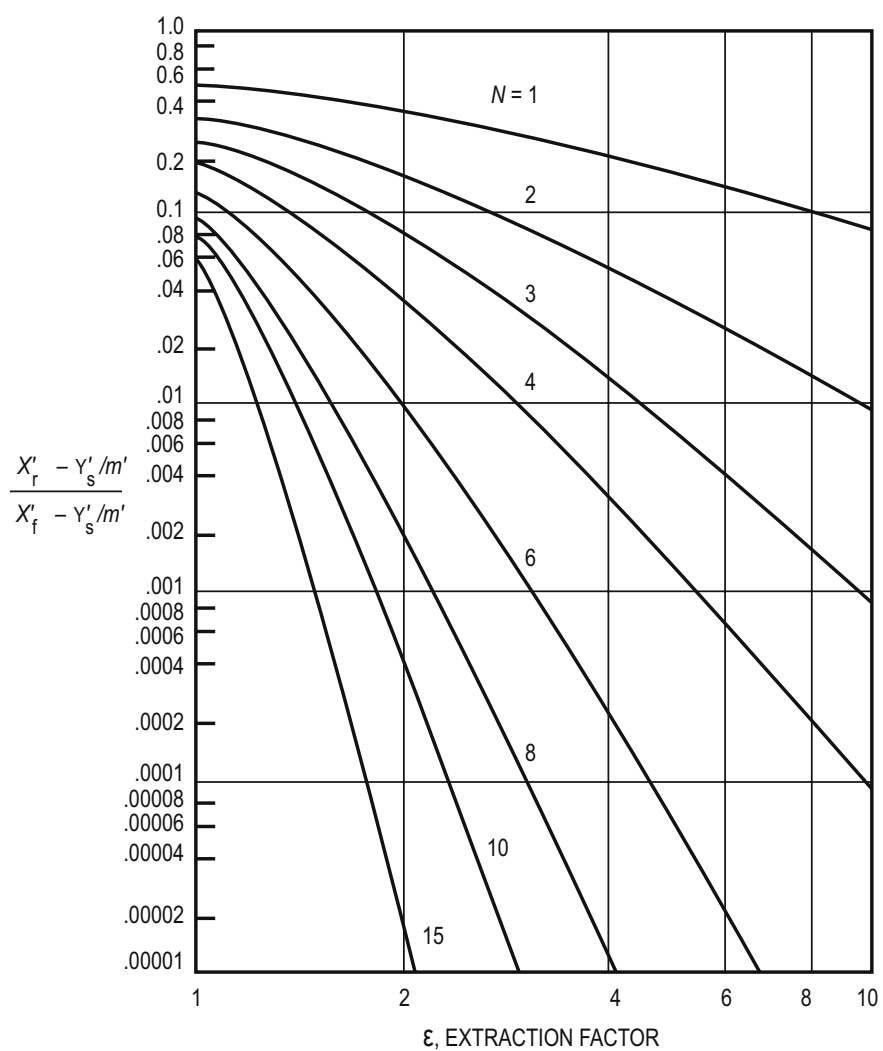
An alternate form is

$$\frac{X_f' - Y_s'/m'}{X_r' - Y_s'/m'} = \frac{\mathcal{E}^N - 1/\mathcal{E}}{1 - 1/\mathcal{E}} \quad \text{for } \mathcal{E} \neq 1$$

$$\frac{X_f' - Y_s'/m'}{X_r' - Y_s'/m'} = N + 1 \quad \text{for } \mathcal{E} = 1$$

Graphical solutions to the *KSB equation* are shown below. Note that the term for the abscissa is the inverse of the term used in the KSB equation.

Graphical Solutions to the KSB Equation



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In general, these equations are valid for any concentration range in which equilibrium can be represented by a linear relationship $Y = mX + b$ (written here in general form for any system of units). For applications that involve dilute feeds, the section of the equilibrium line of interest is a straight line that extends through the origin where $Y_i = 0$ at $X_i = 0$. In this case, $b = 0$, and the slope of the equilibrium line is equal to the partition ratio where $m = K$.

The KSB equation also may be used to represent a linear segment of the equilibrium curve at higher solute concentrations. In this case, the linear segment is represented by a straight line that does not extend through the origin, and m is the local slope of the equilibrium line, so $b \neq 0$ and $m \neq K$.

Furthermore, a series of KSB equations may be used to model a highly curved equilibrium line by dividing the analysis into linear segments and matching concentrations where the segments meet. For equilibrium lines with moderate curvature, an approximate average slope of the equilibrium line may be obtained from the geometric mean of the slopes at low and high solute concentrations:

$$m_{\text{average}} \approx m_{\text{geometric mean}} = \sqrt{m_{\text{low}} m_{\text{high}}}$$

7.5.2.3 Stage Efficiency

$$\xi_o(\%) = \frac{\text{theoretical stages}}{\text{actual stages}} \times 100$$

$$\xi_{\text{md}} = \frac{c_{\text{d},n+1} - c_{\text{d},n}}{c_{\text{d},n+1} - c_{\text{d}}^*}$$

$$\xi_o(\%) = \frac{\ln[1 + \xi_{\text{md}}(\mathcal{E} - 1)]}{\ln \mathcal{E}} \times 100$$

where

ξ_o = overall stage efficiency

ξ_{md} = Murphree stage efficiency based on the dispersed phase

7.5.3 Rate-Based Calculations With Mass-Transfer Units

In most cases, the dominant mass-transfer resistance resides in the feed (raffinate) phase because the slope of the equilibrium line usually is greater than one. In that case, the overall mass-transfer coefficient based on the raffinate phase may be written:

$$\frac{1}{k_{\text{or}}} = \frac{1}{k_{\text{r}}} + \frac{1}{m_{\text{er}}^{\text{vol}} k_{\text{e}}}$$

where

k_{e} = extract-phase mass-transfer coefficient, in $\frac{\text{ft}}{\text{hr}}$ or $\frac{\text{m}}{\text{s}}$

k_{r} = raffinate-phase mass-transfer coefficient, in $\frac{\text{ft}}{\text{hr}}$ or $\frac{\text{m}}{\text{s}}$

k_{or} = overall mass-transfer coefficient based on the raffinate phase, in $\frac{\text{ft}}{\text{hr}}$ or $\frac{\text{m}}{\text{s}}$

$m_{\text{er}}^{\text{vol}}$ = local slope of equilibrium line (volumetric concentration basis)

The required contacting height of an extraction column is related to the height of a transfer unit and the number of transfer units by:

$$Z_t = \frac{V_r}{k_{or} a} \int_{x_{out}}^{x_{in}} \frac{dX}{X - X^{eq}} = HTU_{or} NTU_{or}$$

where

Z_t = total height of extractor

V_r = liquid velocity of raffinate phase, in $\frac{\text{ft}}{\text{sec}}$ or $\frac{\text{m}}{\text{s}}$

a = interfacial area per unit volume, in $\frac{\text{ft}^2}{\text{ft}^3}$ or $\frac{\text{m}^2}{\text{m}^3}$

X^{eq} = mass ratio in equilibrium with composition of extract phase

HTU_{or} = height of overall transfer units (based on raffinate phase)

NTU_{or} = number of transfer units (based on raffinate phase)

For straight equilibrium and operating lines, the number of transfer units is approximated by the *Colburn equation*:

$$NTU_{or} = \frac{\ln \left[\left(\frac{X_f' - \frac{Y_s'}{m'}}{X_r' - \frac{Y_s'}{m'}} \right) \left(1 - \frac{1}{\mathcal{E}} \right) + \frac{1}{\mathcal{E}} \right]}{1 - \frac{1}{\mathcal{E}}}$$

where

$$\mathcal{E} = m' \frac{S'}{F'}, \quad \mathcal{E} \neq 1$$

An alternate form is

$$\frac{X_f' - \frac{Y_s'}{m'}}{X_r' - \frac{Y_s'}{m'}} = \frac{\exp \left[NTU_{or} \left(1 - \frac{1}{\mathcal{E}} \right) \right] - \frac{1}{\mathcal{E}}}{1 - \frac{1}{\mathcal{E}}}$$

The height of a transfer unit is

$$HTU_{or} = HTU_r + \frac{HTU_e}{\mathcal{E}}$$

$$HTU_r = \frac{Q_r}{A_{col} k_r a}$$

$$HTU_e = \frac{Q_e}{A_{col} k_e a}$$

where

HTU_r = height of a transfer unit due to resistance in the raffinate phase, in ft or m

HTU_e = height of a transfer unit due to resistance in the extract phase, in ft or m

A_{col} = column cross-sectional area, in ft^2 or m^2

Q_r = volumetric flow rate of the raffinate phase, in $\frac{\text{ft}^3}{\text{min}}$ or $\frac{\text{m}^3}{\text{s}}$

Q_e = volumetric flow rate of the extract phase, in $\frac{\text{ft}^3}{\text{min}}$ or $\frac{\text{m}^3}{\text{s}}$

The relation between overall raffinate-phase transfer units from the Colburn equation and the number of theoretical stages from the KSB equation is

$$NTU_{\text{or}} = N \times \frac{\ln \mathcal{E}}{1 - \frac{1}{\mathcal{E}}} \quad \text{for } \mathcal{E} \neq 1$$

$$NTU_{\text{or}} = N = \frac{X'_f - \frac{Y'_s}{m'}}{X'_r - \frac{Y'_s}{m'}} - 1 \quad \text{for } \mathcal{E} = 1$$

7.5.3.1 Solute Reduction Factor, F_R

The solute reduction factor F_R , is an indication of process performance.

For a single-stage batch process or for one theoretical stage of a continuous process, the solute reduction factor is

$$F_R = \frac{X_{\text{in}}}{X_{\text{out}}} = \frac{\left(\mathcal{E} - \frac{1}{\mathcal{E}}\right)}{\left(1 - \frac{1}{\mathcal{E}}\right)} \quad \text{for } N = 1$$

The required solvent-to-feed ratio is approximated by

$$\frac{S}{F} = \frac{F_R - 1}{K} \quad \text{for } N = 1$$

where

K = distribution coefficient for phase equilibrium

S = mass flow rate of the solvent phase (solute-free basis)

F = mass flow rate of the feed phase (solute-free basis)

X_{in} = ratio of mass solute i to mass raffinate solvent in the raffinate phase at the inlet of the raffinate

X_{out} = ratio of mass solute i to mass raffinate solvent in the raffinate phase at the outlet of the raffinate

Y_{out} = ratio of mass solute i to mass extract solvent in the extract phase at the outlet of the solvent

Y_{in} = ratio of mass solute i to mass extract solvent in the extract phase at the inlet of the solvent

For any extraction configuration, the concentration of solute in the extract is

$$Y_{\text{out}} = \frac{X_{\text{in}}}{\left(\frac{S}{F}\right)} \left(1 - \frac{1}{F_R}\right) \quad \text{for } Y_{\text{in}} = 0$$

For cross-flow extraction, in which the raffinate phase for each stage is contacted with fresh solvent, the solute reduction factor is

$$F_R = \left(1 + \frac{\mathcal{E}}{N}\right)^{\xi_0 N}$$

$$\frac{S}{F} = \frac{N}{K} \left(F_R^{\xi_0 N} - 1\right)$$

For multistage countercurrent extraction, the solute reduction factor is

$$F_R = \frac{\left(\mathcal{E}^{\xi_0 N} - \frac{1}{\mathcal{E}}\right)}{\left(1 - \frac{1}{\mathcal{E}}\right)}$$

For countercurrent extraction without discrete stages, the solute reduction factor is

$$F_R = \frac{\exp\left[NTU_{\text{or}}\left(1 - \frac{1}{\mathcal{E}}\right)\right] - \frac{1}{\mathcal{E}}}{1 - \frac{1}{\mathcal{E}}}$$

7.5.4 Liquid-Liquid Extraction Equipment

7.5.4.1 Spray Columns

Liquid Dispersion

For liquid distributors, the liquid should issue from the hole as a jet that breaks up into drops. As a general guideline, the maximum recommended design velocity corresponds to a *Weber number* (We) of about 12. The minimum Weber number that ensures jetting in all the holes is about 2. It is common practice to specify a Weber number between 8 and 12 for a new design.

$$u_{o,\text{max}} \approx \left(\frac{We \gamma}{d_o \rho_d}\right)^{0.5}$$

where

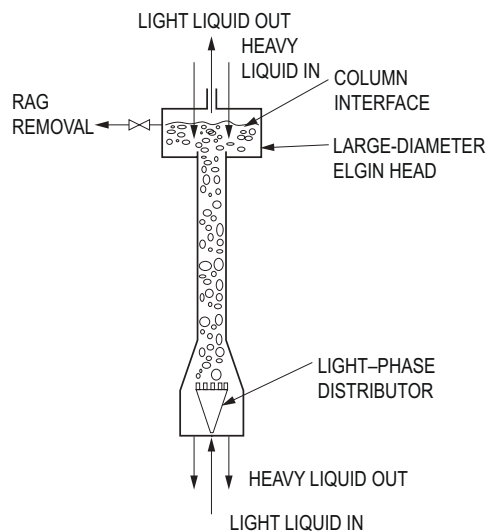
$u_{o,\text{max}}$ = maximum velocity through an orifice or nozzle

We = Weber number

γ = surface tension

d_o = orifice or nozzle diameter

ρ_d = density of the dispersed phase



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Drop Size, Dispersed-Phase Holdup, and Interfacial Area

For the general case where the dispersed phase travels through the column as drops, an average liquid-liquid interfacial area can be calculated from the Sauter mean drop diameter and dispersed-phase holdup.

The drop diameter is

$$d_p = 1.15 \eta \sqrt{\frac{\gamma}{\Delta \rho g}}$$

where

d_p = Sauter mean drop diameter

$\Delta \rho$ = density difference between the raffinate and the extract

η = parameter, specifically:

$\eta = 1.0$ for no mass transfer

$\eta = 1.0$ for transfer from continuous to dispersed phase

$\eta = 1.4$ for transfer from dispersed to continuous phase

The dispersed-phase holdup is

$$\phi_d = \frac{u_d \left[\cos \left(\frac{\pi \zeta}{4} \right) \right]^{-2}}{\varepsilon \left[u_{so} \exp \left(\frac{-6 \phi_d}{\pi} \right) - \frac{u_c}{\varepsilon (1 - \phi_d)} \right]}, \quad \zeta = \frac{a_p d_p}{2}$$

where

ϕ_d = volume fraction of the dispersed phase (holdup)

ζ = tortuosity factor

u_d = liquid velocity of the dispersed phase

u_c = liquid velocity of the continuous phase

u_{so} = slip velocity at low dispersed-phase flow rate

ε = void fraction

a_p = interfacial area

The interfacial area is

$$a_p = \frac{6 \varepsilon \phi_d}{d_p}$$

Drop Velocity

The average velocity of a dispersed drop u_{drop} is

$$u_{\text{drop}} = \frac{u_d}{\varepsilon \phi_d}$$

Interstitial Velocity of Continuous Phase

The interstitial velocity of the continuous phase u_{ic} is

$$u_{ic} = \frac{u_c}{\varepsilon (1 - \phi_d)}$$

Slip Velocity and Characteristic Slip Velocity

The relative velocity between the counterflowing phases is referred to as the slip velocity u_s :

$$u_s = u_{\text{drop}} + u_{\text{ic}}$$

The characteristic slip velocity u_{so} obtained at low dispersed-phase flow rate is

$$Re_{\text{Stokes}} = \frac{\rho_c \Delta \rho g d_p^3}{18 \mu_c^2}$$

where

Re = Reynolds Number

ρ_c = density of the continuous phase

$\Delta \rho$ = density difference between the two phases

μ_c = viscosity of the continuous phase

For $Re_{\text{Stokes}} < 2$:

$$u_{\text{so}} = \frac{\Delta \rho g d_p^2}{18 \mu_c}$$

For $Re_{\text{Stokes}} > 2$:

$$u_{\text{so}} = \frac{Re \mu_c}{d_p \rho_c}$$

where

$$\frac{Re}{P^{0.149}} = 0.94H^{0.757} - 0.857 \quad H \leq 59.3$$

$$\frac{Re}{P^{0.149}} = 3.42H^{0.441} - 0.857 \quad H > 59.3$$

$$P = \frac{\rho_c^2 \gamma^3}{\mu_c^4 g \Delta \rho}$$

$$H = \left(\frac{4d_p^2 g \Delta \rho}{3\gamma} \right) \left(\frac{\mu_w}{\mu_c} \right)^{0.14} P^{0.149}$$

P, H = dimensionless groups

μ_w = reference viscosity equal to 0.9 cP or $9 \times 10^{-4} \text{ Pa}\cdot\text{s}$

The slip velocity at higher holdup is estimated from:

$$u_s \approx u_{\text{so}} (1 - \phi_d)$$

Flooding Velocity

It is generally recommended that flow velocities be limited to 50 percent of the calculated flooding velocities.

$$u_{cf} = \frac{0.178u_{so}}{1 + 0.925\left(\frac{u_{df}}{u_{cf}}\right)}$$

where

u_{cf} = continuous-phase flooding velocity

u_{df} = dispersed-phase flooding velocity

Drop Coalescence Rate

Problems with coalescence are most likely when the superficial dispersed-phase flooding velocity u_{df} is greater than about 12 percent of the characteristic slip velocity.

Mass-Transfer Coefficients and Efficiency

$$k_{oc} a = m_{dc}^{vol} k_{od} a = 0.08 \times \frac{\phi_d (1 - \phi_d) \left(\frac{g^3 \Delta \rho^3}{\gamma \rho_c^2} \right)^{1/4}}{\left(\frac{\mu_c}{\rho_c D_c} \right)^{1/2} + \left(\frac{1}{m_{dc}} \right) \left(\frac{\mu_d}{\rho_d D_d} \right)^{1/2}}$$

where

D_c = solute diffusion coefficient for the continuous phase

D_d = solute diffusion coefficient for the dispersed phase

k_{oc} = overall mass-transfer coefficient based on the continuous phase

k_{od} = overall mass-transfer coefficient based on the dispersed phase

m_{dc} = local slope of equilibrium line for dispersed-phase concentration plotted versus continuous-phase concentration

m_{dc}^{vol} = local slope of equilibrium line for dispersed-phase concentration plotted versus continuous-phase concentration on volumetric concentration basis

γ = interfacial tension

μ_c = viscosity of continuous phase

μ_d = viscosity of dispersed phase

ρ_c = density of continuous phase

ρ_d = density of dispersed phase

ϕ_d = volume fraction of dispersed phase (holdup)

With the height of one transfer unit (based on the continuous phase):

$$HTU_{oc} = \frac{u_c}{k_{oc} a}$$

7.5.4.2 Packed Columns

Liquid Redistribution

Little benefit is gained from a packed height greater than 10 ft (3 m). Redistributing the dispersed phase about every 5 to 10 ft (1.5 to 3 m) is recommended to generate new droplets and constrain backmixing.

Source: Green, Don W., and Robert H. Perry, *Perry's Chemical Engineers' Handbook*, 8th ed., New York: McGraw-Hill, 2008, p. 15-64.

Minimum Packing Size

For a given application, a minimum packing size or dimension exists below which random packing is too small for good extraction performance. The critical packing dimension d_c is

$$d_c = 2.4 \sqrt{\frac{\gamma}{\Delta \rho g}}$$

where

γ = interfacial tension

Packing Holdup

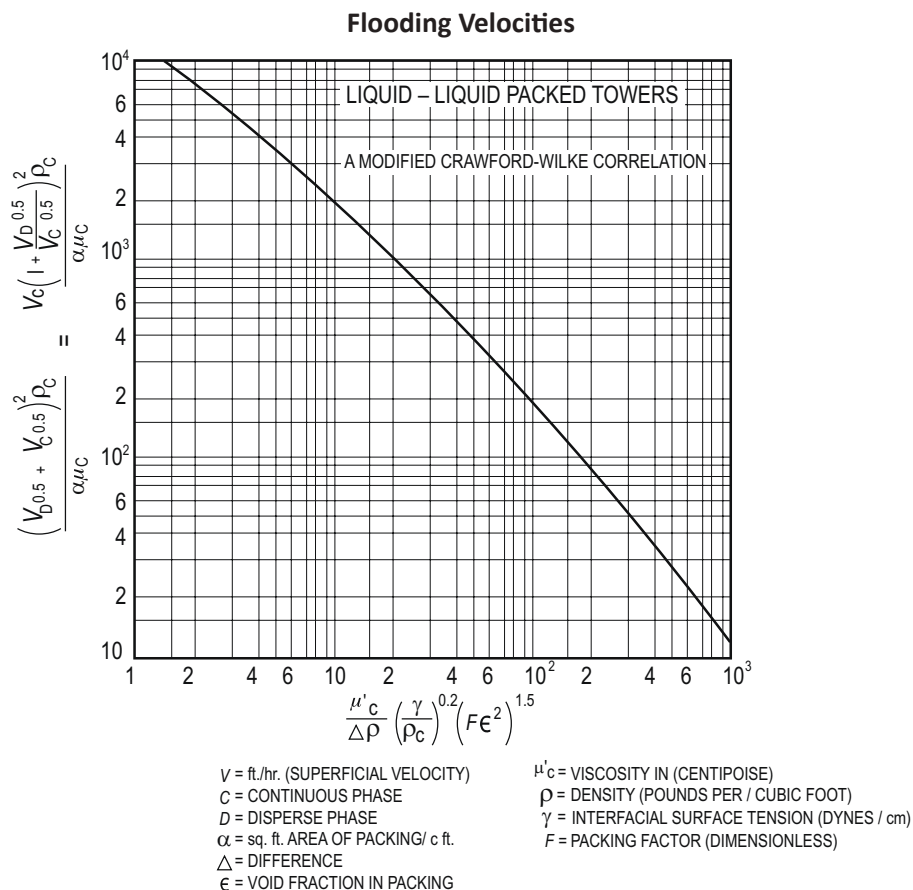
For standard commercial packings of 0.5 in (1.27 cm) and larger, ϕ_d varies linearly with the liquid velocity of the dispersed phase (u_d) up to values of $\phi_d = 0.10$ (for low values of u_d). As u_d increases further, ϕ_d increases sharply up to a "lower transition point" resembling loading in gas-liquid contact. At still higher values of u_d , an upper transition point occurs, the drops of dispersed phase tend to coalesce, and u_d can increase without a corresponding increase in ϕ_d . This regime ends in flooding. Below the upper transition point, the dispersed-phase holdup is

$$\frac{u_d}{\phi_d} + \frac{u_c}{1 - \phi_d} = \varepsilon u_{so} (1 - \phi_d)$$

Packing Flooding: Siebert, Reeves, and Fair Correlation

$$u_{cf} = \frac{0.178 \varepsilon u_{so}}{1 + 0.925 \left(\frac{u_{df}}{u_{cf}} \right) \left\{ \frac{1}{\left[\cos \left(\frac{\pi \zeta}{4} \right) \right]^2} \right\}} \quad \zeta = \frac{a_p d_p}{2}$$

Packing Flooding: Modified Crawford-Wilke Correlation



Pressure Drop

In general, the pressure drop through a packed extractor is due to the hydrostatic head pressure. The resistance to flow caused by the packing itself normally is negligible; typical packings are large and flooding velocities are much lower than those needed to develop significant ΔP from resistance to flow between the packing elements.

Mass-Transfer Coefficients

$$\Phi = \frac{\left(\frac{\mu_d}{\rho_d D_d}\right)^{\frac{1}{2}}}{\left(1 + \frac{\mu_d}{\mu_c}\right)}$$

For $\Phi < 6$:

$$k_d = \frac{0.00375 u_s}{\left(1 + \frac{\mu_d}{\mu_c}\right)}$$

For $\Phi > 6$:

$$k_d = 0.023 u_s \left(\frac{\mu_d}{\rho_d D_d} \right)^{-\frac{1}{2}}$$

$$\frac{k_c d_p}{D_c} = 0.698 \left(\frac{\mu_c}{\rho_c D_c} \right)^{\frac{2}{5}} \left(\frac{d_p u_s \rho_c}{\mu_c} \right)^{\frac{1}{2}} (1 - \phi_d)$$

$$\frac{1}{k_{od}} = \frac{1}{k_d} + \frac{m_{dc}^{vol}}{k_c}$$

where

k_c = continuous-phase mass-transfer coefficient

k_d = dispersed-phase mass-transfer coefficient

Packing Data

Random and Structured Packings Used in Packed Extractors

Packing	Surface Area a_p ¹ $\frac{m^2}{m^3}$	Void Fraction ¹ (ϵ)
Metal Random Packing		
Koch-Glitsch IMTP® 25	224	0.964
Koch-Glitsch IMTP® 40	151	0.980
Koch-Glitsch IMTP® 50	102	0.979
Koch-Glitsch IMTP® 60	84	0.983
Sulzer I-Ring #25	224	0.964
Sulzer I-Ring #40	151	0.980
Sulzer I-Ring #50	102	0.979
Nutter Ring® NR 0.7	226	0.977
Nutter Ring® NR 1	168	0.977
Nutter Ring® NR 1.5	124	0.976
Nutter Ring® NR 2	96	0.982
Nutter Ring® NR 2.5	83	0.984
HY-PAK® #1 in.	172	0.965
HY-PAK® #1-1/2 in.	118	0.976
HY-PAK® #2 in.	84	0.979
FLEXIRING® 1 in.	200	0.959
FLEXIRING® 1-1/2 in.	128	0.974
FLEXIRING® 2 in.	97	0.975
CMR® 1	246	0.973
CMR® 2	157	0.970
CMR® 3	102	0.980
BETARING® #1	186	0.963
BETARING® #2	136	0.973
FLEXIMAX® 200	189	0.973
FLEXIMAX® 300	148	0.979
FLEXIMAX® 400	92	0.983

Random and Structured Packings Used in Packed Extractors (cont'd)

Packing	Surface Area a_p ¹ $\frac{m^2}{m^3}$	Void Fraction ¹ (ϵ)
Plastic Random Packing		
Super INTALOX® Saddles #1	204	0.896
Super INTALOX® Saddles #2	105	0.934
BETARING® #1	167	0.942
BETARING® #2	114	0.940
SNOWFLAKE®	93	0.949
FLEXIRING® 1 in.	205	0.922
FLEXIRING® 1-1/2 in.	119	0.925
FLEXIRING® 2 in.	99	0.932
Ceramic Random Packing		
INTALOX® Saddles 1 in.	256	0.730
INTALOX® Saddles 1-1/2 in.	195	0.750
INTALOX® Saddles 2 in.	118	0.760
Ceramic Structured Packing		
FLEXERAMIC® 28	282	0.720
FLEXERAMIC® 48	157	0.770
FLEXERAMIC® 88	102	0.850
Metal Structured Packing²		
Koch-Glitsch SMV-8	417	0.978
Koch-Glitsch SMV-10	292	0.985
Koch-Glitsch SMV-16	223	0.989
Koch-Glitsch SMV-32	112	0.989
Sulzer SMV 2Y	205	0.990
Sulzer SMV 250Y	256	0.988
Sulzer SMV 350Y	353	0.983
INTALOX® 2T	214	0.989
INTALOX® 3T	170	0.989
INTALOX® 4T	133	0.987
Plastic Structured Packing²		
Koch-Glitsch SMV-8	330	0.802
Koch-Glitsch SMV-16	209	0.875
Koch-Glitsch SMV-32	93	0.944
Sulzer SMV 250Y	256	0.875

1. Typical value for standard wall thickness. Values will vary depending upon thickness.
2. SMV structured packings also are available with horizontal dual-flow perforated plates installed between elements (typically designated SMVP packing). These plates generally reduce backmixing and improve mass-transfer performance at the expense of a reduction in the open cross-sectional area and somewhat reduced capacity.

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7.5.4.3 Sieve Tray Columns

Sieve Tray Perforated Area

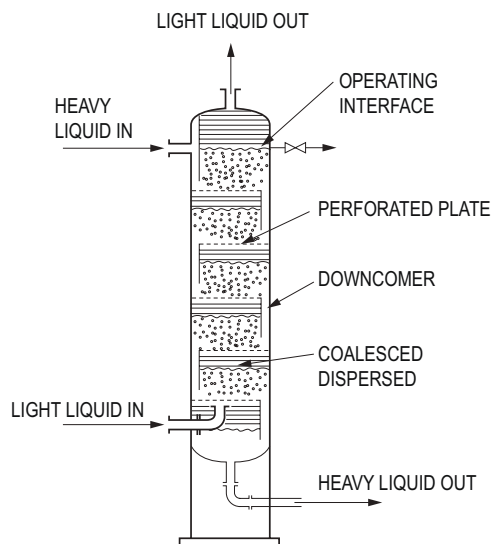
Perforations usually are in the range of 0.125 to 0.25 in (0.32 to 0.64 cm) in diameter, set 0.5 to 0.75 in (1.27 to 1.81 cm) apart, on square or triangular pitch. Hole size appears to have relatively little effect on the mass-transfer rate except that, in systems of high interfacial tension, smaller holes produce somewhat better mass transfer. The entire hole area is normally set at 15 to 25 percent of the column cross-section, although adjustments may be needed. It is common practice to set the velocity of liquid exiting the holes to correspond to a Weber number between 8 and 12. This normally gives velocities in the range of 0.5 to 1.0 $\frac{\text{ft}}{\text{sec}}$ (15 to 30 $\frac{\text{cm}}{\text{s}}$).

The velocity of the continuous phase in the downcomer (or upcomer) u_{down} , which sets the downcomer cross-sectional area, should be set lower than the terminal velocity of some arbitrarily small droplet of dispersed phase, such as a diameter of 1/32 or 1/16 in (0.08 or 0.16 cm). Otherwise, recirculation of entrained dispersed phase around a tray will result in flooding. The terminal velocity u_t of these small drops can be calculated using Stokes' Law:

$$u_t = \frac{g d_p^2 \Delta \rho}{18 \mu_c}$$

Downcomer area typically is in the range of 5 to 20 percent of the total cross-sectional area, depending upon the ratio of continuous- to dispersed-phase volumetric flow rates.

For large columns, tray spacing between 18 and 24 in. (45 and 60 cm) is generally recommended.



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The height of the coalesced layer at each tray, h , is

$$h = \frac{\Delta P_o + \Delta P_{\text{down}} - \phi_d g \Delta \rho L}{(1 - \phi_d) g \Delta \rho}$$

where

ΔP_o = orifice pressure drop

ΔP_{down} = pressure drop for flow through a downcomer (or upcomer)

L = downcomer (or upcomer) length

The orifice pressure drop ΔP_o is

$$\Delta P_o = \frac{1}{2} \left(1 - \frac{0.71}{\log Re} \right)^{-2} \rho_d u_o^2 + 3.2 \left(\frac{d_o^2 g \Delta \rho}{\gamma} \right)^{0.2} \frac{\gamma}{d_o} \quad \text{for } Re = \frac{u_o d_o \rho_d}{\mu_d}$$

where d_o = diameter of orifice in ft

The pressure drop through the downcomer is

$$\Delta P_{\text{dow}} = \frac{4.5 u_{\text{dow}}^2 \rho_c}{2}$$

where u_{dow} = velocity in downcomer (or upcomer)

For large columns, the design should specify that the height of the coalesced layer is at least 1 in. (2.5 cm) to ensure that all holes are adequately covered.

For segmental downcomers, the area of the downcomer is

$$A = \frac{H}{6S} (3H^2 + 4S^2)$$

where

A = area of segmental downcomer (or upcomer)

H = height of segmental downcomer (or upcomer)

S = chord length of segmental downcomer (or upcomer)

Chord length S is

$$S = \left[8H \left(\frac{D_{\text{col}}}{2} - \frac{H}{2} \right) \right]^{\frac{1}{2}}$$

where D_{col} = column diameter

Sieve Tray Flooding Velocity

Velocity of the continuous phase at the flood point is

$$u_{\text{cf}} = \left[\frac{L - A}{B \left(\frac{u_{\text{df}}}{u_{\text{cf}}} \right)^2 + C} \right]^{0.5}$$

where

$$A = \frac{6\gamma}{d_o \Delta \rho g} \quad B = \frac{1.11 \rho_d}{g \Delta \rho f_{\text{ha}}^2} \quad C = \frac{2.7 \rho_c}{2g \Delta \rho f_{\text{da}}^2}$$

where

f_{ha} = fractional hole area

f_{da} = fractional downcomer area

The cross-flow velocity of the continuous phase $u_{\text{c flow}}$ is

$$u_{\text{c flow}} \approx \frac{L_{\text{fp}}}{z - h} u_c$$

where

L_{fp} = length of flow path

z = sieve tray spacing

Sieve Tray Efficiency

The sieve tray efficiency is approximated by

$$\xi_o = 0.21 \left(\frac{z^{0.5}}{\gamma a_o^{0.35}} \right) \left(\frac{u_d}{u_c} \right)^{0.42}$$

7.6 Adsorption

7.6.1 Adsorption Equilibrium

For a single adsorbate in a gas stream, the equilibrium capacity of the adsorbent may be related to the concentration of the adsorbate in the bulk stream by the *Freundlich equation*:

$$W = a p^{1/n}$$

where

$$W = \frac{\text{mass of adsorbate}}{\text{unit mass of adsorbent}}$$

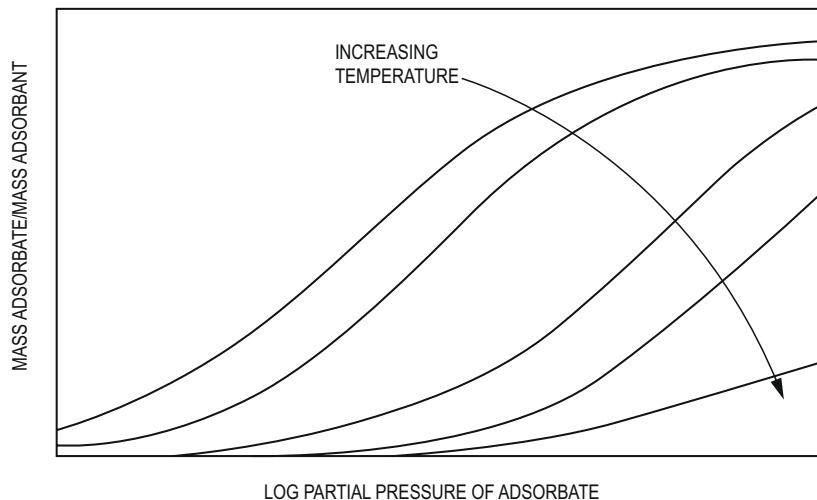
p = partial pressure of adsorbate in the bulk gas stream

a, n = empirical coefficients derived from log-log plot of data for W vs. p

Both coefficients are a function of temperature.

The Freundlich equation can be used for liquid-solid adsorption by entering concentration instead of partial pressure.

Typical Adsorption Isotherms



7.6.2 Adsorption Operation

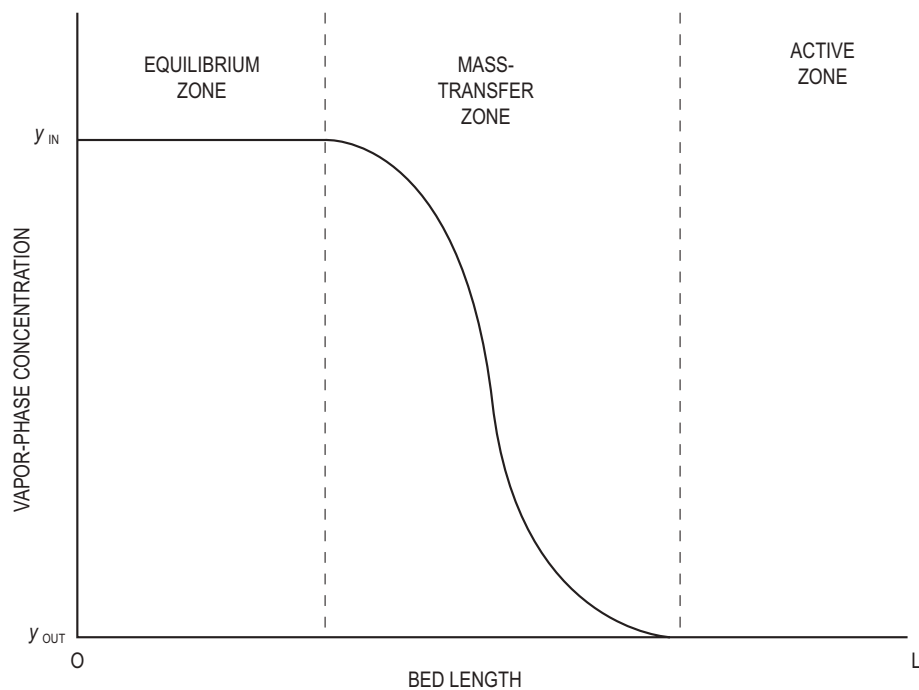
Adsorption in typical commercial operations is conducted by passing the gas or liquid stream through a usually vertical fixed bed of adsorbent particles. Adsorption beds are usually oriented vertically.

Adsorption beds have three zones that characterize the operation:

1. Equilibrium zone where adsorbate is in equilibrium with inlet concentration
2. Mass transfer zone where adsorbate is diffusing into adsorbent
3. Active zone where no adsorption has occurred

The length of the mass transfer zone (MTZ) is a function of the fluid velocity along with adsorbent porosity and uniformity of pore size.

Adsorption Concentration Profiles Across Bed

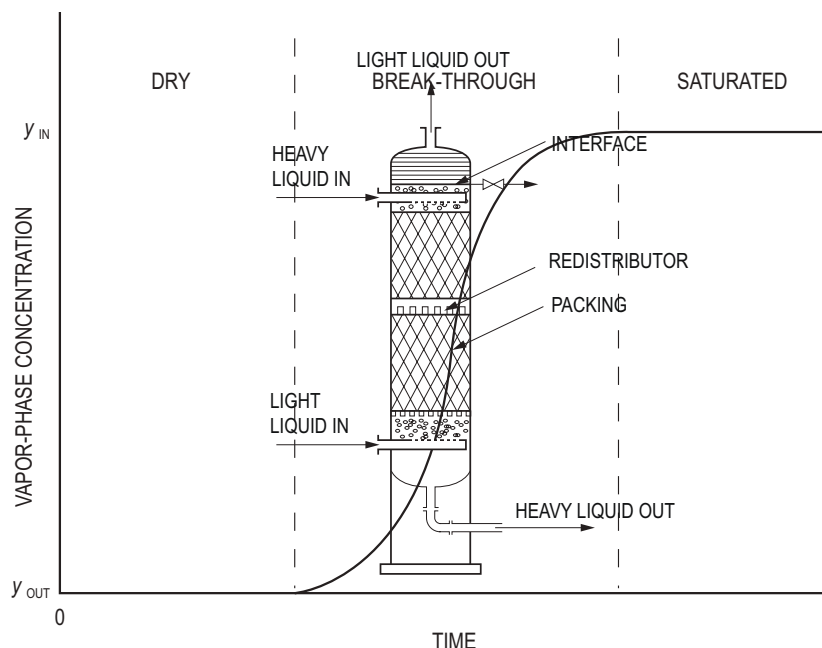


CONCENTRATION PROFILE AT A GIVEN TIME DURING ADSORPTION OPERATION

Three performance regimes for adsorption beds characterize the operation. Considering a given point in a bed:

1. Dry, when the mass transfer zone is below the point in the bed and the concentration has a low value
2. Break-through, when the mass transfer zone reaches the point in the bed and the concentration increases
3. Saturated, when the concentration at the point in the bed increases to the value of the inlet concentration

Adsorption Outlet Composition Versus Time



CONCENTRATION PROFILE AS A FUNCTION OF TIME AT A GIVEN POINT IN THE BED. ADSORPTION STEP.

7.6.3 Adsorption Regeneration

Adsorption processes can be nonregenerative or regenerative. Nonregenerative adsorption is a batch process. For regenerative adsorption, adsorbent beds are cycled between adsorption and desorption (regeneration) modes and multiple beds are required for continuous operation.

During regeneration, stripping the adsorbate is accomplished by passing a pure fluid through the bed at a lower pressure for pressure swing adsorption (PSA) or at a higher temperature for temperature swing adsorption (TSA). For TSA, the pressure may be slightly lowered in addition to the temperature increase. Often a split stream from the fluid exiting the adsorbing bed is used as the pure fluid for regenerating adsorption beds.

The regeneration of adsorption beds leaves a residual concentration of adsorbate in the adsorbent. This reduces the working capacity of regenerated adsorbent in comparison with the capacity of fresh adsorbent.

$$\text{Working capacity } W' = W_{\text{sat}} - W_{\text{regen}}$$

where

W_{sat} = amount adsorbed on the bed at break-through

W_{regen} = amount of adsorbate remaining on the bed after regeneration

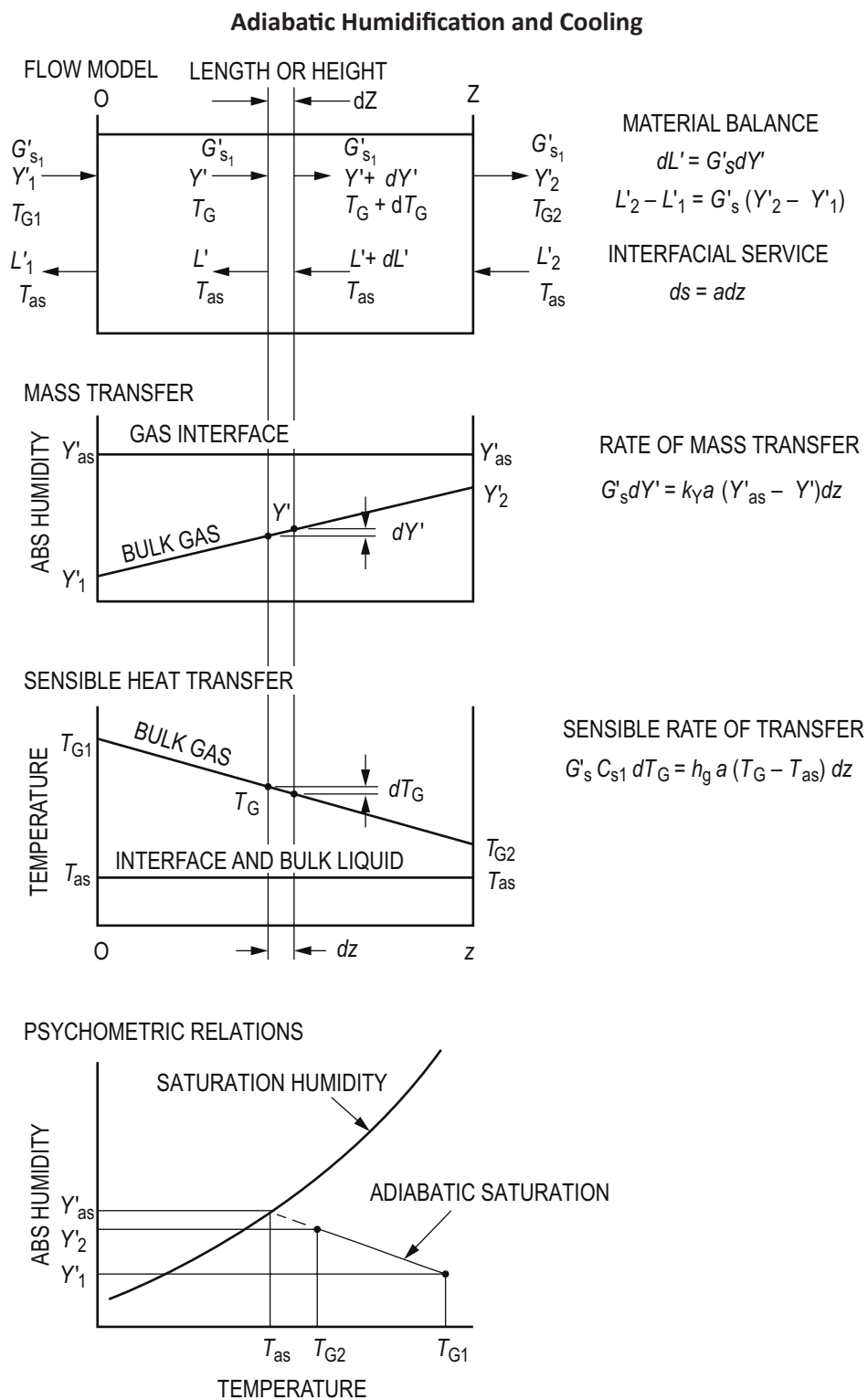
Characteristics of Typical Adsorption Systems

Adsorption System Characteristics

System Type:	TSA		PSA
	Gas Phase	Liquid Phase	Gas Phase
Configuration of system			
Number of beds	2 to 4	2 to 4	2 to 16
Time on adsorption	4 to 8 hours	4 to 8 hours	Minutes to hours
Flow direction on adsorption	Down	Up	Up
Flow direction on regeneration	Up	Down; treated vaporized liquid when feasible	Down
Common adsorbents			
Hydrophobic	Activated carbons for removing VOCs from gas	Activated carbons for water purification	Activated carbon for air separations; heavy hydrocarbons from light hydrocarbons
Hydrophilic	Silica gel, activated alumina, mol sieve for dehydration and removing slightly polar organics		

7.7 Humidification and Drying

7.7.1 Adiabatic Humidification and Cooling



where

L' = solute-free liquid flow rate

G'_s = dry-gas mass flow rate

Y'_1 = initial humidity

Y'_2 = final humidity

Y'_{as} = saturation humidity at liquid-gas interface

T_G = temperature of bulk gas

T_{as} = temperature at liquid-gas interface

C_{s1} = specific heat capacity at the liquid-gas interface

h_g = gas heat-transfer coefficient

Since Y'_{as} is constant:

$$\ln \left(\frac{Y'_{as} - Y'_1}{Y'_{as} - Y'_2} \right) = \frac{k_y a z}{G'_s}$$

where

k_y = overall mass-transfer coefficient

a = interstitial surface per unit volume, in $\frac{\text{ft}^2}{\text{ft}^3}$

z = height, in ft

$$G'_s (Y'_2 - Y'_1) = k_y a z (\Delta Y')_{\text{lm}}$$

where $(\Delta Y')_{\text{lm}}$ = logarithmic mean of humidity difference

or

$$NTU_{tG} = \frac{Y'_2 - Y'_1}{(\Delta Y')_{\text{lm}}} = \ln \left[\frac{Y'_{as} - Y'_1}{Y'_{as} - Y'_2} \right]$$

and

$$HTU_{tG} = \frac{G'_s}{k_y a} = \frac{z}{NTU_{tG}}$$

where

NTU_{tG} = number of gas-phase transfer units

HTU_{tG} = height of transfer unit

Air-Water Systems

y_w = mole fraction of water

y_a = mole fraction of air

$$\bar{Y}_w = \frac{y_w}{y_a} = \frac{y_w}{1 - y_w} = \frac{y_a}{1 - y_a} = \text{molal humidity} = \text{mole water vapor/mole dry air}$$

$$Y_w = \frac{18}{29} \cdot \bar{Y}_w = \text{mass humidity} = \text{mass water vapor/mass dry air}$$

$$\text{Relative humidity} = 100 \frac{\bar{P}_w}{P_w}$$

where $\bar{P}_w$ = partial pressure of water at a given temperature

P_w = vapor pressure of water at a given temperature

$$\bar{Y}_w = \frac{\bar{P}_w}{P_{\text{atm}} - \bar{P}_w} \quad \bar{Y}_{ws} = \frac{P_w}{P_{\text{atm}} - P_w}$$

where $\bar{Y}_{ws}$ = saturation humidity

P_{atm} = atmospheric pressure (14.696 psia or 0.1013 MPa)

$$\% \text{ saturation} = 100 \frac{\bar{Y}_w}{\bar{Y}_{ws}} = \frac{\bar{P}_w (P_{\text{atm}} - P_w)}{P_w (P_{\text{atm}} - \bar{P}_w)} (100) \text{ at total pressure of one atmosphere}$$

$$\text{Humid heat } C_{\text{PH}} = 0.24 + 0.46 Y_w$$

where

C_{PH} = humid heat capacity, Btu/lb

Adiabatic Saturation Temperature

$$t_{\text{AS}} = t_{y0} - \frac{\lambda_R}{C_{\text{PH}}} (Y_{ws} - Y_{w0})$$

where

t_{AS} = adiabatic saturation temperature

t_{y0} = initial inlet temperature

λ_R = latent heat of vaporization at reference temperature

Y_{w0} = initial inlet humidity

C_{PH} = humid heat capacity

Y_{ws} = humidity at saturation

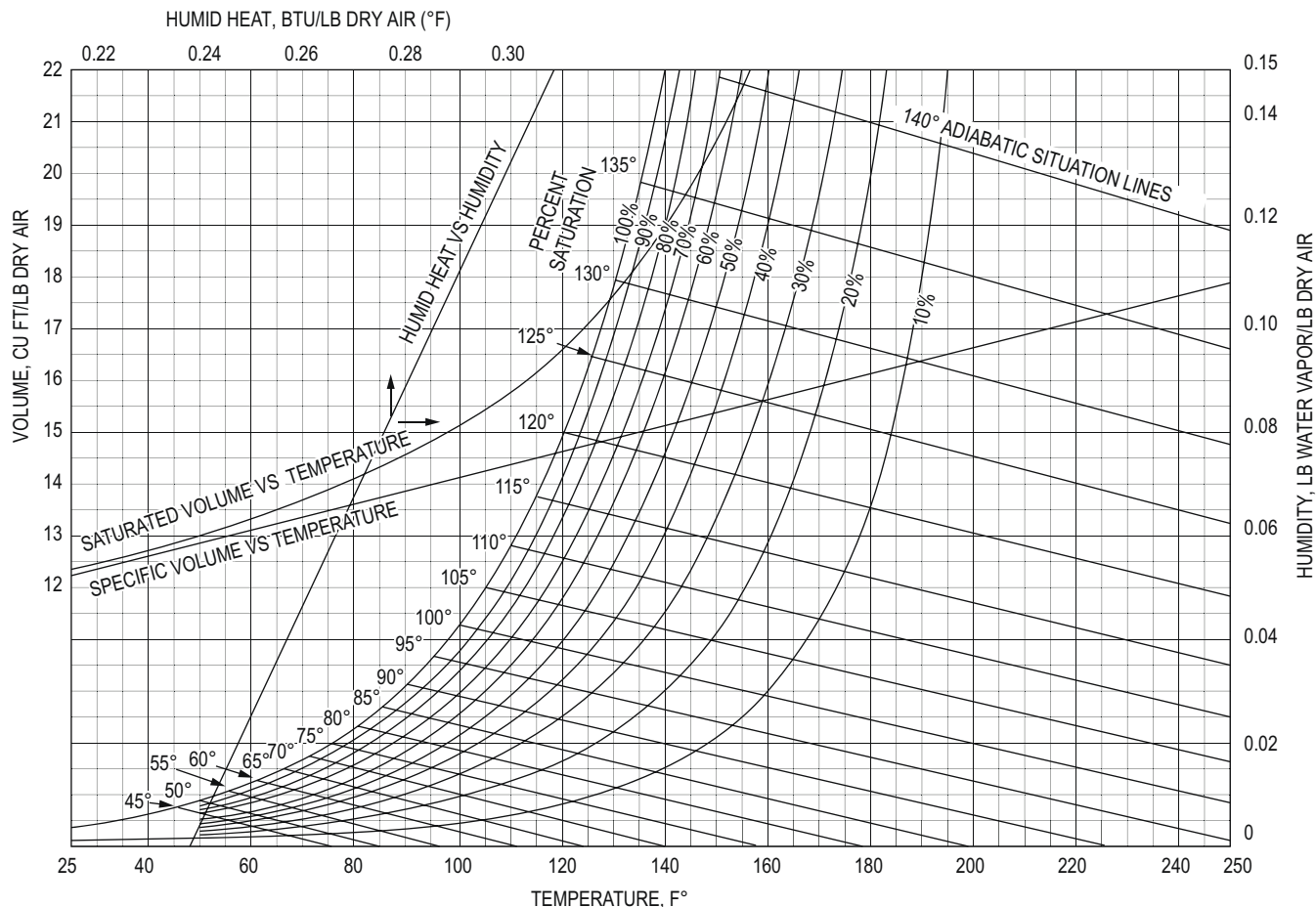
$$t_{\text{WB}} = t_y - \frac{\lambda_R}{C_{\text{PH}}} (Y_{\text{WB}} - Y_w)$$

where

Y_{WB} = humidity at wet bulb temperature

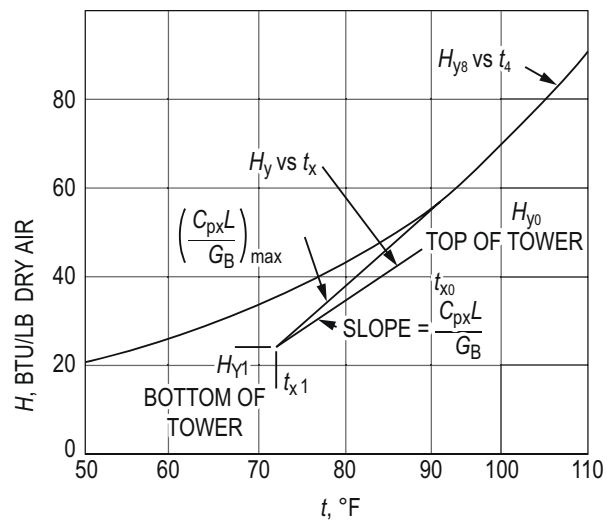
t_{WB} = wet bulb temperature

Humidity Chart for the Air-Water System at One Atmosphere



Source: Brown, G.G., et. al., *Unit Operations*, New York: John Wiley & Sons, Inc., 1950.

Cooling Tower Operating Diagram



$$H_y = \left(\frac{C_{px} L}{G_B} \right) \cdot (t_x - t_{x0}) + H_{y0}$$

where

H_y = enthalpy of vapor phase

C_{px} = specific heat of liquid phase

L = liquid-phase mass velocity

G_B = dry air mass velocity

t_x = liquid-phase temperature

H_{y0} = initial enthalpy of vapor phase

t_{x0} = liquid-phase inlet temperature

7.7.2 Drying of Solids

Moisture (Solvent) Percentage Content

Typically calculated on a dry solid/dry air basis:

$$X = \% \text{ moisture in solid} = \frac{m_w}{m_s}$$

where

X = moisture (solvent) content in solid, moisture mass/dry solid mass

m_w = moisture (solvent) content, mass of water or solvent, in lbm

m_s = mass of dry solid, in lbm

$$Y = \% \text{ moisture in air} = \frac{m_w}{m_a}$$

where

Y = moisture (solvent) content in air, moisture mass/dry air mass

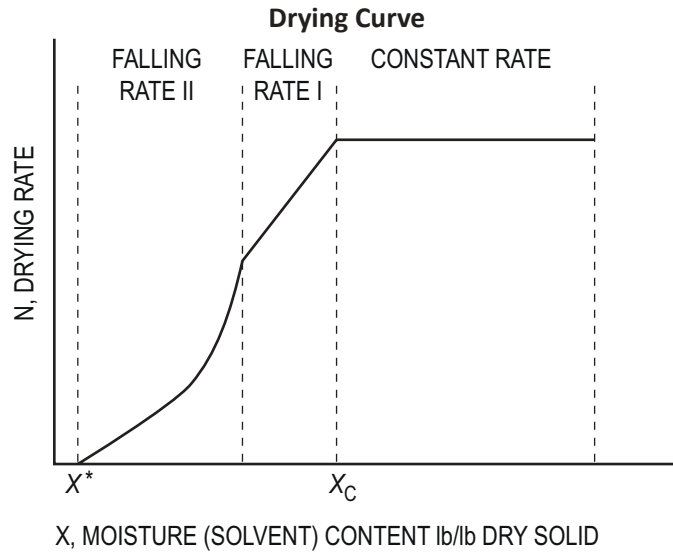
m_w = moisture (solvent) content, mass of water or solvent, in lbm

m_a = mass of dry air, in lbm

Rate of Drying

Rate of drying is dictated by the state of the solvent, such as:

- "Free" solvent on surface of solids
- "Bound" solvent, which must reach the surface through diffusion or capillary action
- "Solvated" solvent, which is chemically bound to the solids (sometimes labile to removal, sometimes not) that are not generally considered in drying analyses



where

X^* = equilibrium moisture content: the moisture content of the solid when it reaches equilibrium with the surrounding air; depending upon the specific conditions of the surrounding air

X_c = critical moisture content: the moisture content that marks the instant when the liquid content on the surface of the solid is no longer sufficient to maintain a continuous liquid film on the surface

Constant Rate: Rate of drying independent of moisture content. During this period the solid is so wet that the entire surface of the solid is covered with a continuous film of liquid.

Falling Rate I: Only part of the solid surface is saturated as the entire solid surface can no longer be maintained at saturation conditions by the movement of moisture within the solid. The rate of drying is linear with regard to X .

Falling Rate II: The entire solid surface is unsaturated and the drying rate is limited by the rate of internal moisture movement.

Source: McCabe, Warren L., and Julian C. Smith, *Unit Operations of Chemical Engineering*, 3rd ed., New York: McGraw-Hill, 1976.

Specific Drying Applications

Drying of slab using gas from one side only:

1. For drying during the constant rate period

Rate of drying can be determined based on the balance between the heat transfer to the material and the rate of vapor removal from the surface.

$$N_C = \frac{h_t \Delta T}{\lambda} = k_g \Delta p$$

where

ΔT = gas dry bulb temperature—temperature at surface of solid

Δp = vapor pressure of water at surface temperature—partial pressure of water vapor in the gas

k_g = mass-transfer coefficient, in $\frac{\text{lbm}}{\text{hr-ft}^2\text{-atm}}$

N_C = constant drying rate, in $\frac{\text{lbm}}{\text{ft}^2\text{-hr}}$

λ = latent heat of evaporation, in $\frac{\text{Btu}}{\text{lbm}}$

h_t = total heat-transfer coefficient, in $\frac{\text{Btu}}{\text{hr-ft}^2\text{-}^\circ\text{F}}$

When the air is flowing parallel to the surface:

$$h_t = 0.0128 G^{0.8}$$

When the air is flowing perpendicular to the surface, the equation is

$$h_t = 0.37 G^{0.37}$$

where G = mass velocity, in $\frac{\text{lbm}}{\text{ft}^2\text{-hr}}$

$$t = \frac{m_s(X_1 - X_2)}{A N_C}$$

where

t = drying time

X_1 = moisture content in solid at time 1

X_2 = moisture content in solid at time 2

- For linear falling rate period I

$$t = \left[\frac{m_s(X_1 - X_2)}{A(N_1 - N_2)} \right] \ln \frac{N_1}{N_2}$$

where

N_1 = drying rate at time 1, in $\frac{\text{lb}}{\text{ft}^2\text{-hr}}$

N_2 = drying rate at time 2, in $\frac{\text{lb}}{\text{ft}^2\text{-hr}}$

m_s = mass of dry solids, in lb

If falling rate I extends all the way to X^* and drying starts at the critical moisture content:

$$t = \frac{m_s(X_C - X^*)}{A \cdot N_C} \cdot \ln \left(\frac{X_C - X^*}{X_2 - X^*} \right)$$

- For falling rate period II, rate curve must be integrated:

$$t = \left(\frac{m_s}{A} \right) \int_{X_2}^{X_1} \frac{dX}{N}$$

$$N_C = \frac{h_t \Delta T}{\lambda} = k_g \Delta p$$

Dryer Design and Performance

- Tray dryers

To determine the tray area for a specific production rate:

$$A = \frac{P(t + t_d)}{L_T}$$

where

P = production rate, in mass of dry solids per hour

t = drying time

t_d = downtime for loading and unloading trays

L_T = tray loading in mass of dry solids per square area of tray, in $\frac{\text{lbm}}{\text{ft}^2}$

2. Continuous through-circulation dryers

To determine required conveyor length:

Required dryer holding capacity C in pounds is

$$C = P t$$

where

P = production rate, in $\frac{\text{lbm dry solid}}{\text{hr}}$

t = drying time, in hr

$$A = \frac{P t}{L}$$

where

A = conveyor area, in ft^2

L = bed loading, in $\frac{\text{lbm dry solid}}{\text{ft}^2 \text{ conveyor area}}$

$$B = \frac{A}{W}$$

where

B = effective dryer length, in ft

W = conveyor width, in ft

3. Rotary dryers

The residence time can be determined empirically using:

for countercurrent flow, sign in the expression below is positive

for concurrent flow, sign in the expression below is negative

$$t = \frac{0.23L}{SN^{0.9}D} \pm 0.6 \frac{5D_p^{-0.5}LG}{F}$$

where

t = retention time, in min

L = dryer length, in ft

D = diameter of shell, in ft

D_p = weighted average particle size of material, in micrometers

N = speed, in $\frac{\text{rev}}{\text{min}}$

S = slope of shell, in $\frac{\text{ft}}{\text{ft}}$

G = air mass velocity in $\frac{\text{lbm}}{\text{hr} \cdot \text{ft}^2}$

F = feed rate in $\frac{\text{lbm dry material}}{\text{hr} \cdot \text{ft}^2 \text{ of dryer cross-sectional area}}$

4. Spray dryers

An estimate of the drying time can be found using:

$$t = \frac{\lambda W \rho_s d_p^2}{12 K_f (T_a - T_s)}$$

where

t = drying time, in min

d_p = drop diameter in ft

W = moisture content in the drop in $\frac{\text{lbm}}{\text{lbm dry solid}}$

K_f = thermal conductivity of the gas film in $\frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$

$T_a - T_s$ = temperature difference between drop and gas in $^\circ\text{F}$

ρ_s = density of the solid

Typical Critical Moisture Content of Various Materials

Approximate Critical Moisture Contents Obtained on the Air Drying of Various Materials, Expressed as Percentage Water on the Dry Basis

Material	Thickness (in.)	Critical Moisture (% Water)
Barium nitrate crystals, on trays	1.0	7
Beaverboard	0.17	Above 120
Brick clay	0.62	14
Carbon pigment	1	40
Celotex	0.44	160
Chrome leather	0.04	125
Copper carbonate, on trays	1–1.5	60
English china clay	1	16
Flint clay refractory brick mix	2.0	13
Gelatin, initially 400% water	0.1–0.2 (wet)	300
Iron blue pigment, on trays	0.25–0.75	110
Kaolin		14
Lithol red	1	50

Approximate Critical Moisture Contents Obtained on the Air Drying of Various Materials, Expressed as Percentage Water on the Dry Basis (cont'd)

Material	Thickness (in.)	Critical Moisture (% Water)
Lithopone press cake, in trays	0.25	6.4
	0.50	8.0
	0.75	12.0
	1.0	16.0
Niter cake fines, on trays		Above 16
Paper, white eggshell	0.0075	41
Fine book	0.005	33
Coated	0.004	34
Newsprint		60–70
Plastic clay brick mix	2.0	19
Poplar wood	0.165	120
Prussian blue		40
Rock salt, in trays	1.0	7
Sand, 50–150 mesh	2.0	5
200–325 mesh	2.0	10
through 325 mesh	2.0	21
Sea sand, on trays	0.25	3
	0.50	4.7
	0.75	5.5
	1.0	5.9
	2.0	6.0
Silica brick mix	2.0	8
Sole leather	0.25	Above 90
Stannic tetrachloride sludge	1	180
Subsoil, clay fraction 55.4%		21
Subsoil, much higher clay content		35
Sulfite pulp	0.25–0.75	60–80
Sulfite pulp (pulp lap)	0.039	110
White lead		11
Whiting	0.25–1.5	6.9
Wool fabric, worsted		31
Wool, undyed serge		8

Source: McCabe, W.L., and J.C. Smith, Unit Operations of Chemical Engineering, 3rd ed., New York: McGraw-Hill, 1976.

7.8 Filtration

Types of filters:

1. Discontinuous pressure filters
2. Continuous filters
3. Centrifugal filters
4. Cartridge filters
5. Bag filters

Factors for Selection of Filter Media

The filter media in any process filter need to meet the following requirements to be of value in a chemical process:

- The septum must obviously be able to retain the solids to be filtered, producing a reasonably clear filtrate
- The removed solids must not plug off the media upon initial or subsequent use.
- The media must be chemically resistant to the chemicals in the filtrate and the filter cake.
- The septum must be strong enough physically to withstand the operating conditions.
- The media must allow the cake to be discharged cleanly and completely.
- The cost of the media must be reasonable enough not to add significantly to the overall plant or production cost.

Filtration Equations

Total pressure drop:

$$\Delta p = p_a - p_b = (p_a - p') + (p' - p_b) = \Delta p_c + \Delta p_m$$

where

Δp = overall pressure drop

p_a = filter inlet pressure

p' = septum inlet pressure

p_b = filter outlet pressure

Δp_c = pressure drop over cake

Δp_m = pressure drop over medium

Filter cake pressure drop:

$$\frac{dp}{dL} = \frac{150\mu u(1-\epsilon)^2}{g_c(\phi_s D_p)^2 \epsilon^3}$$

where

$\frac{dp}{dL}$ = pressure gradient at thickness L

μ = viscosity of filtrate

u = linear velocity of filtrate, based on filter area

ϵ = porosity of cake

D_p = nominal diameter of solid particles

$\phi_s = \frac{6v_p}{D_p s_p}$ for nonspherical particles

$\phi_s = 1$ for spherical particles

or

$$\frac{dp}{dL} = \frac{4.17\mu u(1-\epsilon)^2 \left(\frac{s_p}{v_p}\right)^2}{g_c \epsilon^3}$$

where

s_p = surface of single particle

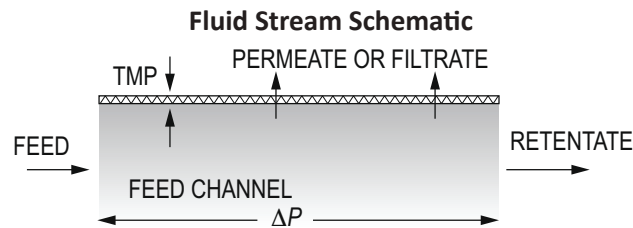
v_p = volume of single particle

Filter medium resistance:

$$R_m = \frac{(p' - p_b)g_c}{\mu u} = \frac{-\Delta p_m g_c}{\mu u}$$

7.9 Membrane Separation Processes

General Background



Normal flow filtration (NFF) refers to the situation in which retentate flow is zero and all the feed stream flows to the membrane surface are normal.

Tangential flow filtration (TFF) refers to the situation in which the feed stream flows are tangential to the membrane surface and exit the module as a retentate stream, creating a velocity gradient at the membrane surface.

Permeation flux J in $\frac{\text{ft}^3}{\text{ft}^2 \text{ day}}$ or $\frac{\text{mol}}{\text{m}^2 \text{ s}}$ indicates the productivity of a membrane:

$$J = \frac{\text{volumetric permeate flow rate}}{\text{membrane area}}$$

Permeability L indicates the sensitivity of productivity or flux to transmembrane pressure (TMP):

$$L = \frac{\text{flux}}{\text{transmembrane pressure}}$$

Transmembrane pressure (TMP) may refer to a module average. *Pure-component permeability* (e.g., water permeability) refers to membrane properties, while the more industrially relevant *process permeability* includes fouling and polarization effects.

The *recovery or conversion ratio* CR indicates the efficiency of a membrane module:

$$CR = \frac{\text{permeate flow rate}}{\text{feed flow rate}}$$

Solutes entrained by the permeate flow are retained by the membrane. They accumulate on the membrane surface and form a region of high concentration called the *polarization boundary layer*. A steady state is reached between back transport away

from the membrane surface, tangential convective transport along the membrane surface, and normal convective flow towards the membrane.

The *local transmission or sieving coefficient* S indicates the passage of a single component through a membrane. The concentrations may change within a module:

$$S = \frac{c_p(\text{local})}{c_f(\text{local})}$$

The *observed passage* S_{obs} indicates the transmission coefficient based on the concentration in the permeate stream exiting a module and in the feed stream entering a module. The observed passage characterizes the module:

$$S_{\text{obs}} = \frac{c_p(\text{module})}{c_f(\text{module})}$$

The *intrinsic passage* S_{int} indicates the transmission coefficient based on the concentration in the permeate stream exiting a module and in the feed stream at the membrane wall. The intrinsic passage characterizes the membrane:

$$S_{\text{int}} = \frac{c_p}{c_w}$$

where

c_f = concentration in feed

c_p = concentration in permeate

c_w = concentration at wall of membrane

The *retention or rejection* R is the complement to the transmission coefficient or passage:

$$R = 1 - S$$

The *multiple-component separation factor* α_{ij} defines the selectivity for component separation:

$$\alpha_{ij} = \frac{\left(\frac{c_{ip}}{c_{if}} \right)}{\left(\frac{c_{jp}}{c_{jf}} \right)} = \frac{S_i}{S_j}$$

where

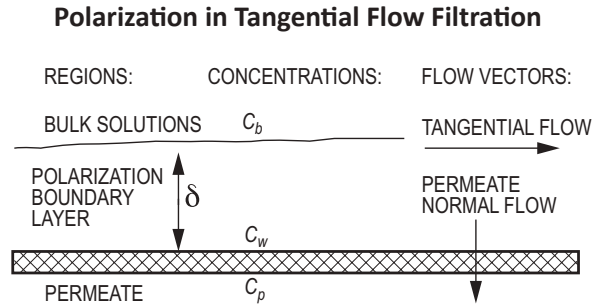
c_{if} = concentration of component i in feed

c_{ip} = concentration of component i in permeate

Component transport through membranes can be considered as mass transfer in series:

1. Transport through a polarization layer above the membrane that may include static or dynamic cake layers
2. Partitioning between the upstream polarization layer and membrane phases at the membrane surface
3. Transport through the membrane
4. Partitioning between the membrane and the downstream fluid

A simplified model of polarization can be used as the basis for analysis:



Source: Republished with permission of McGraw-Hill, Inc., from *Perry's Chemical Engineers' Handbook*, 8th ed., Don W. Green and Robert H. Perry, New York, 2008; permission conveyed through Copyright Clearance Center, Inc.

Gas Separation

The flux for permeation is

$$J_i = \left(\frac{\rho_i}{z} \right) (p_{i,\text{feed}} - p_{i,\text{permeate}})$$

where:

$$J_i = \text{permeation flux of component } i, \text{ in } \frac{\text{ft}^3}{\text{ft}^2 \text{ hr}} \text{ or } \frac{\text{mol}}{\text{m}^2 \text{ s}}$$

$$\rho_i = \text{permeability of component } i, \text{ in } \frac{\text{ft}^3 \text{ ft}}{\text{ft}^2 \text{ hr psi}} \text{ or } \frac{\text{mol}}{\text{m}^2 \text{ s Pa}}$$

z = membrane thickness

p_i = partial pressure of component i

Stage cut θ is defined by

$$\theta = \frac{V}{L} = \frac{\text{permeate volume flow rate}}{\text{feed volume flow rate}}$$

where

$$V = \text{molar permeate flow rate, in } \frac{\text{lb mole}}{\text{hr}} \text{ or } \frac{\text{mol}}{\text{s}}$$

$$L = \text{molar feed flow rate, in } \frac{\text{lb mole}}{\text{hr}} \text{ or } \frac{\text{mol}}{\text{s}}$$

Selectivity is

$$\alpha_{ij} = \frac{\left(\frac{y_i}{y_j} \right)}{\left(\frac{x_i}{x_j} \right)}$$

where

α_{ij} = separation factor

x_i = mole fraction of component i in the feed or reject

y_i = mole fraction of component i in the permeate

The pressure ratio Φ is

$$\Phi = \frac{P_{\text{feed}}}{P_{\text{permeate}}}$$

The ratio of permeation flux for two components i and j is

$$\frac{J_i}{J_j} = \alpha_{ij} \left[\frac{x_i - \left(\frac{y_i}{\Phi} \right)}{x_j - \left(\frac{y_j}{\Phi} \right)} \right]$$

At stage cut $\Phi = 0$, the permeate composition as a function of feed composition is

$$y_i = \left(\frac{\Phi}{2} \right) \left[\left(x_i + \frac{1}{\Phi} + \frac{1}{\alpha - 1} \right) - \sqrt{\left(x_i + \frac{1}{\Phi} + \frac{1}{\alpha - 1} \right)^2 - \frac{4\alpha x_i}{(\alpha - 1)\Phi}} \right]$$

For membrane modules, the partial pressure driving force is a point function dependent on the partial pressures at a point on the membrane and is not constant. To take this into account, the equation may be used in iterative calculations for approximating the performance of membrane modules.

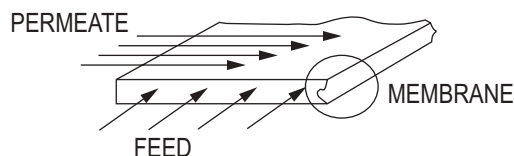
The limiting case for $\alpha \gg \Phi$ is

$$y_i \cong x_i \frac{P_{\text{feed}}}{P_{\text{permeate}}} = x_i \Phi$$

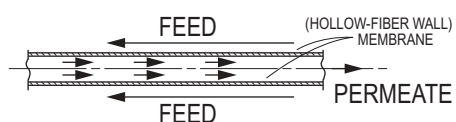
The limiting case for $\alpha \ll \Phi$ is

$$y_i \cong \frac{x_i \alpha}{1 + x_i (\alpha - 1)}$$

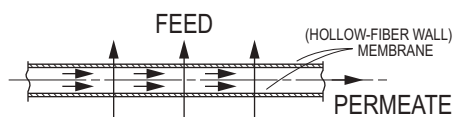
Flow Paths in Gas Permeators



(a) SPIRAL WOUND MODULES

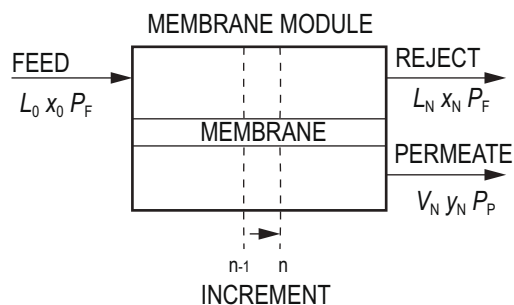


(b) HOLLOW-FIBER MODULES WITH COUNTERCURRENT FLOW



(c) HOLLOW-FIBER MODULES WITH CROSS FLOW

Material Balances for Membrane Modules



Overall and component balances for a module are

$$L_0 = L_N + V_N$$

$$x_0 L_0 = x_N L_N + y_N V_N$$

Overall and component balances for an increment of module area are

$$L_{n-1} + V_{n-1} = L_n + V_n$$

$$x_{n-1} L_{n-1} + y_{n-1} V_{n-1} = x_n L_n + y_n V_n$$

These may be expressed as

$$\Delta V_n = V_n - V_{n-1} = \frac{L_{n-1}(x_{n-1} - x_n)}{\bar{y}_n - x_n}$$

where

$$\bar{y}_n = \frac{1}{2}(y_{n-1} + y_n)$$

$$\Delta V = V_n - V_{n-1}$$

$$\bar{y}_n \Delta V_n \equiv y_n V_n - y_{n-1} V_{n-1}$$

At any point along the membrane, the permeate composition is

$$yV = \sum_1^N y_n \Delta V_n$$

and the permeate composition for the overall module is

$$y_N V_N = \sum_1^N y_n \Delta V_n$$

Area for Membrane Modules

Based on a stepwise incremental solution, the membrane area is

$$A_N = \sum_1^N A_n = \sum_1^N \left[\frac{y_n \Delta V_n}{(J_i)_{avg}} \right]$$

$$\text{where } (J_i)_{avg} = \left(\frac{\rho_i}{z} \right) \left(\frac{1}{2} \right) \left[(x_{n-1} P_F - y_{n-1} P_P) + (x_n P_F - y_n P_P) \right] = \left(\frac{\rho_i}{z} \right) \left(\frac{1}{2} \right) \left[(x_{n-1} + x_n) P_F - (y_{n-1} + y_n) P_P \right]$$

The overall module area can be approximated as

$$A_N = \frac{y_N V_N}{(J_i)_{avg}}$$

$$\text{where } (J_i)_{avg} = \left(\frac{\rho_i}{z} \right) \left(\frac{1}{2} \right) \left[(x_0 P_F - y_0 P_P) + (x_N P_F - y_N P_P) \right] = \left(\frac{\rho_i}{z} \right) \left(\frac{1}{2} \right) \left[(x_0 + x_N) P_F - (y_0 + y_N) P_P \right]$$

Procedure for Incremental Calculation

Given α_{ij} , P_F , P_P , L_0 , and x_0 :

1. Select increment Δx
2. For the initial point 0, calculate y_0 and $(J_i)_0$
3. Determine x_n

4. Calculate y_n and $\bar{y}_n$
5. Calculate ΔV_n and V_n
6. Calculate $\bar{y}_n \Delta V_n$
7. Calculate L_n
8. Calculate $(J_i)_n$, $(J_i)_{\text{avg}}$, and A_n
9. After the final increment, calculate $(J_i)_N$ and A_N

Membrane Separation Processes—Reverse Osmosis

Osmotic Pressure

The osmotic pressure π_s of a solution is

$$\pi_s = \Phi_s i_s c_s R T$$

where

π_s = osmotic pressure in psi or Pa

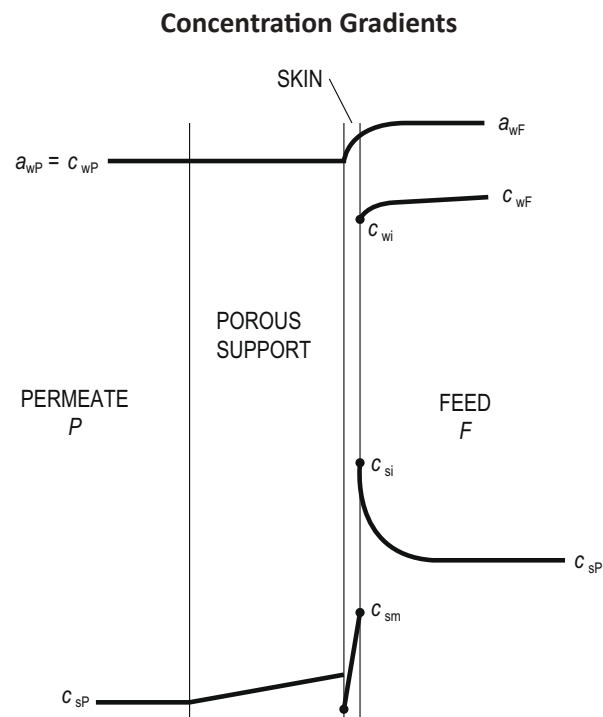
Φ_s = osmotic coefficient

i_s = number of ions formed by solute molecules

c_s = concentration of the solute in $\frac{\text{lb mole}}{\text{ft}^3}$ or $\frac{\text{mol}}{\text{m}^3}$

R = universal gas constant

T = absolute temperature in °R or K



Source: McCabe, Warren L., Julian C. Smith, and Peter Harriott, *Unit Operations of Chemical Engineering*, 5th ed., New York: McGraw-Hill, 1993.

where

a_w = activity of water

c_s = concentration of solute

Flux Across Membrane

The flux of solvent J_w (water, for example) is

$$J_w = \frac{c_w D_w v_w}{RT} \left(\frac{\Delta P - \Delta \pi}{z} \right)$$

where

J = permeation flux in $\frac{\text{ft}^3}{\text{ft}^2 \text{ hr}}$ or $\frac{\text{m}^3}{\text{m}^2 \text{ s}}$

c = concentration in $\frac{\text{lb mole}}{\text{ft}^3}$ or $\frac{\text{mol}}{\text{m}^3}$

D = effective diffusivity in $\frac{\text{ft}^2}{\text{hr}}$ or $\frac{\text{m}^2}{\text{s}}$

v = partial specific volume in $\frac{\text{ft}^3}{\text{lbm}}$ or $\frac{\text{m}^3}{\text{kg}}$

ΔP = friction losses in psi or Pa

z = membrane thickness in ft or m

$\Delta \pi$ = differential osmotic pressure in psi or Pa

The flux of solute is

$$J_s = D_s S_s \left(\frac{\Delta c_s}{z} \right)$$

where:

S_s = distribution coefficient of the solute

Polarization Factor

The polarization factor is the relative concentration difference across the polarization boundary layer and is

$$\Gamma = \frac{c_{si} - c_s}{c_s} = \frac{J_w f}{k_c}$$

where

f = fraction of solute rejected

k_c = mass-transfer coefficient based on concentration, in $\frac{\text{ft}}{\text{hr}}$ or $\frac{\text{m}}{\text{s}}$

Pressure Drop

The internal flow in a hollow-fiber membrane is laminar, and the internal pressure drop ΔP_f with one closed end is

$$\Delta P_f = \frac{128 J_w \mu}{D^3} \frac{L^2}{2}$$

where

L = length

D = diameter

μ = viscosity

7.10 Crystallization

Saturation and Supersaturation

The Solubility-Supersolubility Diagram

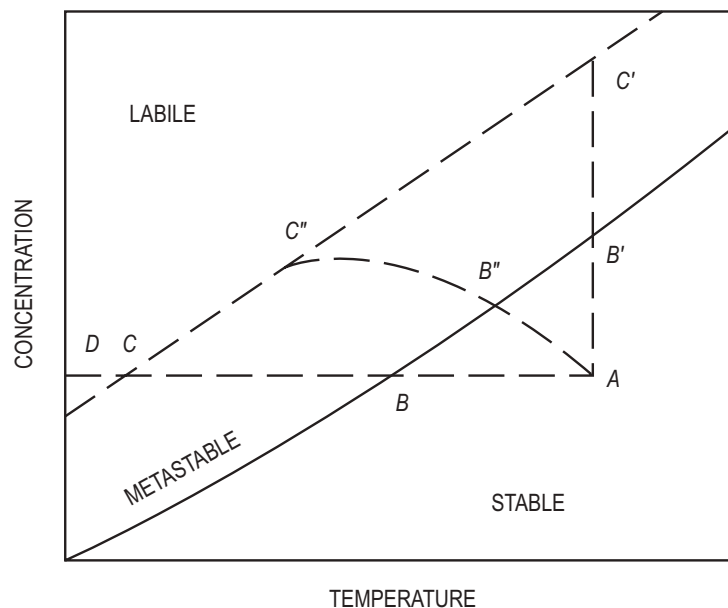


Diagram regions:

- Stable (unsaturated) zone, where crystallization is impossible.
- Metastable (supersaturated) zone, between the solubility and supersolubility curves, where spontaneous crystallization is improbable. However, if a crystal seed were placed in such a metastable solution, growth would occur on it.
- Unstable or Labile (supersaturated) zone, where spontaneous crystallization is probable, but not inevitable.

Expressions of Supersaturation

$$\Delta c = c - c^*$$

where

c = concentration

c^* = saturation concentration

Δc = driving-force concentration

$$S = \frac{c}{c^*}$$

where

S = supersaturation ratio

$$\sigma = \frac{\Delta c}{c^*} = S - 1$$

where σ = relative supersaturation (100 σ is % supersaturation)

$$\Delta\theta = \theta^* - \theta$$

where

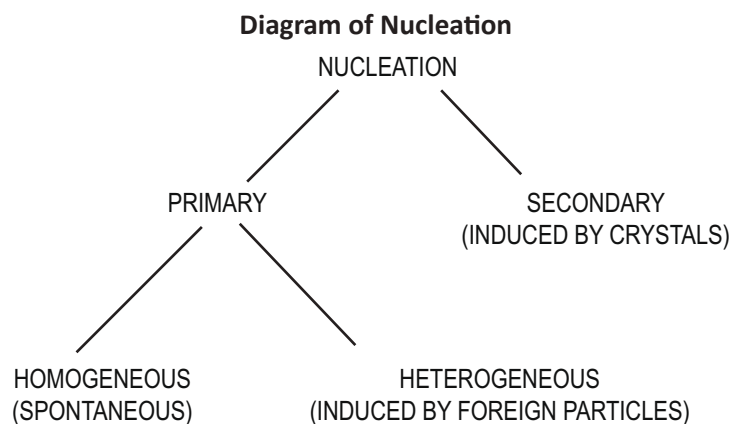
θ = temperature of the solution

θ^* = saturation temperature of the solution

The supersaturation and supercooling are related by the local slope of the solubility curve $\frac{dc^*}{d\theta}$ by

$$\Delta c = \left(\frac{dc^*}{d\theta} \right) \Delta \theta$$

Nucleation



Gibbs Energy of Nucleation

$$\Delta G_{\text{crit}} = \frac{4\pi \gamma r_c^2}{3}$$

where

$$\Delta G_{\text{crit}} = \frac{4\pi \gamma r_c^2}{3} = \text{Gibbs free energy for the critical radius of a stable nucleus}$$

γ = interfacial tension between the developing crystal surface and the supersaturated solution

r_c = critical radius of a stable nucleus

Homogeneous Nucleation Rate (Arrhenius Form)

$$J = A_{\text{EXP}} \left[- \frac{16\pi \gamma^3 v^2}{3k^3 T^3 (\ln S)^2} \right]$$

where

J = nucleation rate

A = rate constant

k = Boltzmann constant ($k = \frac{R_c}{N}$, where N is Avogadro's number)

v = number of moles of ions produced from one mole of electrolyte (for nonelectrolytes, $v = 1$)

T = absolute temperature

S = supersaturation ratio

Crystal Growth

$$R_G = K_G \Delta c^g = \frac{1}{A} \frac{dm}{dt} = \frac{3\alpha}{\beta} \rho_c G = \frac{3\alpha}{\beta} \rho_c \frac{dL}{dt} = \frac{6\alpha}{\beta} \rho_c \frac{dr}{dt} = \frac{6\alpha}{\beta} \rho_c \bar{v}$$

where

R_G = mass deposition rate, in $\frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$

K_G = mass-transfer coefficient with units that are dependent on g (if $g = 1$), in $\frac{\text{m}}{\text{s}}$

g = the order (a fitting parameter)

Δc_g = concentration driving force for mass transfer, in $\frac{\text{kg}}{\text{m}^3}$

$A = \beta L^2$ = particle area, in m^2

$m = \alpha \rho_c L^3$ = particle mass, in kg

t = time, in s

α = volume shape factor

β = surface shape factor

G = overall linear growth rate, in $\frac{\text{m}}{\text{s}}$

ρ_c = crystal density, in $\frac{\text{kg}}{\text{m}^3}$

L = some characteristic size of the crystal, in m

r = radius corresponding to the equivalent sphere, in m

$\bar{v}$ = mean linear velocity of growth, in $\frac{\text{m}}{\text{s}}$

Some Mean Overall Crystal Growth Rates Expressed as a Linear Velocity¹

Crystallizing Substance	°C	S	$\bar{v} \left(\frac{\text{m}}{\text{s}} \right)$
$(\text{NH}_4)_2 \text{SO}_4 \cdot \text{Al}_2(\text{SO}_4)_3 \cdot 24\text{H}_2\text{O}$	15	1.03	$1.1 \times 10^{-8*}$
	30	1.03	$1.3 \times 10^{-8*}$
	30	1.09	$1.0 \times 10^{-7*}$
	40	1.08	$1.2 \times 10^{-7*}$
NH_4NO_3	40	1.05	8.5×10^{-7}
$(\text{NH}_4)_2 \text{SO}_4$	30	1.05	$2.5 \times 10^{-7*}$
	60	1.05	4.0×10^{-7}
	90	1.01	3.0×10^{-8}
$\text{NH}_4\text{H}_2\text{PO}_4$	20	1.06	6.5×10^{-8}
	30	1.02	3.0×10^{-8}
	30	1.05	1.1×10^{-7}
	40	1.02	7.0×10^{-8}
$\text{MgSO}_4 \cdot 7\text{H}_2\text{O}$	20	1.02	$4.5 \times 10^{-8*}$
	30	1.01	$8.0 \times 10^{-8*}$
	30	1.02	$1.5 \times 10^{-7*}$
$\text{NiSO}_4 \cdot (\text{NH}_4)_2 \text{SO}_4 \cdot 6\text{H}_2\text{O}$	25	1.03	5.2×10^{-9}
	25	1.09	2.6×10^{-8}
	25	1.20	4.0×10^{-8}
$\text{K}_2\text{SO}_4 \cdot \text{Al}_2(\text{SO}_4)_3 \cdot 24\text{H}_2\text{O}$	15	1.04	$1.4 \times 10^{-8*}$
	30	1.04	$2.8 \times 10^{-8*}$
	30	1.09	$1.4 \times 10^{-7*}$
	40	1.03	$5.6 \times 10^{-8*}$
KCl	20	1.02	2.0×10^{-7}
	40	1.01	6.0×10^{-7}
KNO_3	20	1.05	4.5×10^{-8}
	40	1.05	1.5×10^{-7}
K_2SO_4	20	1.09	$2.8 \times 10^{-8*}$
	20	1.18	$1.4 \times 10^{-7*}$
	30	1.07	$4.2 \times 10^{-8*}$
	50	1.06	$7.0 \times 10^{-8*}$
	50	1.12	$3.2 \times 10^{-7*}$
KH_2PO_4	30	1.07	3.0×10^{-8}
	30	1.21	2.9×10^{-7}
	40	1.06	5.0×10^{-8}
	40	1.18	4.8×10^{-7}
NaCl	50	1.002	2.5×10^{-8}
	50	1.003	6.5×10^{-8}
	70	1.002	9.0×10^{-8}
	70	1.003	1.5×10^{-7}
$\text{Na}_2\text{S}_2\text{O}_3 \cdot 5\text{H}_2\text{O}$	30	1.02	1.1×10^{-7}
	30	1.08	5.0×10^{-7}
Citric acid monohydrate	25	1.05	3.0×10^{-8}
	30	1.01	1.0×10^{-8}
	30	1.05	4.0×10^{-8}

Some Mean Overall Crystal Growth Rates Expressed as a Linear Velocity¹ (cont'd)

Crystallizing Substance	°C	S	$\bar{v} \left(\frac{\text{m}}{\text{s}} \right)$
Sucrose	30	1.13	$1.1 \times 10^{-8*}$
	30	1.27	$2.1 \times 10^{-8*}$
	70	1.09	9.5×10^{-8}
	70	1.15	1.5×10^{-7}

¹ The supersaturation is expressed by $S = \frac{c}{c'}$, with c and c' as kg of crystallizing substance per kg of free water. The significance of the mean linear growth velocity, $\bar{v} \left(= \frac{1}{2} G \right)$, is explained by equation 6.61 and the values recorded here refer to crystals in the approximate size range 0.5–1 mm growing in the presence of other crystals.

* Denotes that the growth rate is probably size-dependent.

Source: Mullin, J.W., *Crystallization*, 4th ed., Woburn, MA: Reed Educational and Professional Publishing Ltd., 2001.

7.11 Leaching

Leaching is the removal of a soluble substance from an insoluble solid via liquid extraction. The desired component diffuses into the solvent by mass transfer.

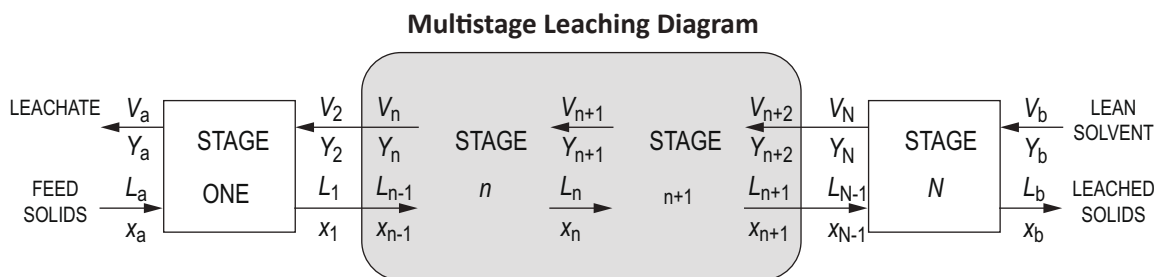
Two common methods of leaching are:

- Percolation of liquids through stationary solid beds
- Dispersion of solids in each leaching stage by mechanical agitation

Multistage Leaching

For multistage leaching processes, the most common setup is continuous countercurrent leaching, where a liquid solvent overflows from stage to stage in a direction opposite to the flow of the solid. The stages are numbered in the direction of flow of the solid.

- The flow rates of contained liquid in the solid slurry streams are shown as L -values.
- The concentrations of solute in the solid slurries are shown as x -values.
- Feed solid slurries enter at stage 1, containing a liquid flow of L_a with a solute concentration of X_a .
- Leached solid slurries exit at stage N , containing a liquid flow of L_b with a solute concentration of X_b .
- It is assumed that the solids flow rate is constant from stage to stage.
- The flow rates of overflow solvent from each stage are shown as V -values.
- The concentrations of solute in the solvent streams are shown as y -values.
- Lean solvent enters the process at stage N , at a mass flow rate of V_b and a solute concentration of Y_b .
- The concentrated solvent, or leachate, exits at a mass flow rate of V_a and a solute concentration of Y_a .



Inputs = Outputs:

$$L_a + V_{n+1} = V_a + L_n$$

Component balance:

$$L_a(x_a) + V_{n+1}(y_{n+1}) = V_a(y_a) + L_n(x_n)$$

Leaching Operating Line

$$Y_{n+1} = \frac{L_n}{V_{n+1}} X_n + \frac{V_a Y_a - L_a X_a}{V_{n+1}}$$

Note: If the density of liquid L_n is constant from stage to stage, then the overflow and underflow rates are both constant and the operating line is straight.

Calculation of the Number of Required Stages in Leaching With Constant Overflow

The equilibrium line for leaching is

$$X_e = Y_e$$

The first stage of the leaching process is calculated initially as a mass balance to set up the flow of slurried solids through the rest of the stages. Therefore, the following calculation determines the total number of stages N , in the format of $N-1$:

$$N-1 = \frac{\ln\left(\frac{y_b - x_b}{y_a - x_a}\right)}{\ln\left(\frac{y_b - y_a}{x_b - x_a}\right)}$$

Langmuir Adsorption of Undissociated Single Species

Reversible adsorption of species A: $A + s \xrightleftharpoons[k_{dA}]{k_{aA}} A \cdot s$

Rate of adsorption of A: $r_{aA} = k_{aA} C_A (1 - \theta_A)$

Rate is proportional to the rate at which molecules strike the surface, concentration in bulk gas, and fraction of unoccupied sites ($1 - \theta_A$)

Rate of desorption of A: $r_{dA} = k_{dA} \theta_A$

At equilibrium, $r_{aA} = r_{dA}$

Langmuir adsorption isotherm:

where $K_A = \frac{k_{aA}}{k_{dA}}$

$$\theta_A = \frac{k_{aA} C_A}{k_{dA} + k_{aA} C_A} = \frac{(k_{aA}/k_{dA}) C_A}{1 + (k_{aA}/k_{dA}) C_A} = \frac{K_A C_A}{1 + K_A C_A}$$

Langmuir Adsorption of Dissociated Single Species

Adsorption of a dissociating diatomic molecule: $B_2 + 2s \xrightleftharpoons[k_{dB_2}]{k_{aB_2}} 2B \cdot s$

Rate of adsorption of B: $r_{aB_2} = k_{aB_2} C_{B_2} (1 - \theta_B)^2$

Rate of desorption of B: $r_{dB_2} = k_{dB_2} \theta_B^2$

Langmuir adsorption isotherm:

where $K_{B_2} = \frac{k_{aB_2}}{k_{dB_2}}$

$$\theta_B = \frac{(K_{B_2} C_{B_2})^{\frac{1}{2}}}{1 + (K_{B_2} C_{B_2})^{\frac{1}{2}}}$$

*If n sites are required for n fragment, the exponent becomes $1/n$.

Source: Missen, Ronald W., Charles A. Mims, and Bradley A. Saville, *Introduction to Chemical Reaction Engineering and Kinetics*, New York: John Wiley & Sons, Inc., 1999.

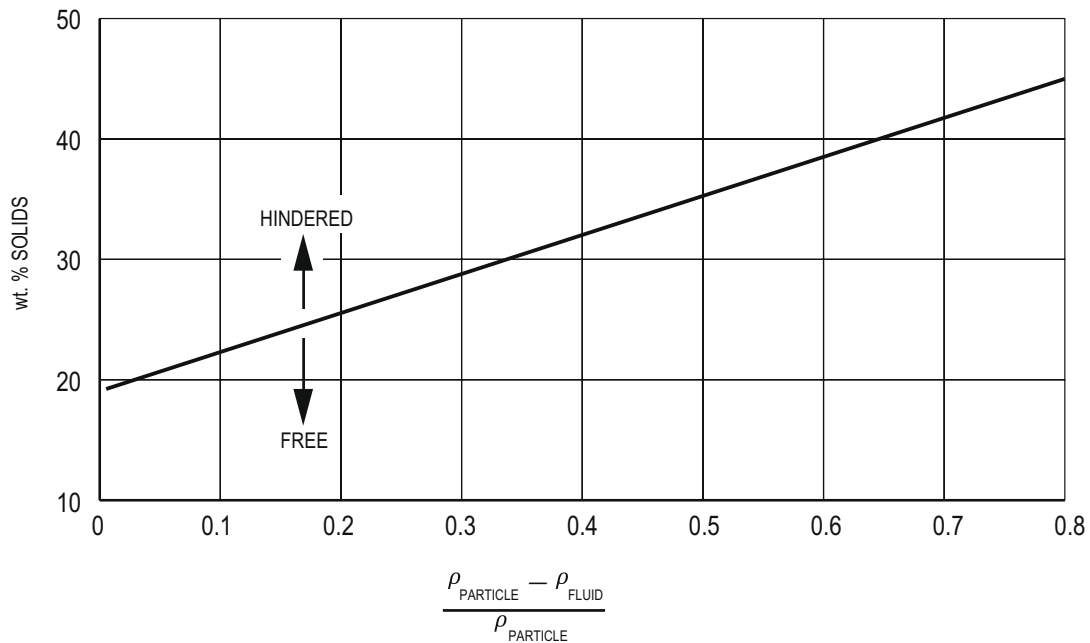
7.12 Particle Settling and Cyclonic Separation

7.12.1 Particle Settling

Free Settling: Particle-to-particle interactions are negligible.

Hindered Settling: Particle settling is at a reduced rate relative to the settling velocity of a single particle caused by interactions with neighboring particles.

Approximate Regions of Free and Hindered Settling for Given Solids' Concentration and Density



If upwards fluid velocity (u_f) is less than the settling velocity of the particle (u_s), then the particles will settle.

For settling operations, the settling velocity (u_s) equals the terminal velocity (u_t).

For Stokes flow, the smallest diameter spherical particle (D_p) that will settle is

$$D_p = \sqrt{\frac{18\mu u_f}{g(\rho_p - \rho_f)}}$$

For general flow up to $Re < 2 \times 10^5$, the smallest diameter spherical particle that will settle is

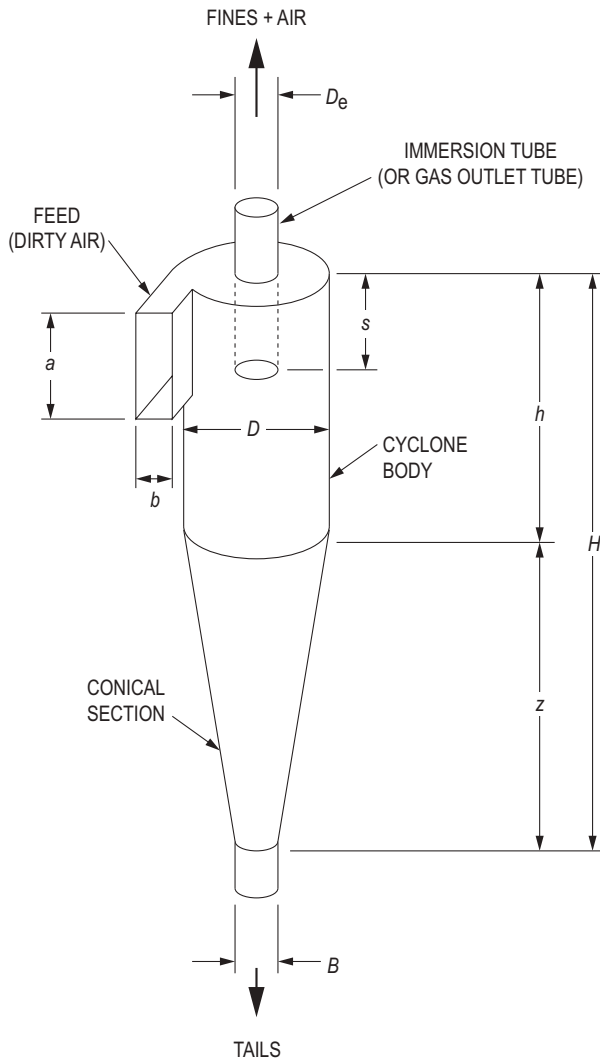
$$D_p = \frac{Re \mu}{u_f \rho_f}$$

where the Reynolds number can be estimated using

$$\frac{1}{\sqrt{Re}} = \left(0.00433 + 0.203 \sqrt{\frac{C_D}{Re}} \right)^{\frac{1}{2}} - 0.0658 \quad \text{where} \quad \frac{C_D}{Re} = \frac{4\mu g(\rho_p - \rho_f)}{3\rho_f^2 u_t^3}$$

7.12.2 Cyclone Separators

Cyclone Separator



where

a = height of tangential inlet

b = width of tangential inlet

D_e = diameter of immersion tube

s = immersion length of outlet tube

D = cyclone diameter

h = length of cylindrical section

z = length of conical section

H = cyclone height

B = diameter of tail outlet

Particle Removal Efficiency

$$\eta = \frac{1}{1 + \left(\frac{D_{pc}}{D_p} \right)^2}$$

where

D_{pc} = diameter of particle collected with 50% efficiency

D_p = diameter of particle of interest

η = fractional particle collection efficiency

Effective Number of Turns

$$N_e = \frac{1}{a} \left(h + \frac{z}{2} \right)$$

where

N_e = number of effective turns the gas makes in the cyclone

h = length of body of cyclone

z = length of cone of cyclone

Cyclone 50% Particle Efficiency for Particle Diameter

$$D_{pc} = \left[\frac{9\mu b}{2\pi N_e u_i (\rho_p - \rho_g)} \right]^{0.5}$$

where

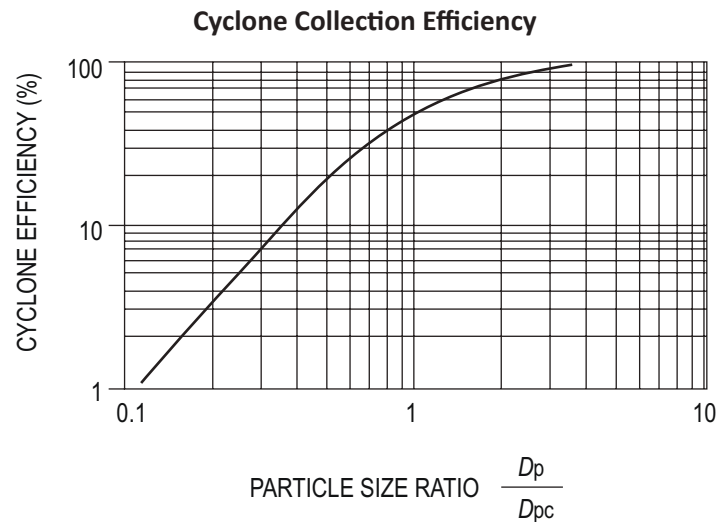
D_{pc} = diameter of particle that is collected with 50% efficiency, in meters

μ = dynamic viscosity of gas

u_i = inlet velocity into cyclone

ρ_p = density of particle

ρ_g = density of gas



Cyclone Ratio of Dimensions to Body Diameter (D) Capacity

Dimension	High Efficiency	Conventional	High Throughput
Inlet height, a	0.44	0.50	0.80
Inlet width, b	0.21	0.25	0.35
Cylindrical section length, h	1.40	1.75	1.70
Cone length, z	2.50	2.00	2.00
Immersion length, s	0.50	0.60	0.85
Gas exit diameter, D	0.40	0.50	0.75
Tails outlet diameter, B	0.40	0.40	0.40
Cyclone height, H	3.90	3.75	3.70

Source: Adapted from Cooper, C. David, and F.C. Alley, *Air Pollution Control: A Design Approach*, 4th ed., Illinois: Waveland Press, 2011.



8 PLANT DESIGN AND OPERATIONS

8.1 Terms and Definitions

Definitions

Term	Description
Boiling point	The temperature at which the vapor pressure of a liquid equals the atmospheric pressure of 14.7 pounds per square inch (psia), 101 kPa, or 760 mm of mercury. For purposes of this classification, when an accurate boiling point is not available for a material or when a mixture does not have a constant boiling point, use the 20%-evaporated point of a distillation performed in accordance with ASTM D 86. Boiling point is commonly expressed in °F or °C.
Combustible dust	A finely divided solid material that is 420 microns or less in diameter and that, when dispersed in air in the proper proportions, can be ignited by a flame, spark, or other source of ignition. Will pass through a U.S. No. 40 standard sieve.
Combustible liquid	A liquid having a closed-cup flash point at or above 100°F (38°C). Subdivided into: Class II: Closed-cup flash point at or above 100°F (38°C) and below 140°F (60°C) Class IIIA: Closed-cup flash point at or above 140°F (60°C) and below 200°F (93°C) Class IIIB: Closed-cup flash point at or above 200°F (93°C) This category does not include compressed gases or cryogenic fluids.
Deflagration	An exothermic reaction, such as the extremely rapid oxidation of a flammable dust or vapor in air, in which the reaction progresses through the unburned material at a rate less than the velocity of sound. A deflagration can have an explosive effect.
Detonation	An exothermic reaction characterized by the presence of a shock wave in the material that establishes and maintains the reaction. The reaction zone progresses through the material at a rate greater than the velocity of sound. The principal heating mechanism is one of shock compression. A detonation has an explosive effect.

Definitions (cont'd)

Term	Description
Explosion	An effect produced by the sudden, violent expansion of gases, which may be accompanied by a shock wave, a disruption of enclosing materials or structures, or both. An explosion could result from: • Chemical changes such as rapid oxidation, deflagration or detonation, decomposition of molecules, or runaway polymerization (usually detonations) • Physical changes such as pressure tank ruptures • Atomic changes such as nuclear fission or fusion
Flammable gas	A material that is a gas at 68°F (20°C) or less at 14.7 psia (101 kPa) of pressure—therefore a material that has a boiling point of 68°F (20°C) or less at 14.7 psia (101 kPa)—and which either: • Ignites at 14.7 psia (101 kPa) when in a mixture of 13% or less by volume with air • Has a flammable range mixed in air at 14.7 psia (101 kPa) and 63°F (20°C) These levels shall be determined at the specified pressure and temperature in accordance with ASTM E 681.
Flammable liquefied gas	A liquefied compressed gas that, under a charged pressure, is partially liquid at a temperature of 68°F (20°C) and that is flammable.
Flammable liquid	A liquid having a closed-cup flash point below 100°F (38°C). Flammable liquids are further categorized into a group known as Class I liquids and subdivided into: Class IA: Closed-cup flash point below 73°F (23°C) and boiling point below 100°F (38°C) Class IB: Closed-cup flash point below 73°F (23°C) and boiling point at or above 100°F (38°C) Class IC: Closed-cup flash point at or above 73°F (23°C) and boiling point below 100°F (38°C). The category of flammable liquids does not include compressed gases or cryogenic fluids.
Flammable material	A material capable of being readily ignited from a common source of heat or at a temperature of 600°F (316°C).
Flammable solid	A solid, other than a blasting agent or explosive, that: Is capable of causing fire through friction, absorption or moisture, spontaneous chemical change, or retained heat from manufacturing or processing or Has an ignition temperature below 212°F (100°C) or Burns so vigorously and persistently when ignited as to create a serious hazard A chemical shall be considered a flammable solid in accordance with the test method of CPSC 16 CFR: Part 1500.44 if it ignites and burns with a self-sustained flame at a rate greater than 0.1 inch (2.5 mm) per second along its major axis.
Flammable vapors or fumes	The concentration of flammable constituents in air that exceeds 25% of their lower flammable limit (LFL).
Flash point	The minimum temperature in degrees Fahrenheit (or Centigrade) at which a liquid will give off sufficient vapors to form an ignitable mixture with air near the surface or in the container, but will not sustain combustion. The flash point of a liquid shall be determined by appropriate test procedure and apparatus as specified in ASTM D 56, ASTM D 93, or ASTM D 3278.

Definitions (cont'd)

Term	Description
Highly toxic	A material that produces a lethal dose or lethal concentration that falls within any of these categories: • A chemical that has a median lethal dose (LD₅₀) of 50 milligrams or less per kilogram of body weight when administered orally to albino rats weighing between 200 and 300 grams each • A chemical that has a median lethal dose (LD₅₀) of 200 milligrams or less per kilogram of body weight when administered by continuous contact for 24 hours (or less if death occurs within 24 hours) with the bare skin of albino rabbits weighing between 2 and 3 kilograms each • A chemical that has a median lethal concentration (LC₅₀) in air of 200 parts per million by volume or less of gas or vapor, or 2 milligrams per liter or less of mist, fume, or dust when administered by continuous inhalation for 1 hour (or less if death occurs within 1 hour) to albino rats weighing between 200 and 300 grams each Mixtures of these materials with ordinary materials, such as water, may not warrant classification as highly toxic.
Immediately dangerous to life and health (IDLH)	The concentration of air-borne contaminants that poses a threat of death, immediate or delayed permanent adverse health effects, or effects that could prevent escape from such an environment. This concentration level of contaminants is established by the National Institute for Occupational Safety and Health (NIOSH) based on both toxicity and flammability. Generally it is expressed in parts per million by volume (ppm/v) or milligrams per cubic meter (mg/m³).
Organic peroxide	An organic compound that contains the bivalent -O-O- structure and that may be considered a structural derivative of hydrogen peroxide in which one or both of the hydrogen atoms have been replaced by an organic radical. Organic peroxides can pose an explosion hazard (detonation or deflagration) or can be shock sensitive. They also can decompose into various unstable compounds over an extended period of time. Class I: Formulations that are capable of deflagration but not detonation Class II: Formulations that burn very rapidly and pose a moderate reactivity hazard Class III: Formulations that burn rapidly and pose a moderate reactivity hazard Class IV: Formulations that burn in the same manner as ordinary combustibles and pose a minimal reactivity hazard Class V: Formulations that burn with less intensity than ordinary combustibles or do not sustain combustion and pose no reactivity hazard Unclassified detonable: Organic peroxides that are capable of detonation. These pose an extremely high explosion hazard through rapid explosive decomposition.

Definitions (cont'd)

Term	Description
Oxidizer	A material that readily yields oxygen or other oxidizing gas or that readily reacts to promote or initiate combustion of combustible materials and, if heated or contaminated, can result in vigorous self-sustained decomposition. Class 4: An oxidizer that can undergo an explosive reaction due to contamination or exposure to thermal or physical shock and that causes a severe increase in the burning rate of combustible materials with which it comes into contact. Additionally, the oxidizer causes a severe increase in the burning rate and can cause spontaneous ignition of combustibles. Class 3: An oxidizer that causes a severe increase in the burning rate of combustible materials with which it comes into contact. Class 2: An oxidizer that causes a moderate increase in the burning rate of combustible materials with which it comes into contact. Class 1: An oxidizer that does not moderately increase the burning rate of combustible materials.
Oxidizing gas	A gas that can support and accelerate combustion of other materials more than air does.
Physical hazard	A chemical for which there is evidence that it is one of the following: • Combustible liquid • Cryogenic fluid • Explosive or flammable solid, liquid, or gas • Solid or liquid organic peroxide • Solid or liquid oxidizer • Oxidizing gas • Pyrophoric solid, liquid, or gas • Unstable (reactive) solid, liquid, or gas material • Water-reactive solid or liquid material
Toxic	A chemical falling within any of these categories: • Has a median lethal dose (LD_{50}) of more than 50 milligrams per kilogram but not more than 500 milligrams per kilogram of body weight when administered orally to albino rats weighing between 200 and 300 grams each. • A chemical that has a median lethal dose (LD_{50}) of more than 200 milligrams per kilogram but not more than 1000 milligrams per kilogram of body weight when administered by continuous contact for 24 hours (or less if death occurs within 24 hours) with the bare skin of albino rabbits weighing between 2 and 3 kilograms each. • A chemical that has a median lethal concentration (LC_{50}) in air of more than 200 parts per million but not more than 2000 part per million by volume or less of gas or vapor, or more than 2 milligrams per liter but not more than 20 milligrams per liter of mist, fume, or dust, when administered by continuous inhalation for 1 hour (or less if death occurs within 1 hour) to albino rats weighing between 200 and 300 grams each.

Definitions (cont'd)

Term	Description
Unstable (reactive) material	A material, other than an explosive, that in the pure state or as commercially produced will vigorously polymerize, decompose, condense, or become self-reactive and undergo other violent chemical changes, including explosion, when it is either: • Exposed to heat, friction, or shock • In the absence of an inhibitor • In the presence of contaminants • In contact with incompatible materials Unstable (reactive) materials are subdivided into: Class 4: Materials that in themselves are readily capable of detonation or explosive decomposition or explosive reaction at normal temperatures and pressures. Includes materials that are sensitive to mechanical or localized thermal shock at normal temperatures and pressures Class 3: Materials that in themselves are capable of detonation or of explosive decomposition or explosive reaction but which require a strong initiating source or which must be heated under confinement before initiation. Includes materials that are sensitive to thermal or mechanical shock at elevated temperatures and pressures Class 2: Materials that in themselves are normally unstable and readily undergo violent chemical change but do not detonate; includes materials that can undergo chemical change with rapid release of energy at normal temperatures and pressures and that can undergo violent chemical change at elevated temperatures and pressures Class 1: Materials that in themselves are normally stable but that can become unstable at elevated temperatures and pressures
Water-reactive material	A material that explodes; violently reacts; produces flammable, toxic, or other hazardous gases; or evolves enough heat to cause autoignition or ignition of combustibles upon exposure to water or moisture. Water-reactive materials are subdivided into: Class 3: Materials that react explosively with water without requiring heat or confinement Class 2: Materials that react violently with water or have the ability to boil water. Materials that produce flammable, toxic, or other hazardous gases, or evolve enough heat to cause autoignition or ignition of combustibles upon exposure to water or moisture Class 1: Materials that react with water with some release of energy, but not violently

Source: 2015 *International Building Code*, Country Club Hills, Illinois: International Code Council.

8.2 Safety, Health, and Environment

8.2.1 Hazard Identification

8.2.1.1 Definition of Safety

Safety is the condition of protecting people from threats or failure that could harm their physical, emotional, occupational, psychological, or financial well-being. Safety is also the control of known threats to attain an acceptable level of risk. The United States relies on public codes and standards, engineering designs, and corporate policies to ensure that a structure or place does what it should do to maintain a steady state of safety—that is, long-term stability and reliability. Some safety/regulatory agencies that develop codes and standards commonly used in the United States are shown below.

Insurance, Safety, and Regulatory Agencies

Abbreviation	Name	Jurisdiction
ANSI	American National Standards Institute	Nonprofit standards organization
CGA	Compressed Gas Association	Nonprofit trade association
CSA	Canadian Standards Association	Nonprofit standards organization
FAA	Federal Aviation Administration	U.S. federal regulatory agency
FMG	FM Global	Insurance
IEC	International Electrotechnical Commission	Nonprofit standards organization
ITSNA	Intertek Testing Services NA (formerly Edison Testing Labs)	Nationally recognized testing laboratory
MSHA	Mine Safety and Health Administration	Federal regulatory agency
NFPA	National Fire Protection Association	Nonprofit trade organization
OSHA	Occupational Health and Safety Administration	Federal regulatory agency
UL	Underwriters Laboratories	Nationally recognized testing laboratory
USCG	United States Coast Guard	Federal regulatory agency
USDOT	United States Department of Transportation	Federal regulatory agency
USEPA	United States Environmental Protection Agency	Federal regulatory agency

8.2.1.2 Definition of Risk

A traditional preventive approach to both accidents and occupational illness involves recognizing, evaluating, and controlling hazards and work conditions that may cause physical injuries or adverse health effects.

Hazard is the capacity to cause harm. It is an inherent quality of a material or a condition. For example, a rotating saw blade or an uncontrolled high-pressure jet of water has the capability (hazard) to slice through flesh. A toxic chemical or a pathogen has the capability (hazard) to cause illness.

Risk is the chance or the probability that a person will experience harm and is not the same as a hazard. Risk always involves both probability and severity elements. The hazard associated with a rotating saw blade or the water jet continues to exist, but the probability of causing harm, and thus the risk, can be reduced by installing a guard or by controlling the jet's path. Risk is expressed by the equation:

$$\text{Risk} = \text{Severity/Consequence} \times \text{Probability}$$

When people discuss the hazards of disease-causing agents, the term *exposure* is typically used more than *probability*. If a certain type of chemical has a toxicity hazard, the risk of illness rises with the degree to which that chemical contacts your body or enters your lungs. In that case, the equation becomes:

$$\text{Risk} = \text{Hazard} \times \text{Exposure}$$

Organizations evaluate hazards using multiple techniques and data sources.

8.2.1.3 Job Safety Analysis

Job safety analysis (JSA) is known by many names, including activity hazard analysis (AHA), or job hazard analysis (JHA). Hazard analysis helps integrate accepted safety and health principles and a specific task. In a JSA, each basic step of the job is reviewed, potential hazards identified, and recommendations documented as to the safest way to do the job. JSA techniques work well when used on a task that the analysts understand well. JSA analysts look for specific types of potential accidents and ask basic questions about each step, such as these:

- Can the employee strike against or otherwise make injurious contact with the object?
- Can the employee be caught in, on, or between objects?
- Can the employee strain muscles by pushing, pulling, or lifting?
- Is exposure to toxic gases, vapors, dust, heat, electrical currents, or radiation possible?

8.2.1.4 OSHA Highly Hazardous Chemicals

The following is from 29 CFR 1910.119, Appendix A. It contains a list of toxic and reactive highly hazardous chemicals that present a potential for a catastrophic event at or above the threshold quantity.

Highly Hazardous Chemicals

Chemical Name	CAS*	TQ**
Acetaldehyde	75-07-0	2500
Acrolein (2-Propenal)	107-02-8	150
Acrylyl Chloride	814-68-6	250
Allyl Chloride	107-05-1	1000
Allylamine	107-11-9	1000
Alkylaluminum	Varies	5000
Ammonia, Anhydrous	7664-41-7	10,000
Ammonia solutions (greater than 44% ammonia by weight)	7664-41-7	15,000
Ammonium Perchlorate	7790-98-9	7500
Ammonium Permanganate	7787-36-2	7500
Arsine (also called Arsenic Hydride)	7784-42-1	100
Bis (Chloromethyl) Ether	542-88-1	100
Boron Trichloride	10294-34-5	2500
Boron Trifluoride	7637-07-2	250
Bromine	7726-95-6	1500
Bromine Chloride	13863-41-7	1500
Bromine Pentafluoride	7789-30-2	2500
Bromine Trifluoride	7787-71-5	15,000
3-Bromopropyne (also called Propargyl Bromide)	106-96-7	100
Butyl Hydroperoxide (Tertiary)	75-91-2	5000
Butyl Perbenzoate (Tertiary)	614-45-9	7500
Carbonyl Chloride (see Phosgene)	75-44-5	100
Carbonyl Fluoride	353-50-4	2500
Cellulose Nitrate (concentration greater than 12.6% nitrogen)	9004-70-0	2500
Chlorine	7782-50-5	1500
Chlorine Dioxide	10049-04-4	1000
Chlorine Pentafluoride	13637-63-3	1000
Chlorine Trifluoride	7790-91-2	1000

Highly Hazardous Chemicals (cont'd)

Chemical Name	CAS*	TQ**
Chlorodiethylaluminum (also called Diethylaluminum Chloride)	96-10-6	5000
1-Chloro-2,4-Dinitrobenzene	97-00-7	5000
Chloromethyl Methyl Ether	107-30-2	500
Chloropicrin	76-06-2	500
Chloropicrin and Methyl Bromide mixture	None	1500
Chloropicrin and Methyl Chloride mixture	None	1500
Cumene Hydroperoxide	80-15-9	5000
Cyanogen	460-19-5	2500
Cyanogen Chloride	506-77-4	500
Cyanuric Fluoride	675-14-9	100
Diacetyl Peroxide (concentration greater than 70%)	110-22-5	5000
Diazomethane	334-88-3	500
Dibenzoyl Peroxide	94-36-0	7500
Diborane	19287-45-7	100
Dibutyl Peroxide (tertiary)	110-05-4	5000
Dichloro Acetylene	7572-29-4	250
Dichlorosilane	4109-96-0	2500
Diethylzinc	557-20-0	10,000
Diisopropyl Peroxydicarbonate	105-64-6	7500
Dilauroyl Peroxide	105-74-8	7500
Dimethyldichlorosilane	75-78-5	1000
1,1-Dimethylhydrazine	57-14-7	1000
Dimethylamine, Anhydrous	124-40-3	2500
2,4-Dinitroaniline	97-02-9	5000
Ethyl Methyl Ketone Peroxide (also Methyl Ethyl Ketone Peroxide; concentration greater than 60%)	1338-23-4	5000
Ethyl Nitrite	109-95-5	5000
Ethylamine	75-04-7	7500
Ethylene Fluorohydrin	371-62-0	100
Ethylene Oxide	75-21-8	5000
Ethyleneimine	151-56-4	1000
Fluorine	7782-41-4	1000
Formaldehyde (Formalin)	50-00-0	1000
Furan	110-00-9	500
Hexafluoroacetone	684-16-2	5000
Hydrochloric Acid, Anhydrous	7647-01-0	5000
Hydrofluoric Acid, Anhydrous	7664-39-3	1000
Hydrogen Bromide	10035-10-6	5000
Hydrogen Chloride	7647-01-0	5000
Hydrogen Cyanide, Anhydrous	74-90-8	1000
Hydrogen Fluoride	7664-39-3	1000
Hydrogen Peroxide (52% by weight or greater)	7722-84-1	7500
Hydrogen Selenide	7783-07-5	150

Highly Hazardous Chemicals (cont'd)

Chemical Name	CAS*	TQ**
Hydrogen Sulfide	7783-06-4	1500
Hydroxylamine	7803-49-8	2500
Iron, Pentacarbonyl	13463-40-6	250
Isopropylamine	75-31-0	5000
Ketene	463-51-4	100
Methacrylaldehyde	78-85-3	1000
Methacryloyl Chloride	920-46-7	150
Methacryloyloxyethyl Isocyanate	30674-80-7	100
Methyl Acrylonitrile	126-98-7	250
Methylamine, Anhydrous	74-89-5	1000
Methyl Bromide	74-83-9	2500
Methyl Chloride	74-87-3	15,000
Methyl Chloroformate	79-22-1	500
Methyl Ethyl Ketone Peroxide (concentration greater than 60%)	1338-23-4	5000
Methyl Fluoroacetate	453-18-9	100
Methyl Fluorosulfate	421-20-5	100
Methyl Hydrazine	60-34-4	100
Methyl Iodide	74-88-4	7500
Methyl Isocyanate	624-83-9	250
Methyl Mercaptan	74-93-1	5000
Methyl Vinyl Ketone	79-84-4	100
Methyltrichlorosilane	75-79-6	500
Nickel Carbonyl (Nickel Tetracarbonyl)	13463-39-3	150
Nitric Acid (94.5% by weight or greater)	7697-37-2	500
Nitric Oxide	10102-43-9	250
Nitroaniline (para Nitroaniline)	100-01-6	5000
Nitromethane	75-52-5	2500
Nitrogen Dioxide	10102-44-0	250
Nitrogen Oxides (NO; NO(2); N2O4; N2O3)	10102-44-0	250
Nitrogen Tetroxide (also called Nitrogen Peroxide)	10544-72-6	250
Nitrogen Trifluoride	7783-54-2	5000
Nitrogen Trioxide	10544-73-7	250
Oleum (65% to 80% by weight; also called Fuming Sulfuric Acid)	8014-95-7	1000
Osmium Tetroxide	20816-12-0	100
Oxygen Difluoride (Fluorine Monoxide)	7783-41-7	100
Ozone	10028-15-6	100
Pentaborane	19624-22-7	100
Peracetic Acid (concentration greater 60% Acetic Acid; also called Peroxyacetic Acid)	79-21-0	1000
Perchloric Acid (concentration greater than 60% by weight)	7601-90-3	5000
Perchloromethyl Mercaptan	594-42-3	150

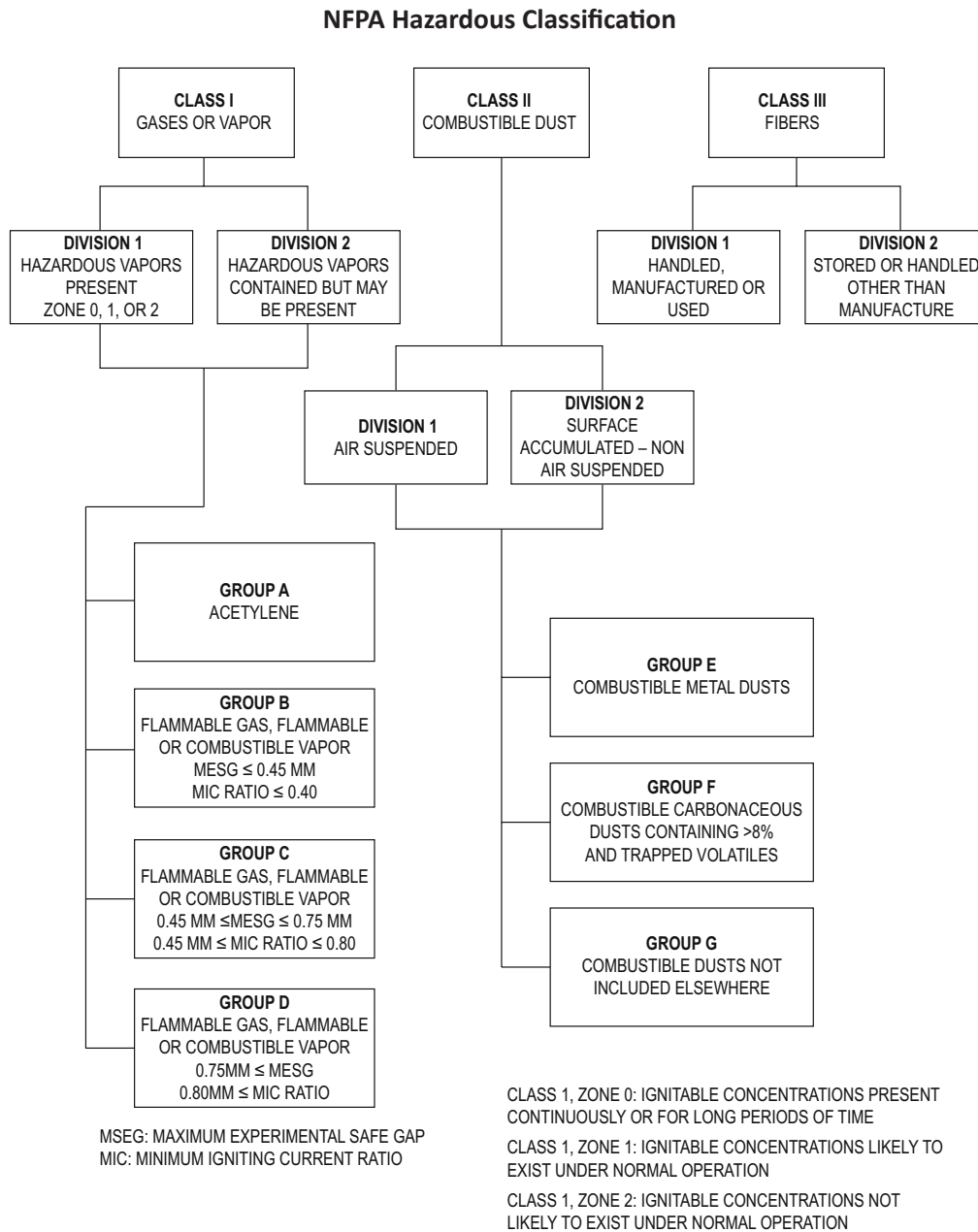
Highly Hazardous Chemicals (cont'd)

Chemical Name	CAS*	TQ**
Perchloryl Fluoride	7616-94-6	5000
Peroxyacetic Acid (concentration greater than 60% Acetic Acid; also called Peracetic Acid)	79-21-0	1000
Phosgene (also called Carbonyl Chloride)	75-44-5	100
Phosphine (Hydrogen Phosphide)	7803-51-2	100
Phosphorus Oxychloride (also called Phosphoryl Chloride)	10025-87-3	1000
Phosphorus Trichloride	7719-12-2	1000
Phosphoryl Chloride (also called Phosphorus Oxychloride)	10025-87-3	1000
Propargyl Bromide	106-96-7	100
Propyl Nitrate	627-3-4	2500
Sarin	107-44-8	100
Selenium Hexafluoride	7783-79-1	1000
Stibine (Antimony Hydride)	7803-52-3	500
Sulfur Dioxide (liquid)	7446-09-5	1000
Sulfur Pentafluoride	5714-22-7	250
Sulfur Tetrafluoride	7783-60-0	250
Sulfur Trioxide (also called Sulfuric Anhydride)	7446-11-9	1000
Sulfuric Anhydride (also called Sulfur Trioxide)	7446-11-9	1000
Tellurium Hexafluoride	7783-80-4	250
Tetrafluoroethylene	116-14-3	5000
Tetrafluorohydrazine	10036-47-2	5000
Tetramethyl Lead	75-74-1	1000
Thionyl Chloride	7719-09-7	250
Trichloro (chloromethyl) Silane	1558-25-4	100
Trichloro (dichlorophenyl) Silane	27137-85-5	2500
Trichlorosilane	10025-78-2	5000
Trifluorochloroethylene	79-38-9	10,000
Trimethoxysilane	2487-90-3	1500

* Chemical abstract service number

** Threshold quantity in pounds (amount necessary to be covered by OSHA CFR 1910.119 standard)

8.2.1.5 Hazardous Classification Based on NFPA 70



Source: Reproduced with permission from NFPA 70®, *National Electric Code*®, © 2011, National Fire Protection Association. This is not the complete and official position of the NFPA on the referenced subject, which is represented only by the standard in its entirety. National Electrical Code and NEC are registered trademarks of the National Fire Protection Association, Quincy, MA.

Maximum Experimental Safe Gap (MESG): The maximum clearance between two parallel metal surfaces that has been found, under specified test conditions, to prevent an explosion in a test chamber from being propagated to a secondary chamber containing the same gas or vapor at the same concentration.

Minimum Igniting Current (MIC) Ratio: The ratio of the minimum current required from an inductive spark discharge to ignite the most easily ignitable mixture of a gas or vapor, divided by the minimum current required from an inductive spark discharge to ignite methane under the same test conditions.

8.2.1.6 Flammability

Flammable describes any solid, liquid, vapor, or gas that will ignite easily and burn rapidly. A flammable liquid is defined by NFPA and USDOT as a liquid with a flash point below 100°F (38°C). Flammability is further defined with lower and upper limits:

LFL = lower flammability limit (volume % in air)

UFL = upper flammability limit (volume % in air)

A vapor-air mixture will only ignite and burn over the range of concentrations between LFL and UFL. Flammability data is shown at the end of this chapter.

Predicting Lower Flammable Limits of Mixtures of Flammable Gases (Le Chatelier's Rule)

Based on an empirical rule developed by Le Chatelier, the lower flammable limit of mixtures of multiple flammable gases in air can be determined. A generalization of Le Chatelier's rule is

$$\sum_{i=1}^n \left(\frac{C_i}{LFL_i} \right) \geq 1$$

where

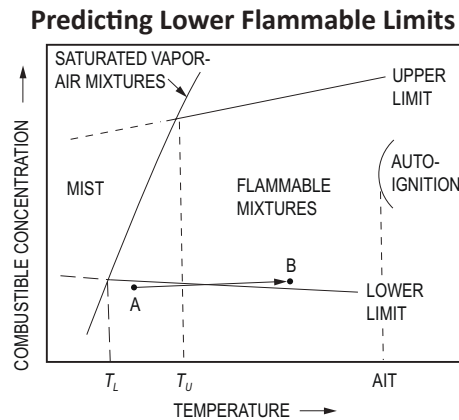
C_i = the volume percent of fuel gas i in the fuel/air mixture

LFL_i = the volume percent of fuel gas i at its lower flammable limit in air alone

If the indicated sum is greater than unity, the mixture is above the lower flammable limit. This can be restated in terms of the lower flammable limit concentration of the fuel mixture (LFL_m):

$$LFL_m = \frac{100}{\sum_{i=1}^n \left(\frac{C_{fi}}{LFL_i} \right)}$$

where C_{fi} = the volume percent of fuel gas i in the fuel gas mixture.



Source: DiNenno, Philip J., *The SFPE Handbook of Fire Protection Engineering*, 1st ed., Gaithersburg: Society of Fire Protection Engineers, 1988.

8.2.1.7 Fundamental Burning Velocities

Fundamental Burning Velocities of Selected Gases and Vapors

Gas	Fundamental Burning Velocity ($\frac{\text{cm}}{\text{s}}$)	Gas	Fundamental Burning Velocity ($\frac{\text{cm}}{\text{s}}$)
Acetone	54	Cyclobutane	67
Acetylene	166*	ethyl-	53
Acrolein	66	isopropyl-	46
Acrylonitrile	50	methyl-	52
Allene (propadiene)	87	Cyclohexane	46
Benzene	48	methyl-	44
<i>n</i> -butyl-	37	Cyclopentadiene	46
tert-butyl-	39	Cyclopentane	44
1,2-dimethyl-	37	methyl-	42
1,2,4-trimethyl-	39	Cyclopropane	56
1,2-Butadiene (methylallene)	68	cis-1,2-dimethyl-	55
1,3-Butadiene	64	trans-1,2-dimethyl-	55
2,3-dimethyl-	52	ethyl-	56
2-methyl-	55	methyl-	58
<i>n</i> -Butane	45	1,1,2-trimethyl-	52
2-cyclopropyl-	47	trans-Decalin (decahydronaphthalene)	36
2,2-dimethyl-	42	<i>n</i> -Decane	43
2,3-dimethyl-	43	1-Decene	44
2-methyl-	43	Diethyl ether	47
2,2,3-trimethyl-	42	Dimethyl ether	54
Butanone	42	Ethane	47
1-Butene	51	Ethene (ethylene)	80*
2-cyclopropyl-	50	Ethyl acetate	38
2,3-dimethyl	46	Ethylene oxide	108
2-ethyl-	46	Ethylenimine	46
2-methyl-	46	Gasoline (100-octane)	40
3-methyl-	49	<i>n</i> -Heptane	46
2,3-dimethyl-2-butene	44	Hexadecane	44
2-Buten 1-yne (vinylacetylene)	89	1,5-Hexadiene	52
1-Butyne	68	<i>n</i> -Hexane	46
3,3-dimethyl-	56	1-Hexene	50
2-Butyne	61	1-Hexyne	57
Carbon disulfide	58	3-Hexyne	53
Carbon monoxide	46	HFC-23 (Difluoromethane)	6.7

Fundamental Burning Velocities of Selected Gases and Vapors (cont'd)

Gas	Fundamental Burning Velocity ($\frac{\text{cm}}{\text{s}}$)	Gas	Fundamental Burning Velocity ($\frac{\text{cm}}{\text{s}}$)
HFC-143 (1,1,2-Trifluoroethane)	13.1	1-Pentene	50
HFC-143a (1,1,1-Trifluoroethane)	7.1	2-methyl-	47
HFC-152a (1,1-Difluoroethane)	23.6	4-methyl-	48
Hydrogen	312*	1-Pentene	63
Isopropyl alcohol	41	4-methyl-	53
Isopropylamine	31	cis-2-Pentene	51
Jet fuel, grade JP-1 (average)	40	2-Pentyne	61
Methane	40*	4-methyl-	54
diphenyl-	35	Propane	46*
Methyl alcohol	56	2-cyclopropyl-	50
Methylene	61	1-deutero-	40
1,2-Pentadiene (ethylallene)	61	1-deutero-2-methyl-	40
cis-1,3-Pentadiene	55	2-deutero-2-methyl-	40
trans-1,3-Pentadiene (piperylene)	54	2,2-dimethyl-	39
2-methyl-(cis or trans)	46	2-methyl-	41
1,4-Pentadiene	55	2-cyclopropyl-	53
2,3-Pentadiene	60	2-methyl-	44
n-Pentane	46	Propionaldehyde	58
2,2-dimethyl-	41	Propylene oxide (1,2-epoxypropane)	82
2,3-dimethyl-	43	1-Propyne	82
2,4-dimethyl-	42	Spiropentane	71
2-methyl-	43	Tetrahydropyran	48
3-methyl-	43	Tetralin (tetrahydronaphthalene)	39
2,2-trimethyl-	41	Toluene (methylbenzene)	41

* Gases that were critically examined as to their fundamental burning velocities, in studies by Andrews and Bradley (Andrews, G.E., and D. Bradley, "Determination of Burning Velocities: a Critical Review," *Combustion and Flame*, Vol. 18, New York: Elsevier Scientific Publishing Co., 1972, pp. 133–153) or by France and Pritchard (France, D.H., and R. Pritchard, "Burning Velocity Measurements of Multicomponent Fuel Gas Mixtures," *Gas Warnie International*, Vol. 26, No. 12, 1977).

Source: Reproduced with permission from NFPA 68, *Standard on Explosion Protection by Deflagration Venting*, © 2013, National Fire Protection Association. This is not the complete and official position of the NFPA on the referenced subject, which is represented only by the standard in its entirety.

The table below compares values from the Andrews/Bradley and France/Pritchard studies to those in the preceding table.

Comparison of Fundamental Burning Velocities for Selected Gases

Fundamental Burning Velocity ($\frac{\text{cm}}{\text{s}}$)				
Gas	From Table Above	Andrews and Bradley		France and Pritchard (in Air)
		In Air	In Oxygen	
Acetylene	166	158	1140	-
Ethylene	80	79	-	0
Hydrogen	312	310	1400	347
Methane	40	45	450	43
Propane	46	-	-	46

Flammability Properties of Gases
Gas Volume: 5L (0.005 m³) Sphere
Energy of the Ignition Source : E = 10J

Flammable Material	P_{max} (bar-g)
Acetophenone ^a	7.6
Acetylene	10.6
Ammonia ^b	5.4
β -Naphthol ^c	4.4
Butane	8.0
Carbon disulfide	6.4
Diethyl ether	8.1
Dimethyl formamide ^a	8.4
Dimethyl sulfoxide ^a	7.3
Ethane ^a	7.8
Ethyl alcohol	7.0
Ethyl benzene ^a	7.4
Hydrogen	6.8
Hydrogen sulfide	7.4
Isopropanol ^a	7.8
Methane	7.1
Methanol ^a	7.5
Methylene chloride	5.0
Methyl nitrite	11.4
Neopentane	7.8
Octanol ^a	6.7
Octyl chloride ^a	8.0

Flammability Properties of Gases
Gas Volume: 5L (0.005 m³) Sphere
Energy of the Ignition Source : E = 10J (cont'd)

Flammable Material	$P_{\max}$ (bar)
Pentane ^a	7.8
Propane	7.9
South African crude oil	6.8–7.6
Toluene ^a	7.8

- a. Measured at elevated temperatures and extrapolated to 25°C (77°F) at normal conditions
- b. E = 100J – 200J
- c. 200°C (392°F)

Source: Reproduced with permission from NFPA 68, *Standard on Explosion Protection by Deflagration Venting*, © 2013, National Fire Protection Association. This is not the complete and official position of the NFPA on the referenced subject, which is represented only by the standard in its entirety.

8.2.1.8 Combustible Dust

Combustible dust is a solid material composed of distinct particles or pieces, regardless of size, shape, or chemical composition, that presents a fire or deflagration hazard when suspended in air or some other oxidizing medium over a range of concentrations. Combustible dusts are often either organic or metal dusts that are finely ground into very small particles, fibers, fines, chips, chunks, flakes, or a small mixture of these.

According to OSHA's Safety and Health Information Bulletin (SHIB) "Combustible Dust in Industry: Preventing and Mitigating the Effects of Fire and Explosions," dust particles with an effective diameter of less than 420 microns (those passing through a U.S. No. 40 standard sieve) should be deemed to meet the criterion of the definition. However, larger particles can still pose a deflagration hazard (for instance, as larger particles are moved, they can abrade each other, creating smaller particles). In addition, particles can stick together (agglomerate) due to electrostatic charges accumulated through handling, causing them to become explosible when dispersed.

Types of dusts include, but are not limited to:

- Metal dust, such as aluminum and magnesium
- Wood dust
- Plastic or rubber dust
- Biosolids
- Coal dust
- Organic dust, such as flour, sugar, paper, soap, and dried blood
- Dusts from certain textiles

K_{st} is the *dust deflagration index*, which measures relative explosion severity compared to other dusts. The larger the value for K_{st} , the more severe the explosion. K_{st} provides the best "single number" estimate of the anticipated behavior of a dust deflagration.

MIE, the minimum ignition energy, predicts the ease and likelihood of ignition of a dispersed dust cloud.

MEC, the minimum explosible concentration, measures the minimum amount of dust dispersed in air required to spread an explosion. The MEC is analogous to the Lower Flammable Limit (LFL) or Lower Explosive Limit (LEL) for gases and vapors in air.

Examples of K_{st} Values for Different Types of Dusts

Dust Explosion Class*	$K_{st} \left(\text{bar} \cdot \frac{\text{m}}{\text{s}} \right)^*$	Characteristic	Typical materials**
St 0	0	No explosion	Silica
St 1	> 0 and ≤ 200	Weak explosion	Powered milk, charcoal, sulfur, sugar, zinc
St 2	> 200 and ≤ 300	Strong explosion	Cellulose, wood flour, poly methyl acrylate
St 3	> 300	Very strong explosion	Anthraquinone, aluminum, magnesium

* OSHA CPL 03-00-008 – *Combustible Dust National Emphasis Program*

** NFPA 68, *Standard on Explosion Prevention by Deflagration Venting*

Source: OSHA 3371-08: *Hazard Communication Guidance for Combustible Dust*: Occupational Safety and Health Administration, 2009, p. 8-9.

The actual class is sample-specific and will depend on varying characteristics of the material, such as particle size or moisture.

Source for next five tables: Reproduced with permission from NFPA 68, *Standard on Explosion Protection by Deflagration Venting*, © 2013, National Fire Protection Association. This is not the complete and official position of the NFPA on the referenced subject, which is represented only by the standard in its entirety.

Agricultural Products

Material	Mass Median Diameter (μm)	Minimum Flammable Concentration $\left(\frac{\text{g}}{\text{m}^3} \right)$	$P_{\max}$ (bar)	$K_{st} \left(\text{bar} \cdot \frac{\text{m}}{\text{s}} \right)$	Dust Hazard Class
Cellulose	33	60	9.7	229	2
Cellulose pulp	42	30	9.9	62	1
Cork	42	30	9.6	202	2
Corn	28	60	9.4	75	1
Egg white	17	125	8.3	38	1
Milk, powdered	83	60	5.8	28	1
Milk, nonfat, dry	60	–	8.8	125	1
Soy flour	20	200	9.2	110	1
Starch, corn	7	–	10.3	202	2
Starch, rice	18	60	9.2	101	1
Starch, wheat	22	30	9.9	115	1
Sugar	30	200	8.5	138	1
Sugar, milk	27	60	8.3	82	1
Sugar, beet	29	60	8.2	59	1
Tapioca	22	125	9.4	62	1
Whey	41	125	9.8	140	1
Wood Flour	29	–	10.5	205	2

Carbonaceous Dusts

Material	Mass Median Diameter (μm)	Minimum Flammable Concentration ($\frac{\text{g}}{\text{m}^3}$)	P_{max} (bar)	K_{st} ($\text{bar} \cdot \frac{\text{m}}{\text{s}}$)	Dust Hazard Class
Charcoal, activated	28	60	7.7	14	1
Charcoal, wood	14	60	9.0	10	1
Coal, bituminous	24	60	9.2	129	1
Coke, petroleum	15	125	7.6	47	1
Lampblack	<10	60	8.4	121	1
Lignite	32	60	10.0	151	1
Peat, 22% H_2O	–	125	84.0	67	1
Soot, pine	<10	–	7.9	26	1

Chemical Dusts

Material	Mass Median Diameter (μm)	Minimum Flammable Concentration ($\frac{\text{g}}{\text{m}^3}$)	P_{max} (bar)	K_{st} ($\text{bar} \cdot \frac{\text{m}}{\text{s}}$)	Dust Hazard Class
Adipic acid	<10	60	8.0	97	1
Anthraquinone	<10	–	10.6	364	3
Absorbic acid	39	60	9.0	111	1
Calcium acetate	92	500	5.2	9	1
Calcium acetate	85	250	6.5	21	1
Calcium stearate	12	30	9.1	132	1
Carbonyl- methyl- cellulose	24	125	9.2	136	1
Dextrin	41	60	8.8	106	1
Lactose	23	60	7.7	81	1
Lead stearate	12	30	9.2	152	1
Methyl cellulose	75	60	9.5	134	1
Paraformaldehyde	23	60	9.9	178	1
Sodium ascorbate	23	60	8.4	119	1
Sodium stearate	22	30	8.8	123	1
Sulfur	20	30	6.8	151	1

Metal Dusts

Material	Mass Median Diameter (μm)	Minimum Flammable Concentration ($\frac{\text{g}}{\text{m}^3}$)	P_{max} (bar)	K_{st} ($\text{bar} \cdot \frac{\text{m}}{\text{s}}$)	Dust Hazard Class
Aluminum	29	30	12.4	415	3
Bronze	18	750	4.1	31	1
Iron, carbonyl	<10	125	6.1	111	1
Magnesium	28	30	17.5	508	3
Phenolic resin	55	–	7.9	269	2
Zinc	10	250	6.7	125	1
Zinc	<10	125	7.3	176	1

Plastic Dusts

Material	Mass Median Diameter (μm)	Minimum Flammable Concentration ($\frac{\text{g}}{\text{m}^3}$)	P_{max} (bar)	K_{st} ($\text{bar} \cdot \frac{\text{m}}{\text{s}}$)	Dust Hazard Class
(poly) Acrylamide	10	250	5.9	12	1
(poly) Acrylonitrile	25	–	8.5	121	1
(poly) Ethylene (low-pressure process)	<10	30	8.0	156	1
Epoxy resin	26	30	7.9	129	1
Melamine resin	18	125	10.2	110	1
Melamine, molded (wood flour and mineral filled phenol-formaldehyde)	15	60	7.5	41	1
Melamine, molded (phenol-cellulose)	12	60	10.0	127	1
(poly) Methyl acrylate	21	30	9.4	269	2
(poly) Methyl acrylate, emulsion polymer	18	30	10.1	202	2
Phenolic resin	<10	15	9.3	129	1
(poly) Propylene	25	30	8.4	101	1
Terpene-phenol resin	10	15	8.7	143	1
Urea-formaldehyde/cellulose, molded	13	60	10.2	136	1
(poly) Vinyl acetate/ethylene copolymer	32	30	8.6	119	1
(poly) Vinyl alcohol	26	60	8.9	128	1
(poly) Vinyl butyral	65	30	8.9	147	1
(poly) Vinyl chloride	107	200	7.6	46	1
(poly) Vinyl chloride/vinyl acetylene emulsion copolymer	35	60	8.2	95	1
(poly) Vinyl chloride/ethylene/vinyl acetylene suspension copolymer	60	60	8.3	98	1

Ignition and Reaction Temperatures of Metal-Powder Layers in an Air, Carbon Dioxide, or Nitrogen Atmosphere

Line No.	Sample No.	Material	Ignition Temperature, °C			Reaction Temperature, °C	
			Air	Carbon Dioxide	Nitrogen	Carbon Dioxide	Nitrogen
35	701	Aluminum, atomized	900	--	--	900	750
21	897	Aluminum, atomized	490	540	--	--	800
80	702	Aluminum, flake	590	660	--	--	700
102	705	Chromium	670	--	--	⁽¹⁾	700
110	706	Copper	270	--	--	--	700
120	712	Iron, hydrogen-reduced	290	--	--	⁽¹⁾	200
136	716	Lead	210	--	--	870	400
151	725	Magnesium	490	630	530	--	--
154	727	Magnesium	510	--	510	--	--
156	729	Magnesium	520	--	520	--	--
157	730	Magnesium	490	--	500	--	--
159	1020	Magnesium	490	600	550	--	--
164	734	Magnesium	480	--	490	--	--
165	733	Magnesium	480	--	500	--	--
168	736	Magnesium	420	--	--	700	900
176	737	Silicon	950	--	--	1000	1000
190	1652	Thorium	280	450	500	--	--
191	1653	Thorium hydride	20	340	330	--	--
193	739	Tin	430	--	900	720	--
195	1555	Titanium	480	900	900	--	--
198	740	Titanium	460	680	--	--	900
199	1556	Titanium, copper-coated	430	900	900	--	--
201	864	Titanium	470	470	500	--	--
204	1649	Titanium hydride	500	710	750	--	--
211	1625	Uranium	100	350	410	--	--
212	1626	Uranium hydride	20	360	210	--	--
217	744	Zinc	460	480	--	--	600
225	745	Zirconium	210	560	530	--	--
228	1632	Zirconium	190	620	790	--	--
229	1633	Zirconium	300	710	--	--	--
233	1627	Zirconium hydride	340	650	--	--	--
247	1021	Aluminum-magnesium	460	660	550	--	--
248	746	Aluminum-magnesium	470	700	550	--	--
249	748	Aluminum-magnesium	480	670	630	--	--

¹ No reaction at 850°C

Source: Jacobson, M., A.R. Cooper, and J. Nagy, *Explosibility of Metal Powders*, Bureau of Mines Report of Investigations 6516, Washington, D.C.: United States Department of the Interior, Bureau of Mines, 1964.

8.2.1.9 Hazardous Waste

Characteristic Hazardous Waste

A waste is a *characteristic waste* if it meets any of the characteristics identified in 40 CFR 261 Subpart C (D code waste).

Hazardous Waste Characteristics

Characteristic (D Code) [Subpart #]	Definition
Ignitability (D001) [40 CFR 261.21]	(1) A liquid (other than an aqueous solution containing <24% alcohol by volume) that has flash point <140°F [Method Pensky-Martens or Setaflash]. (2) A nonliquid that is capable (under STP) of causing fire through friction, absorption of moisture, or spontaneous chemical changes and, when ignited, burns so vigorously and persistently that it creates a hazard. (3) An ignitable compressed gas.
Corrosivity (D002) [40 CFR 261.22]	(1) An aqueous solution with a pH ≤ 2 or ≥ 12.5 [Method 9040C in SW-846]. (2) A liquid that corrodes steel (SAE 1020) at a rate of $> 1/4$ inch per year at a test temperature of 130°F [Method 1110A in SW-846].
Reactivity (D003) [40 CFR 261.23]	(1) Normally unstable and readily undergoes violent change without detonating. (2) Reacts violently with water. (3) Forms potentially explosive mixtures with water. (4) When mixed with water, generates toxic gases, vapors, or fumes in a quantity sufficient to present a danger to human health or the environment. (5) A cyanide- or sulfide-bearing waste that, when exposed to pH conditions between 2 and 12.5, can generate toxic gases, vapors, or fumes in a quantity sufficient to present a danger to human health or the environment. (6) Capable of detonation or explosive reaction if subjected to a strong initiating source or if heated under confinement. (7) Readily capable of detonation or explosive decomposition or reaction at standard temperature and pressure. (8) A forbidden explosive as defined in 49 CFR 173.54, or a Division 1.1, 1.2, or 1.3 explosive as defined in 49 CFR 173.50 and 173.53.
Toxicity (D004 to D043) [40 CFR 261.24]	A waste that contains constituents above the regulatory threshold listed in Table 1 of 40 CFR 261.24 using the Toxicity Characteristic Leaching Procedure (TCLP) test [Method 1311 in SW846]. Constituents: arsenic, barium, benzene, cadmium, carbon tetrachloride, chlordane, chlorobenzene, chloroform, chromium, o-cresol, m-cresol, p-cresol, total cresols, 2,4-D, 1,4-dichlorobenzene, 1,2-dichloroethane, 1,1-dichloroethylene, 2,4-dinitrotoluene, endrin, heptachlor (and its epoxide), hexachlorobenzene, hexachlorobutadiene, hexachloroethane, lead, lindane, mercury, methoxychlor, methyl ethyl ketone, nitrobenzene, pentachlorophenol, pyridine, selenium, silver, tetrachloroethylene, toxaphene, trichloro-ethylene, 2,4,5-trichlorophenol, 2,4,6-trichlorophenol, 2,4,5-TP (silvex), and vinyl chloride.

Hazardous Waste Compatibility Chart

Source: Hatayama, H.K., et. al., *A Method for Determining the Compatibility of Hazardous Wastes*, Cincinnati: U.S. Environmental Protection Agency, 1980.

8.2.1.10 Elements of Process Safety Management (PSM)

The U.S. Occupational Safety and Health Administration (OSHA) 1910.119 defines all 14 elements of a process safety management plan:

1. **Employee Participation**—Consult with employees and their representatives on the development and conduct of hazard assessments and the development of chemical accident prevention plans, and provide access to these and other records required under the standard.
2. **Process Safety Information**—Develop and maintain written safety information identifying workplace chemical and process hazards, equipment used in the processes, and technology used in the processes.
3. **Process Hazard Analysis**—Perform a workplace hazard assessment including, as appropriate, identification of potential sources of accidental releases, identification of any previous release within the facility that had a potential for catastrophic consequences in the workplace, estimation of workplace effects of a range of releases, and estimation of the health and safety effects of such a range on employees. Establish a system to respond to the workplace hazard assessment findings, which shall address prevention, mitigation, and emergency responses.
4. **Operating Procedures**—Develop and implement written operating procedures for the chemical processes, including procedures for each operating phase, operating limitations, and safety and health considerations.
5. **Training**—Provide written safety and operating information for employees and employee training in operating procedures, by emphasizing hazards and safe practices that must be developed and made available.
6. **Contractors**—Ensure contractors and contract employees are provided with appropriate information and training.
7. **Pre-startup Safety Review**—Conduct pre-startup safety reviews of all newly installed or modified equipment.
8. **Mechanical Integrity**—Establish maintenance systems for critical process-related equipment, including written procedures, employee training, appropriate inspections, and testing of such equipment to ensure ongoing mechanical integrity. Establish a quality-assurance program to ensure that initial process-related equipment, maintenance materials, and spare parts are fabricated and installed consistent with design specifications.
9. **Hot-Work Permit**—A permit must be issued for hot-work operations conducted on or near a covered process. The permit must document that the fire prevention and protection requirements have been implemented prior to beginning the hot-work operations; it must indicate the date(s) authorized for hot work and identify the object on which hot work is to be performed. The permit must be kept on file until completion of the hot work.
10. **Management of Change**—Establish and implement written procedures managing change to process chemicals, technology, equipment, and facilities.
11. **Incident Investigation**—Investigate every incident that results in or could have resulted in a major accident in the workplace, with any findings to be reviewed by operating personnel and modifications made if appropriate.
12. **Emergency Planning and Response**—Develop and implement an emergency action plan for the entire plant in accordance with the provisions of other OSHA rules. Include in the emergency action plan procedures for handling small releases of hazardous chemicals.
13. **Compliance Audits**—Employers must certify that they have evaluated compliance with the provisions of PSM at least every three years. This will verify that the procedures and practices developed under the standards are adequate and are being followed.
14. **Trade Secrets**—Employers must make available all information necessary to comply with PSM to those persons responsible for compiling the process safety information, those developing the process hazard analysis, those responsible for developing the operating procedures, and those performing incident investigations, emergency planning and response, and compliance audits, without regard to the possible trade-secret status of such information.

8.2.2 Hazard Management

Commitment to process safety	Process safety culture Compliance with standards Workforce involvement Stakeholder outreach
Understanding hazards and risk	Process knowledge management Hazards identification and risk analysis
Managing risk	Operating procedures Safe work practices Asset integrity and reliability Contractors management Training and performance assurance Management of change Operational readiness Conduct of operations Emergency management
Learning from experience	Incident investigation Measurements and metrics Auditing Management review and continuous improvement

8.2.2.1 Management of Change

Management of Change (MOC)—A management system to identify, review, and approve all modifications to equipment, procedures, raw materials, and processing conditions, other than replacement in kind, prior to implementation, to help ensure that changes to processes are properly analyzed (for example, for potential adverse impacts), documented, and communicated to affected employees.

Key Principles:

- Maintain a dependable practice
- Identify potential change situations
- Evaluate possible impacts
- Decide whether to allow the change
- Complete follow-up activities

8.2.2.2 Process Hazard Analysis

Terms and Definitions for Hazards Identification and Management

Term	Definition
Alarm	An audible and/or visible means of indication to the operator an equipment malfunction, process deviation, or abnormal condition requiring a timely response.
As Low as Reasonably Practicable (ALARP)	The concept that efforts to reduce risk should be continued until the incremental sacrifice (in terms of cost, time, effort, or other expenditure of resources) is grossly disproportionate to the incremental risk reduction achieved. The term as low as reasonably achievable (ALARA) is often used synonymously.

Terms and Definitions for Hazards Identification and Management (cont'd)

Term	Definition
Consequences	The direct, undesirable result of an accident sequence usually involving a fire, explosion, or release of toxic material. Consequence descriptions may be qualitative or quantitative estimates of the effects of an accident.
Frequency	Number of occurrences of an event per unit time $\left(\text{e.g., } 1 \text{ event in } 1000 \text{ yr.} = 1 \times 10^{-3} \frac{\text{events}}{\text{yr}} \right).$
Failure Mode and Effect Analysis (FMEA)	A hazard identification technique in which all known failure modes of components or features of a system are considered in turn and undesired outcomes are noted. It is usually used in combination with fault tree analysis. It is a complicated procedure, usually carried out by experienced risk analysts.
Hazards and Operability Analysis (HAZOP)	A systematic qualitative technique to identify process hazards and potential operating problems using a series of guide words to study process deviations. A HAZOP is used to question every part of a process to discover what deviations from the intention of the design can occur and what their causes and consequences may be. This is done systematically by applying suitable guide words. This is a systematic detailed review technique, for both batch and continuous plants, which can be applied to new or existing processes to identify hazards.
Independent Protection Layer (IPL)	A device, system, or action that is capable of preventing a postulated accident sequence from proceeding to a defined, undesirable endpoint. An IPL is independent of the event that initiated the accident sequence and independent of any other IPLs. IPLs are normally identified during layer of protection analysis.
Initiating Event	The minimum combination of failures or errors necessary to start the propagation of an incident sequence. It can be comprised of a single initiating cause, multiple causes, or initiating causes in the presence of enabling conditions. (The term initiating event is the usual term employed in Layer of Protection Analysis to denote an initiating cause or where appropriate, an aggregation of initiating causes with the same immediate effect, such as "BPCS failure resulting in high reactant flow.")
Layer of Protection Analysis (LOPA)	An approach that analyzes one incident scenario (cause-consequence pair) at a time, using predefined values for the initiating event frequency, independent protection layer failure probabilities, and consequence severity, in order to compare a scenario risk estimate to risk criteria for determining where additional risk reduction or more detailed analysis is needed. Scenarios are identified elsewhere, typically using a scenario-based hazard evaluation procedure such as the HAZOP study.
Lock-Out/Tag-Out (LOTO)	Specific practices and procedures to safeguard employees from the unexpected energization or startup of machinery and equipment, or the release of hazardous energy during service or maintenance activities.
Process Hazard Analysis (PHA)	An organized effort to identify and evaluate hazards associated with processes and operations to enable their control. This review normally involves the use of qualitative techniques to identify and assess the significance of hazards. Conclusions and appropriate recommendations are developed. Occasionally, quantitative methods are used to help prioritize risk reduction.
Process Safety Management (PSM)	A management system that is focused on prevention of, preparedness for, mitigation of, response to, and restoration from catastrophic releases of chemicals or energy from a process associated with a facility.

Terms and Definitions for Hazards Identification and Management (cont'd)

Term	Definition
Quantitative Risk Analysis (QRA)	QRA is a technique that provides advanced quantitative means to supplement other hazard identification, analysis, assessment, control, and management methods to identify the potential for such incidents and to evaluate risk reduction and control strategies. QRA identifies those areas where operation, engineering, or management systems may be modified to reduce risk and may identify the most economical way to do it. The primary goal of QRA is that appropriate management actions, based on results from a QRA study, help to make facilities handling hazardous chemicals safer. QRA is one component of an organization's total process risk management. It allows the quantitative assessment of risk alternatives that can be balanced against other considerations.
Qualitative Risk Analysis (QRA)	The systematic development of numerical estimates of the expected frequency and/or consequence of potential accidents associated with a facility or operation. Using consequence and probability analyses and other factors such as population density and expected weather conditions, QRA predicts the fatality rate for a given event. This methodology is useful for evaluation of alternatives, but its value as an absolute measure of risk should be considered carefully.
Risk	A measure of human injury, environmental damage, or economic loss in terms of both the incident likelihood and the magnitude of the loss or injury. A simplified version of this relationship expresses risk as the product of the likelihood and the consequences (i.e., Risk = Consequence x Likelihood) of an incident.

8.2.2.3 Layers of Protection

Independent protective layer (IPL) requirements:

- Effective: Must be capable of preventing the undesired event all by itself
- Independent:
 - Independent of the initiating event
 - Sensor, logic solver, and final acting element are all independent of any other sensor, logic solver, and final acting element protecting against the same undesired event
- Auditable: Must be able to check that the IPL has not failed in the dangerous (silent) mode

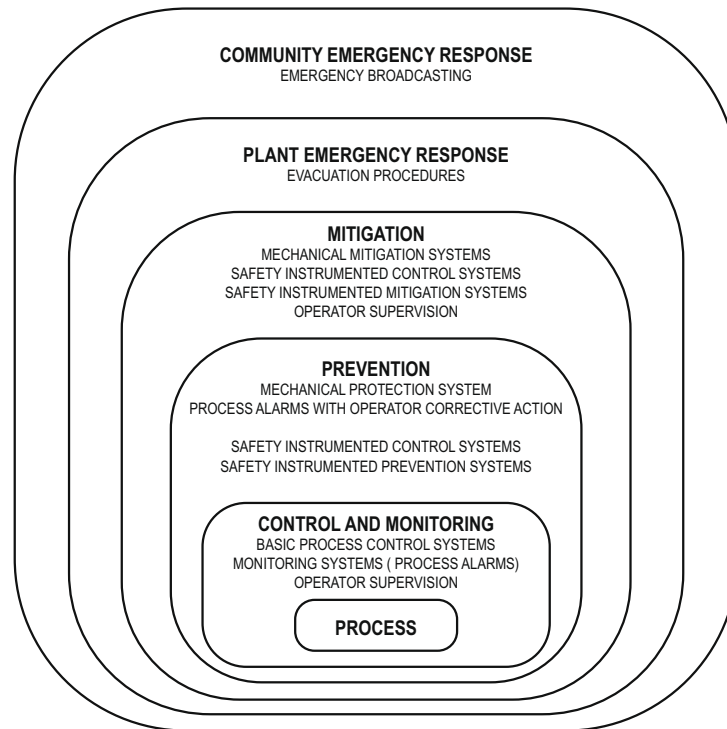
IPLs may be

1. An alarm followed by operator intervention
2. A basic process control system (BPCS) continuously modulating control loop, independent of the cause of the undesired event
3. A safety instrumented function (SIF)
4. A mechanical device; for example, a pressure relief valve

IPL	Probability of Failure on Demand (PFD)
Alarm and operator intervention, routine task, not stressed	Once every 10 to 100 years
BPCS control loop	Once every 10 to 100 years
Pressure relief valve, non-fouling	Typically, once every 100 to 1,000 years
SIF	Greater than every 10 to 100,000 years

Risk reduction from IPLs is cumulative. For example, if layers of protection analysis (LOPA) requires 3 IPLs, this can be accomplished with one SIL-3 SIF, or with one alarm and operator intervention plus one BPCS control loop plus one SIL-1 SIF. Many other combinations of IPLs can be used to meet this requirement. PFD for SIFs are defined further in the Safety Integrity Levels: Probability of Failure on Demand section.

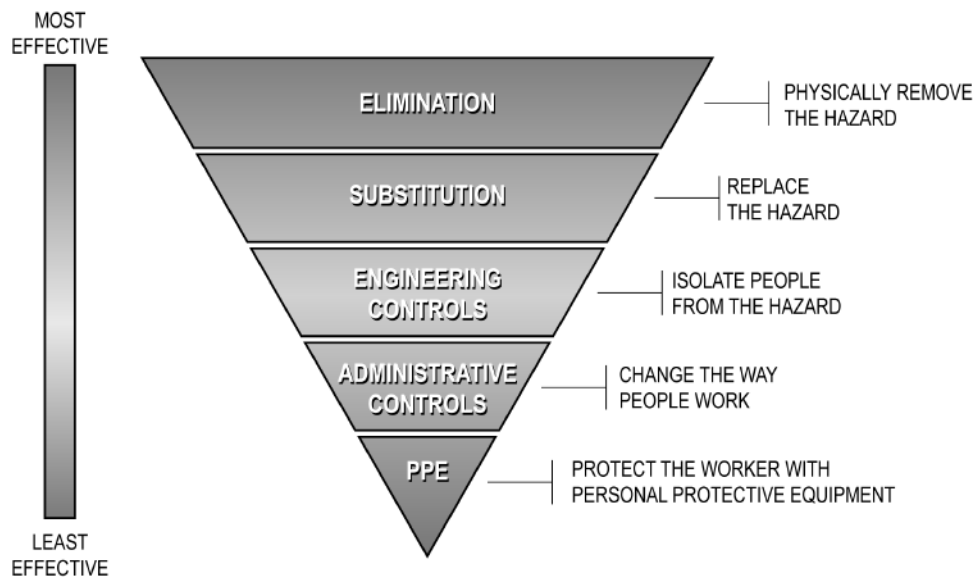
Typical Risk Reduction Methods Found in Process Plants



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Hierarchy of Controls



Source: *Controls for Noise Exposure*, Atlanta: The National Institute for Occupational Safety and Health (NIOSH), 2016.

8.2.3 Hazard Communication

8.2.3.1 Safety Data Sheets (SDS)

Source: Appendix D to OSHA CFR 1910.1200 - Safety Data Sheets (Mandatory).

A safety data sheet (SDS) must include the information in the table below under the section number and heading indicated for Sections 1–11 and 16. If no relevant information is found for any given subheading within a section, the SDS must clearly indicate that no applicable information is available. Sections 12–15 may be included in the SDS, but are not mandatory.

Minimum Information for a Safety Data Sheet

	Heading	Subheading
1.	Identification	a. Product identifier used on the label b. Other means of identification c. Recommended use of the chemical and restrictions on use d. Name, address, and telephone number of the chemical manufacturer, importer, or other responsible party e. Emergency phone number
2.	Hazard(s) Identification	a. Classification on the chemical in accordance with paragraph (d) of §1910.1200 b. Signal word, hazard statement(s), symbol(s), and precautionary statement(s) in accordance with paragraph (f) of §1910.1200. (Hazard symbols may be provided as graphical reproductions in black and white or the name of the symbol, e.g., flame, skull and crossbones) c. Describe any hazards not otherwise classified that have been identified during the classification process d. Where an ingredient with unknown acute toxicity is used in a mixture at a concentration = 1% and the mixture is not classified based on testing of the mixture as a whole, a statement that X % of the mixture consists of ingredient(s) of unknown acute toxicity is required

Minimum Information for a Safety Data Sheet (cont'd)

	Heading	Subheading
3.	Composition and Information on Ingredients	Except as provided for in paragraph (i) of §1910.1200 on trade secrets: For Substances - Chemical name - Common name and synonyms - CAS number and other unique identifiers - Impurities and stabilizing additives that are themselves classified and that contribute to the classification of the substance For Mixtures In addition to the information required for substances: - The chemical name and concentration (exact percentage) or concentration ranges of all ingredients that are classified as health hazards in accordance with paragraph (d) of §1910.1200 and either - Are present above their cut-off/concentration limits - Present a health risk below the cut-off/concentration limits - The concentration (exact percentage) shall be specified unless a trade secret claim is made in accordance with paragraph (i) of §1910.1200, when there is batch-to-batch variability in the production of a mixture, or for a group of substantially similar mixtures (See A.0.5.1.2) with similar chemical composition. In these cases, concentration ranges may be used. For All Chemicals for Which a Trade Secret Is Claimed When a trade secret is claimed in accordance with paragraph (i) of §1910.1200, a statement that the specific chemical identity and/or exact percentage (concentration) of composition has been withheld as a trade secret is required.
4.	First-aid Measures	- Description of necessary measures, subdivided according to the different routes of exposure, i.e., inhalation, skin and eye contact, and ingestion - Most important symptoms/ effects, acute and delayed - Indication of immediate medical attention and special treatment needed, if necessary
5.	Fire-fighting Measures	- Suitable (and unsuitable) extinguishing media - Specific hazards arising from the chemical (e.g., nature of any hazardous combustion products) - Special protective equipment and precautions for fire-fighters
6.	Accidental Release Measures	- Personal precautions, protective equipment, and emergency procedures - Methods and materials for containment and cleaning up
7.	Handling and Storage	- Precautions for safe handling - Conditions for safe storage, including any incompatibilities
8.	Exposure Controls and Personal Protection	- OSHA permissible exposure limit (PEL), American Conference of Governmental Industrial Hygienists' (ACGIH) threshold limit value (TLV), and any other exposure limit used or recommended by the chemical manufacturer, importer, or employer preparing the safety data sheet, where available - Appropriate engineering controls - Individual protection measures, such as personal protective equipment

Minimum Information for a Safety Data Sheet (cont'd)

	Heading	Subheading
9.	Physical and Chemical Properties	a. Appearance (physical state, color, etc.) b. Odor c. Odor threshold d. pH e. Melting point/freezing point f. Initial boiling point and boiling range g. Flash point h. Evaporation rate i. Flammability (solid, gas) j. Upper/lower flammability or explosive limits k. Vapor pressure l. Vapor density m. Relative density n. Solubility(ies) o. Partition coefficient: <i>n</i>-octanol/water p. Auto-ignition temperature q. Decomposition temperature r. Viscosity
10.	Stability and Reactivity	a. Reactivity b. Chemical stability c. Possibility of hazards reactions d. Conditions to avoid (e.g., static discharge, shock, or vibration) e. Incompatible materials f. Hazardous decomposition products
11.	Toxicological Information	Description of the various toxicological (health) effects and the available data used to identify those effects, including: a. Information on the likely routes of exposure (inhalation, ingestion, skin and eye contact) b. Symptoms related to the physical, chemical, and toxicological characteristics c. Delayed and immediate effects and also chronic effects from short- and long-term exposure d. Numerical measures of toxicity (such as acute toxicity estimates) e. Whether the hazardous chemical is listed in the National Toxicology Program (NTP) Report on Carcinogens (latest edition) or has been found to be a potential carcinogen in the International Agency for Research on Cancer (IARC) Monographs (latest editions), or by OSHA
12.	Ecological Information (Non-mandatory)	a. Ecotoxicity (aquatic and terrestrial, where available) b. Persistence and degradability c. Bioaccumulative potential d. Mobility in soil e. Other adverse effects (such as hazardous to the ozone layer)
13.	Disposal Considerations (Non-mandatory)	Description of waste residues and information on their safe handling and method of disposal, including disposal of any contaminated packaging.

Minimum Information for a Safety Data Sheet (cont'd)

	Heading	Subheading
14.	Transport Information (Non-mandatory)	a. UN number b. UN proper shipping name c. Transport hazard class(es) d. Packing group, if applicable e. Environmental hazards (e.g., Marine pollutant [Yes/No]) f. Transport in bulk (according to Annex II of MARPOL 73/78 and IBC Code) g. Special precautions which a user needs to be aware of, or needs to comply with, in connection with transport or conveyance either within or outside their premises
15.	Regulatory Information (Non-mandatory)	Safety, health, and environmental regulations specific for the product in question
16.	Other Information, Including Date of Preparation or Last Revision	The date of preparation of the SDS or the last change to it

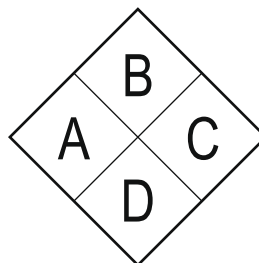
Source: Hazard Communication, "Safety Data Sheets," Washington, D.C.: Occupational Safety and Health Administration.

8.2.3.2 NFPA Hazard Identification System

The fire/hazard diamond below summarizes common hazard data available on the Safety Data Sheet (SDS) and is frequently shown on chemical labels.

Position A – Health Hazard (Blue)

- 0 = Normal material
- 1 = Slightly hazardous
- 2 = Hazardous
- 3 = Extreme danger
- 4 = Deadly



Position B – Flammability (Red)

- 0 = Will not burn
- 1 = Will ignite if preheated
- 2 = Will ignite if moderately heated
- 3 = Will ignite at most ambient temperature
- 4 = Burns readily at ambient conditions

Position C – Reactivity (Yellow)

- 0 = Stable and not reactive with water
- 1 = Unstable if heated
- 2 = Violent chemical change
- 3 = Shock short may detonate
- 4 = May detonate

Position D – (White)

- ALKALI = Alkali
- OXY = Oxidizer
- ACID = Acid
- Cor = Corrosive
- W = Use no water



= Radiation

8.2.3.3 OSHA Globally Harmonized System (GHS)

The *Globally Harmonized System of Classification and Labeling of Chemicals*, or GHS, is a system for standardizing and harmonizing the classification and labeling of chemicals.

GHS is a comprehensive approach to:

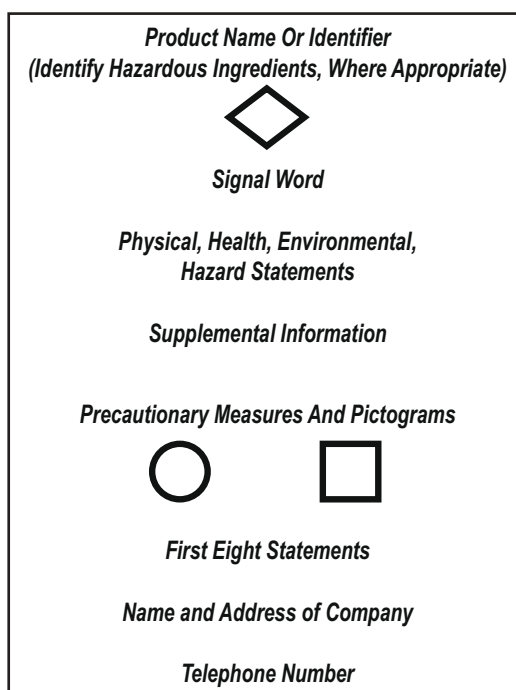
- Defining health, physical, and environmental hazards of chemicals
- Creating classification processes that use available data on chemicals for comparison with the defined hazard criteria
- Communicating hazard information, as well as protective measures, on labels and Safety Data Sheets (SDSs), formerly called Material Safety Data Sheets (MSDSs).

GHS label elements include:

- Precautionary statements and pictograms: Measures to minimize or prevent adverse effects
- Product identifier (ingredient disclosure): Name or number used for a hazardous product on a label or in the SDS
- Supplier identification: The name, address, and telephone number of the supplier
- Supplemental information: nonharmonized information

Other label elements include symbols, signal words, and hazard statements.

GHS Label Elements



Note: Pictograms for hazard statements must have red borders.

Source: *A Guide to the Globally Harmonized System of Classification and Labeling of Chemicals (GHS)*, Washington, D.C.: Occupational Safety and Health Administration, 2009, p. 38.

HCS Quick Card



Hazard Communication Standard Labels

OSHA has updated the requirements for labeling of hazardous chemicals under its Hazard Communication Standard (HCS). All labels are required to have pictograms, a signal word, hazard and precautionary statements, the product identifier, and supplier identification. A sample revised HCS label, identifying the required label elements, is shown on the right. Supplemental information can also be provided on the label as needed.



For more information:



Occupational Safety and Health Administration

www.osha.gov (800) 321-OSHA (6742)

SAMPLE LABEL

CODE _____
Product Name _____

Company Name _____
Street Address _____
City _____ State _____
Postal Code _____ Country _____
Emergency Phone Number _____

Product Identifier

Supplier Identification

Hazard Pictograms

Signal Word
Danger

Hazard Statements

Precautionary Statements

Supplemental Information

Directions for Use _____

Fill weight: _____ Lot Number: _____
Gross weight: _____ Fill Date: _____
Expiration Date: _____

Keep container tightly closed. Store in a cool, well-ventilated place that is locked.
Keep away from heat/sparks/open flame. No smoking.
Only use non-sparking tools.
Use explosion-proof electrical equipment.
Take precautionary measures against static discharge.
Ground and bond container and receiving equipment.
Do not breathe vapors.
Wear protective gloves.
Do not eat, drink or smoke when using this product.
Wash hands thoroughly after handling.
Dispose of in accordance with local, regional, national, international regulations as specified.

In Case of Fire: use dry chemical (BC) or Carbon Dioxide (CO₂) fire extinguisher to extinguish.

First Aid
If exposed call Poison Center.
If on skin (or hair): Take off immediately any contaminated clothing. Rinse skin with water.

Highly flammable liquid and vapor.
May cause liver and kidney damage.

Note: Pictograms for hazard statements must have red borders.

Source: *A Guide to the Globally Harmonized System of Classification and Labeling of Chemicals (GHS)*, Washington, D.C.: Occupational Safety and Health Administration, 2009, p. 40.

HCS Pictograms and Hazards Card

Health Hazard  • Carcinogen • Mutagenicity • Reproductive Toxicity • Respiratory Sensitizer • Target Organ Toxicity • Aspiration Toxicity	Flame  • Flammables • Pyrophorics • Self-Heating • Emits Flammable Gas • Self-Reactives • Organic Peroxides	Exclamation Mark  • Irritant (skin and eye) • Skin Sensitizer • Acute Toxicity (harmful) • Narcotic Effects • Respiratory Tract Irritant • Hazardous to Ozone Layer (Non-Mandatory)
Gas Cylinder  • Gases Under Pressure	Corrosion  • Skin Corrosion/ Burns • Eye Damage • Corrosive to Metals	Exploding Bomb  • Explosives • Self-Reactives • Organic Peroxides
Flame Over Circle  • Oxidizers	Environment (Non-Mandatory)  • Aquatic Toxicity	Skull and Crossbones  • Acute Toxicity (fatal or toxic)
For more information:  Occupational Safety and Health Administration U.S. Department of Labor www.osha.gov (800) 321-OSHA (6742) OSHA 3491-02 2012		

Note: Pictograms for hazard statements must be printed with red borders.

Source: *A Guide to the Globally Harmonized System of Classification and Labeling of Chemicals (GHS)*, Washington, D.C., Occupational Safety and Health Administration.

	Acute Oral Toxicity				
LD ₅₀	CATEGORY 1 ≤ 5 mg/kg	CATEGORY 2 > 5 < 50 mg/kg	CATEGORY 3 50 < 300 mg/kg	CATEGORY 4 300 < 2000 mg/kg	CATEGORY 5 2000 > 5000 mg/kg
PICTOGRAM					NO SYMBOL
SIGNAL WORD	DANGER	DANGER	DANGER	WARNING	WARNING
HAZARD STATEMENT	FATAL IF SWALLOWED	FATAL IF SWALLOWED	TOXIC IF SWALLOWED	HARMFUL IF SWALLOWED	MAY BE HARMFUL IF SWALLOWED

Source: *A Guide to the Globally Harmonized System of Classification and Labeling of Chemicals (GHS)*, Washington, D.C.: Occupational Safety and Health Administration, 2009, p. 41.

8.2.4 Personal Safety and Industrial Hygiene

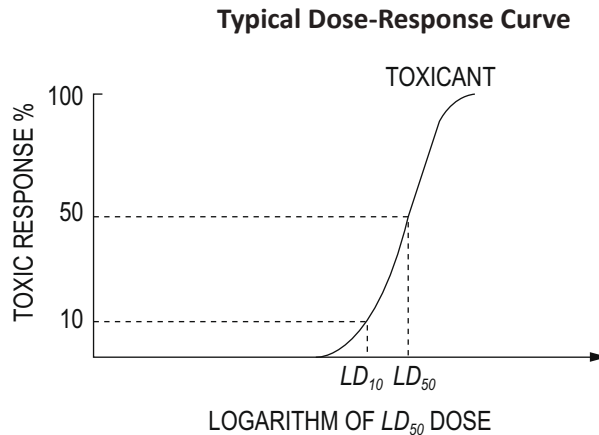
8.2.4.1 Industrial Hygiene

Personal protective equipment (PPE) is designed to protect employees from serious injuries or illnesses resulting from contact with chemical, radiological, physical, electrical, mechanical, or other workplace hazards. Besides face shields, safety glasses, hard hats, and safety shoes, PPE includes a variety of devices and garments, such as goggles, coveralls, gloves, vests, earplugs, and respirators.

8.2.4.2 Toxicology

The Dose-Response Curve

The dose-response curve relates toxic response (i.e., percentage of test population exhibiting a specified symptom or dying) to the logarithm of the dosage (i.e., $\frac{\text{mg}}{\text{kg} \cdot \text{day}}$ ingested). A typical dose-response curve is shown below.



where

LC_{50} = Median lethal concentration in air that, based on laboratory tests, is expected to kill 50% of a group of test animals when administered as a single exposure over one or four hours.

LD_{50} = Median lethal single dose, based on laboratory tests, expected to kill 50% of a group of test animals, usually by oral or skin exposure.

Similar definitions exist for LC_{10} and LD_{10} , where the corresponding percentages are 10%.

Chapter 8: Plant Design and Operations

The following table lists the LD_{50} values for several chemicals:

Comparative Acutely Lethal Doses

Actual Ranking Number	$LD_{50} \left(\frac{mg}{kg} \right)$	Toxic Chemical
1	15,000	PCBs
2	10,000	Alcohol (ethanol)
3	4000	Table salt—sodium chloride
4	1500	Ferrous sulfate—an iron supplement
5	1375	Malathion—a pesticide
6	900	Morphine
7	150	Phenobarbital—a sedative
8	142	Tylenol (acetaminophen)
9	2	Strychnine
10	1	Nicotine
11	0.5	Curare—an arrow poison
12	0.001	2,3,7,8-TCDD (dioxin)
13	0.00001	Botulinum toxin (food poison)

Adapted from Loomis, T.A., and A.W. Hayes. *Loomis's Essentials of Toxicology*, 4th ed., San Diego: Academic Press, 1996.

Selected Chemical Interaction Effects

Effect	Relative toxicity (hypothetical)	Example
Additive	$2 + 3 = 5$	Organophosphate pesticides
Synergistic	$2 + 3 = 20$	Cigarette smoking + asbestos
Antagonistic	$6 + 6 = 8$	Toluene + benzene or caffeine + alcohol

Adapted from Williams, P.L., R.C. James, and S.M. Roberts. *Principles of Toxicology: Environmental and Industrial Applications*, 2nd ed., New York: John Wiley & Sons, Inc., 2000.

Exposure Limits for Selected Compounds^(a)

Substance	CAS No. ^(c)	Regulatory Limits		Recommended Limits	
		OSHA PEL ^(b)		NIOSH REL ^(g) (as of 4/26/13)	ACGIH® 2015 TLV® ^(h)
		ppm ^(d)	$\frac{\text{mg}}{\text{m}^3}$ ^(e)	Up to 10-hour TWA, (ST) STEL, (C) Ceiling ^(f)	8-hour TWA, (ST) STEL, (C) Ceiling
Acetic acid	64-19-7	10	25	10 ppm (ST) 15 ppm	10 ppm (ST) 15 ppm
Acetone	67-64-1	1000	2400	250 ppm	250 ppm (ST) 500 ppm
Benzoyl peroxide	94-36-0		5	$5 \frac{\text{mg}}{\text{m}^3}$	$5 \frac{\text{mg}}{\text{m}^3}$
Bromine	7726-95-6	0.1	0.7	0.1 ppm (ST) 0.3 ppm	0.1 ppm (ST) 0.2 ppm
Butyl mercaptan	109-79-5	10	35	(C) 0.5 ppm [15-min]	0.5 ppm
Carbon dioxide	124-38-9	5000	9000	5000 ppm (ST) 30,000 ppm	5000 ppm (ST) 30,000 ppm
Carbon monoxide	630-08-0	50	55	35 ppm (C) 200 ppm	25 ppm
Chlorine	7782-50-5	(C) 1	(C) 3	(C) 0.5 ppm [15-min]	0.5 ppm (ST) 1 ppm
Chloroform (trichloromethane)	67-66-3	(C) 50	(C) 240	(ST) 2 ppm [60-min]	10 ppm
Cresol, all isomers	1319-77-3	5	22	2.3 ppm	$20 \frac{\text{mg}}{\text{m}^3}$ (IFV)
Cumene	98-82-8	50	245	50 ppm	50 ppm
Ethyl alcohol (ethanol)	64-17-5	1000	1900	1000 ppm	(ST) 1000 ppm
Ethyl ether	60-29-7	400	1200	N/A	400 ppm (ST) 500 ppm
Iodine	7553-56-2	(C) 0.1	(C) 1	(C) 0.1 ppm	0.01 ppm (IFV) (ST) 0.1 ppm (V)
Isopropyl ether	108-20-3	500	2100	500 ppm	250 ppm (ST) 310 ppm
L.P.G. (liquefied petroleum gas)	68476-85-7	1000	1800	1000 ppm	N/A
Methyl mercaptan	74-93-1	(C) 10	(C) 20	(C) 0.5 ppm [15-min]	0.5 ppm
Naphthalene	91-20-3	10	50	10 ppm (ST) 15 ppm	10 ppm (ST) 15 ppm
Ozone	10028-15-6	0.1	0.2	(C) 0.1 ppm	0.05–0.20 ppm depending on workload and time
Phosphoric acid	7664-38-2	N/A	1	$1 \frac{\text{mg}}{\text{m}^3}$ (ST) $3 \frac{\text{mg}}{\text{m}^3}$	$1 \frac{\text{mg}}{\text{m}^3}$ (ST) $3 \frac{\text{mg}}{\text{m}^3}$

Exposure Limits for Selected Compounds^(a) (cont'd)

Substance	CAS No. ^(c)	Regulatory Limits		Recommended Limits	
		OSHA PEL ^(b)		NIOSH REL ^(g) (as of 4/26/13)	ACGIH® 2015 TLV® ^(h)
		ppm ^(d)	$\frac{\text{mg}}{\text{m}^3}$ ^(e)	Up to 10-hour TWA, (ST) STEL, (C) Ceiling ^(f)	8-hour TWA, (ST) STEL, (C) Ceiling
Propane	74-98-6	1000	1800	1000 ppm	N/A
n-Propyl alcohol	71-23-8	200	500	200 ppm (ST) 250 ppm	100 ppm
Sulfuric acid	7664-93-9		1	$1 \frac{\text{mg}}{\text{m}^3}$	$0.2 \frac{\text{mg}}{\text{m}^3}$ (Thoracic-size fraction)
1,1,1-Trichloroethane (methyl chloroform)	71-55-6	350	1900	350 ppm (ST) 450 ppm (C) 800 ppm	350 ppm (ST) 450 ppm

- Columns 3 and 4 list PELs from OSHA Table Z-1 in 29 CFR 1910.1000. Columns 5 and 6 list other occupational exposure limits (OELs) from NIOSH and ACGIH®.
- Occupational Safety and Health Administration (OSHA) Permissible Exposure Limits (PELs) from 29 CFR 1910.1000 Z-1 Table [58 FR 35340, June 30, 1993; 58 FR 40191, July 27, 1993, as amended at 61 FR 56831, Nov. 4, 1996; 62 FR 1600, Jan 10, 1997; 62 FR 42018, Aug. 4, 1997; 71 FR 10373, Feb. 28, 2006; 71 FR 16673, Apr. 3, 2006; 71 FR 36008, June 23, 2006.]. PELs are 8-hour time-weighted averages (TWAs), unless otherwise indicated. OSHA enforces these limits under section 5(a)(2) of the OSH Act. In addition to the values listed in this table, the Z tables in 29 CFR 1910.1000 list skin absorption designations.
- The CAS number is for information only. Enforcement is based on the substance name. For an entry covering more than one metal compound measured as the metal, the CAS number for the metal is given—not CAS numbers for the individual compounds.
- Parts of vapor or gas per million parts of contaminated air by volume at 25 °C and 760 torr.
- Milligrams of substance per cubic meter of air. When entry is in this column only, the value is exact; when listed with a ppm entry, it is approximate.
- TWA indicates a time-weighted average concentration. A short-term exposure limit (STEL) is designated by ST preceding the value; unless noted otherwise, the STEL is a 15-minute TWA exposure that should not be exceeded at any time during a work day. A ceiling REL is designated by C preceding the value; unless noted otherwise, the ceiling value should not be exceeded at any time.
- National Institute for Occupational Safety and Health (NIOSH) Recommended Exposure Limits (RELs) from the *NIOSH Pocket Guide to Chemical Hazards* (<http://www.cdc.gov/niosh/npg>) (NIOSH 2007). RELs are for up to 10-hour time weighted averages (TWAs) during a 40-hour work week, unless otherwise indicated. NIOSH has established occupational exposure limits for compounds not included in the OSHA Z Tables. Please see the *NIOSH Pocket Guide* for additional limits, skin absorption and other designations, and explanations.
- ACGIH® Threshold Limit Values (TLVs®) (ACGIH® 2015). TLVs® are listed in the order of 8-hour time-weighted averages (TWAs), STELs (ST), and Ceilings (C), if available. ACGIH® has established TLVs® for compounds not included in the OSHA Z Tables. Please see ACGIH® *Documentation* for additional limits, skin absorption and other designations, and explanations. The 2015 *TLV® and BEI® Book and Documentation of the Threshold Limit Values on Chemical Substances*, 7th Edition, are available through the ACGIH® website at <http://www.acgih.org>. The TLVs® and BEIs® are copyrighted by ACGIH® and are not publicly available. Permission must be requested from ACGIH® to reproduce the TLVs® and BEIs®.

Carcinogens

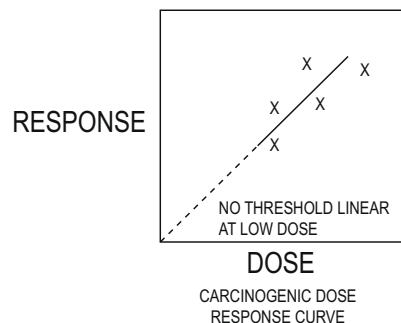
For carcinogens, the EPA considers an acceptable risk to an individual to be a lifetime excess cancer risk within the range of 10^{-4} to 10^{-6} . The added risk of cancer is calculated as follows:

$$\text{Risk} = \text{dose} \times \text{toxicity} = CDI \times CSF$$

where

CDI = Chronic daily intake

CSF = Cancer slope factor, the slope of the dose-response curve for carcinogenic materials



Noncarcinogens

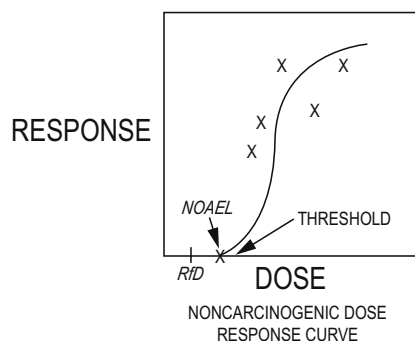
For noncarcinogens, a hazard index (HI) characterizes the risk from all pathways and exposure routes. The EPA considers that an $HI > 1.0$ represents an unacceptable risk of an adverse effect occurring.

$$HI = \frac{CDI_{\text{noncarcinogen}}}{RfD}$$

where

$CDI_{\text{noncarcinogen}}$ = chronic daily intake of noncarcinogenic compound

RfD = reference dose



Dose is expressed as $\left(\frac{\text{mass of chemical}}{\text{body weight} \times \text{exposure time}} \right)$

$NOAEL$ = No observable adverse effect level (the dose below which no harmful effects are apparent)

Reference Dose

Reference dose (RfD) is determined from the noncarcinogenic dose-response curve using the $NOAEL$:

RfD = lifetime (i.e., chronic) dose that a healthy person could be exposed to daily without adverse effects

$$RfD = \frac{NOAEL}{UF}$$

and

$$SHD = RfD \times W = \frac{NOAEL \times W}{UF}$$

where

SHD = safe human dose (mg/day)

$NOAEL$ = threshold dose per kg of test animal $\frac{\text{mg}}{\text{kg day}}$ from the dose-response curve

UF = the total uncertainty factor, depending on nature and reliability of the animal test data

W = the weight of the adult male (typically 70 kg)

Exposure

Residential Exposure Equations for Various Pathways

Pathway	Exposure Equation
Ingestion in drinking water	$CDI = \frac{(CW)(IR)(EF)(ED)}{(BW)(AT)}$
Ingestion while swimming	$CDI = \frac{(CW)(CR)(ET)(EF)(ED)}{(BW)(AT)}$
Dermal contact with water	$AD = \frac{(CW)(SA)(PC)(ET)(EF)(ED)(CF)}{(BW)(AT)}$
Ingestion of chemicals in soil	$CDI = \frac{(CS)(IR)(CF)(FI)(EF)(ED)}{(BW)(AT)}$
Dermal contact with soil	$AD = \frac{(CW)(CF)(SA)(AF)(ABS)(EF)(ED)}{(BW)(AT)}$
Inhalation of airborne (vapor phase) chemicals	$CDI = \frac{(CA)(IR)(ET)(EF)(ED)}{(BW)(AT)}$
Ingestion of contaminated fruits, vegetables, fish, and shellfish	$CDI = \frac{(CF)(IR)(FI)(EF)(ED)}{(BW)(AT)}$

Source: *Risk Assessment Guidance for Superfund: Volume I, Human Health Evaluation Manual (Part A)*, 1st ed., Washington, D.C.: U.S. Environmental Protection Agency, 1989.

where

ABS = absorption factor for soil contaminant is unitless

AD = absorbed dose in $\frac{\text{mg}}{\text{kg} \cdot \text{day}}$

AF = soil-to-skin adherence factor in $\frac{\text{mg}}{\text{cm}^2}$

AT = averaging time in days

BW = body weight in kg

CA = contaminant concentration in air in $\frac{\text{mg}}{\text{m}^3}$

CDI = chronic daily intake in $\frac{\text{mg}}{\text{kg} \cdot \text{day}}$

CF = volumetric conversion factor for water is $\frac{1 \text{ L}}{1000 \text{ cm}^3}$
 = conversion factor for soil in $\frac{10^{-6} \text{ kg}}{\text{mg}}$

CR = contact rate in $\frac{\text{L}}{\text{hr}}$

CS = chemical concentration in soil in $\frac{\text{mg}}{\text{kg}}$

CW = chemical concentration in water in $\frac{\text{mg}}{\text{L}}$

ED = exposure duration in years

EF = exposure frequency in $\frac{\text{days}}{\text{year}}$ or $\frac{\text{events}}{\text{year}}$

ET = exposure time in $\frac{\text{hr}}{\text{day}}$ or $\frac{\text{hr}}{\text{event}}$

FI = fraction ingested is unitless

IR = ingestion rate in $\frac{\text{L}}{\text{day}}$ or $\frac{\text{mg soil}}{\text{day}}$ or $\frac{\text{kg}}{\text{meal}}$
 = inhalation rate in $\frac{\text{m}^3}{\text{hr}}$

PC = Chemical-specific dermal permeability constant in $\frac{\text{cm}}{\text{hr}}$

SA = skin surface area available for contact in cm^2

Source: Risk Assessment Guidance for Superfund: Volume 1, Human Health Evaluation Manual (Part A),
 1st ed., Washington, D.C.: U.S. Environmental Protection Agency, 1989.

Intake Rates

EPA-Recommended Values for Estimating Intake

Parameter	Standard Value
Average body weight, female adult	65.4 kg
Average body weight, male adult	78 kg
Average body weight, child ^a	
6–11 months	9 kg
1–5 years	16 kg
6–12 years	33 kg
Amount of water ingested, adult	2.3 L/day
Amount of water ingested, child	1.5 L/day
Amount of air breathed, female adult	11.3 m ³ /day
Amount of air breathed, male adult	15.2 m ³ /day
Amount of air breathed, child (3–5 years)	8.3 m ³ /day
Amount of fish consumed, adult	6 g/day
Water swallowing rate, while swimming	50 mL/hr
Inhalation rates	
adult (6-hr day)	0.98 m ³ /hr
adult (2-hr day)	1.47 m ³ /hr
child	0.46 m ³ /hr
Skin surface available, adult male	1.94 m ²
Skin surface available, adult female	1.69 m ²
Skin surface available, child	
3–6 years (average for male and female)	0.720 m ²
6–9 years (average for male and female)	0.925 m ²
9–12 years (average for male and female)	1.16 m ²
12–15 years (average for male and female)	1.49 m ²
15–18 years (female)	1.60 m ²
15–18 years (male)	1.75 m ²
Soil ingestion rate, child 1–6 years	>100 mg/day
Soil ingestion rate, persons > 6 years	50 mg/day
Skin adherence factor, gardener's hands	0.07 mg/cm ²
Skin adherence factor, wet soil	0.2 mg/cm ²
Exposure duration	
Lifetime (carcinogens; for noncarcinogens use, actual exposure duration)	75 years
At one residence, 90th percentile	30 years
National median	5 years
Averaging time	(ED) (365 days/year)

EPA-Recommended Values for Estimating Intake (cont'd)

Parameter	Standard Value
Exposure Frequency (EF)	
Swimming	7 days/year
Eating fish and shellfish	48 days/year
Oral ingestion	350 days/year
Exposure time (ET)	
Shower, 90th percentile	12 min
Shower, 50th percentile	7 min

^aData in this category taken from Copeland, T., A. M. Holbrow, J. M. Otan, et al. "Use of probabilistic methods to understand the conservatism in California's approach to assessing health risks posed by contaminants." *Journal of the Air and Waste Management Association*, Vol. 44, pp. 1399–1413, 1994.

Source: U.S. Environmental Protection Agency, *Risk Assessment Guidance for Superfund: Volume 1, Human Health Evaluation Manual* (part A), EPA/540/1-89/002, 1989.

8.2.4.3 Respirators

Assigned protection factors (APFs)—Per 29 CFR 1910.134, employers must use the assigned protection factors (listed in the table that follows) to select a respirator that meets or exceeds the required level of employee protection. When using a combination respirator (e.g., airline respirators with an air-purifying filter), employers must ensure that the assigned protection factor is appropriate to the mode of operation in which the respirator is being used.

Immediately dangerous to life or health (IDLH)—An atmosphere that poses an immediate threat to life, would cause irreversible adverse health effects, or would impair an individual's ability to escape from a dangerous atmosphere.

Powered air-purifying respirator (PAPR)—An air-purifying respirator that uses a blower to force the ambient air through air-purifying elements to the inlet covering.

Supplied-air respirator (SAR) or airline respirator—An atmosphere-supplying respirator for which the source of breathing air is not designed to be carried by the user.

Workplace protection factor (WPF) study—A study, conducted under actual conditions of use in the workplace, that measures the protection provided by a properly selected, fit-tested, and functioning respirator, when the respirator is worn correctly and used as part of a comprehensive respirator program that is in compliance with OSHA's Respiratory Protection Standard at 29 CFR 1910.134. Measurements of Co and Ci are obtained only while the respirator is being worn during performance of normal work tasks (that is, samples are not collected when the respirator is not being worn). As the degree of protection afforded by the respirator increases, the WPF increases.

Simulated workplace protection factor (SWPF) study—A study, conducted in a controlled laboratory setting, in which Co and Ci sampling is performed while the respirator user performs a series of set exercises. The laboratory setting is used to control many of the variables found in workplace studies, while the exercises simulate the work activities of respirator users. This type of study is designed to determine the optimum performance of respirators by reducing the impact of sources of variability through maintenance of tightly controlled study conditions.

Assigned Protection Factors⁵

Type of Respirator ^{1,2}	Quarter Mask	Half Mask	Full Face Piece	Helmet/Hood	Loose-Fitting Face Piece
1. Air-purifying respirator	5	10 ³	50	—	—
2. Powered air-purifying respirator (PAPR)	—	50	1000	25/1000 ⁴	25
3. Supplied-air respirator (SAR) or airline respirator					
• Demand mode	—	10	50	—	—
• Continuous flow mode	—	50	1000	25/1000 ⁴	25
• Pressure-demand or other positive-pressure mode	—	50	1000	—	—
4. Self-contained breathing apparatus (SCBA)					
• Demand mode	—	10	50	50	—
• Pressure-demand or other positive-pressure mode (e.g., open or closed)	—	—	10,000	10,000	—

Notes:

- Employers may select respirators assigned for use in higher workplaces concentration of a hazardous substance for use at lower concentrations of that substance, or when required respirator use is independent of concentration.
- The assigned protection factors in this table are only effective when the employer implements a continuing, effective respirator program as required by this section (29 CFR 1910.134), including training, fit-testing, maintenance, and use requirements.
- This APF category includes filtering face pieces, and half masks with elastomeric face pieces.
- The employer must have evidence provided by the respirator manufacturer that testing of these respirators demonstrates performance at a level of protection of 1000 or greater to receive an APF of 1000. This level of performance can best be demonstrated by performing a WPF or SWPF study or equivalent testing. Absent such testing, all other PAPRs and SARs with helmets/hoods are to be treated as loose-fitting face piece respirators, and receive an APF of 25.
- These APFs do not apply to respirators used solely for escape. For escape respirators used in association with specific substances covered by 29 CFR 1910 subpart Z, employers must refer to the appropriate substance-specific standards in that subpart. Escape respirators for other IDLH atmospheres are specified by 29 CFR 1910.134(d)(2)(ii).

Source: Assigned Protection Factors for the Revised Respiratory Protection Standard, Washington, D.C.: Occupational Safety and Health Administration, 2009.

8.2.4.4 Toxicity of Pesticides

This section establishes four toxicity categories for acute hazards of pesticide products. Category I is the highest category. Most human hazard, precautionary statements, and human personal protective equipment statements are based on the toxicity category of the pesticide product as sold or distributed. In addition, toxicity categories may be used for regulatory purposes other than labeling, such as classification for restricted use and requirements for child-resistant packaging. In certain cases, statements based on the toxicity category of the product as diluted for use are also permitted. A toxicity category is assigned for each of five types of acute exposure, as specified in the table below.

Acute Toxicity Categories for Pesticide Products

Hazard Indicators	I	II	III	IV
Oral LD ₅₀	Up to and including 50 mg/kg	>50 through 500 mg/kg	>500 through 5000 mg/kg	> 5000 mg/kg
Dermal LD ₅₀	Up to and including 200 mg/kg	>200 through 2000 mg/kg	>2000 through 20,000 mg/kg	>20,000 mg/kg
Inhalation LC ₅₀	Up to and including 0.2 mg/liter	>0.2 through 2 mg/liter	>2 through 20 mg/liter	>20 mg/liter
Eye irritation	Corrosive: corneal opacity not reversible within 7 days	Corneal opacity reversible within 7 days; irritation persisting for 7 days	No corneal opacity; irritation reversible within 7 days	No irritation
Skin irritation	Corrosive	Severe irritation at 72 hours	Moderate irritation at 72 hours	Mild or slight irritation at 72 hours

Source: From *Regulating Pesticides*, U.S. Environmental Protection Agency.

Pesticide Toxicity Categories

Toxicity Category	Signal Word
I	Poison
II	Warning
III	Caution
IV	Caution

Source: 40 CFR 156: *Labeling Requirements for Pesticides and Devices*: U.S. Environmental Protection Agency.

8.2.4.5 Noise Pollution

$$SPL \text{ (dB)} = 10 \log_{10} \left(\frac{P^2}{P_0^2} \right)$$

$$SPL_{\text{total}} = 10 \log_{10} \sum 10^{SPL/10}$$

$$\text{Point Source Attenuation: } \Delta SPL \text{ (dB)} = 10 \log_{10} \left(\frac{r_2}{r_1} \right)^2 \text{ or } 20 \log_{10} \left(\frac{r_2}{r_1} \right)$$

$$\text{Line Source Attenuation: } \Delta SPL \text{ (dB)} = 10 \log_{10} \left(\frac{r_2}{r_1} \right)^2$$

where

SPL (dB) = sound pressure level, measured in decibels

P = sound pressure (Pa)

P_0 = reference sound pressure (2×10^{-5} Pa)

SPL_{total} = sum of multiple sources

ΔSPL (dB) = change in sound pressure level with distance, measured in decibels

r_1 = distance from source to receptor at point 1

r_2 = distance from source to receptor at point 2

8.2.4.6 Permissible Noise Exposure (per OSHA Regulations)

Noise dose D should not exceed 100%.

$$D = 100\% \times \sum \frac{C_i}{T_i}$$

where

C_i = time spent at specified sound pressure level (SPL) in hours

T_i = time permitted at SPL in hours

$\sum C_i = 8$ (hours)

Permissible Noise Level vs. Permissible Time of Exposure

Noise Level (dBA)	Permissible Time (hr)
80	32
85	16
90	8
95	4
100	2
105	1
110	0.5
115	0.25
120	0.125
125	0.063
130	0.031

If $D > 100\%$, noise abatement is required.

If $50\% \leq D \leq 100\%$, hearing conservation program is required.

Note: $D = 100\%$ is equivalent to 90 dBA time-weighted average (TWA). $D = 50\%$ is equivalent to TWA of 85 dBA.

Hearing conservation program requires: (1) testing employee hearing, (2) providing hearing protection at employee's request, and (3) monitoring noise exposure.

Exposure to impulsive or impact noise should not exceed 140 dB sound pressure level (SPL).

8.2.4.7 Electrical Safety

Probable Effects of Various Levels of Current on the Human Body

Level of Current (milliamperes)	Probable Effect
1 mA	Perception level. Slight tingling sensation. Still dangerous under certain conditions.
5 mA	Slight shock felt; not painful but disturbing. Average individual can let go. However, strong involuntary reactions to shocks in this range may lead to injuries.
6 mA–16 mA	Painful shock; begin to lose muscular control. Commonly referred to as the freezing current or "let-go" range.
17 mA–99 mA	Extreme pain, respiratory arrest, severe muscular contractions. Individual cannot let go. Death is possible.
100 mA–2000 mA	Ventricular fibrillation (uneven, uncoordinated pumping of the heart). Muscular contraction and nerve damage begins to occur. Death is likely.
>2000 mA	Cardiac arrest, internal organ damage, and severe burns. Death is probable.

Sources: NIOSH, *Worker Deaths by Electrocution; A Summary of NIOSH Surveillance and Investigative Findings*, Ohio: U.S. Health and Human Services, 1998. And Greenwald, E.K., *Electrical Hazards and Accidents—Their Cause and Prevention*, New York: Van Nostrand Reinhold, 1991.

8.2.4.8 Fundamentals of Ventilation

Ventilation Definitions

Aerosol: An assemblage of small particles, solid or liquid, suspended in air. The diameter of the particles may vary from 100 microns down to 0.01 micron or less, e.g., dust, fog, smoke.

Air cleaner: A device designed for the purpose of removing atmospheric airborne impurities such as dusts, gases, mists, vapors, fumes, and smoke. (Air cleaners include air washers, air filters, electrostatic precipitators, and charcoal filters.)

Air filters: An air-cleaning device that removes light particulate loadings from normal atmospheric air before introducing into the building. Usual range: loadings up to 3 grains per thousand cubic feet (0.003 grains per cubic foot). Note: Atmospheric air in heavy industrial areas and in-plant air in many industries have higher loadings than this, and dust collectors are then indicated for proper air cleaning.

Aspect ratio: The ratio of the width (W) to the length (L); $AR = \frac{W}{L}$.

Aspect ratio of an elbow: The width (W) along the axis of the bend divided by the depth (D) in the plane of the bend;
 $AR = \frac{W}{D}$.

Blast gate: Sliding damper.

Capture velocity: The air velocity at any point in front of the hood or at the hood opening necessary to overcome opposing air currents and capture the contaminated air at that point by causing it to flow into the hood.

Density factor: The ratio of actual air density to density of standard air. The product of the density factor and the density of standard air ($0.075 \frac{\text{lb}}{\text{ft}^3}$) gives the actual air density in pounds per cubic foot; $\text{Density} = \text{df} \times 0.075 \frac{\text{lb}}{\text{ft}^3}$.

Dust: Small solid particles created by breakup of larger particles by processes, such as crushing, grinding, drilling, and explosions. Dust particles already in existence in a mixture of materials may escape into the air through such operations as shoveling, conveying, screening, or sweeping.

Dust collector: An air-cleaning device to remove heavy particulate loadings from exhaust systems. Usual range of particulate loading: 0.003 grains per cubic foot or higher.

Entry loss: Loss in pressure caused by air flowing into a duct or hood (inches H₂O).

Fumes: Small solid particles formed by the condensation of vapors of solid materials.

Gases: Formless fluids that tend to occupy an entire space uniformly at ordinary temperatures and pressures.

Hood: A shaped inlet designed to capture contaminated air and conduct it into the exhaust duct system.

Hood flow coefficient: The ratio of flow caused by a given hood static pressure compared to the theoretical flow that would result if the static pressure could be converted to velocity pressure with 100 percent efficiency.

Inch of water: A unit of pressure equal to the pressure exerted by a column of liquid water one inch high at a standard temperature.

Minimum design duct velocity: Minimum air velocity required to move the particles in the air stream (fpm).

Mists: Small droplets of materials that are ordinarily liquid at normal temperature and pressure.

Pressure, static: The potential pressure exerted in all directions by a fluid at rest. For a fluid in motion, it is measured in a direction normal to the direction of flow. Usually expressed in inches of water gauge when dealing with air. (The tendency to either burst or collapse the pipe.)

Pressure, total: The algebraic sum of the velocity pressure and the static pressure (with due regard to sign).

Pressure, velocity: The kinetic pressure in the direction of flow necessary to cause a fluid at rest to flow at a given velocity. Usually expressed in inches of water gauge.

Replacement air: A ventilation term used to indicate the volume of controlled outdoor air supplied to a building to replace air being exhausted.

Slot velocity: Linear flow rate of contaminated air through a slot, fpm.

Smoke: An air suspension (aerosol) of particles, usually but not necessarily solid, often originating in a solid nucleus, formed from combustion or sublimation.

Standard air: Dry air at 70°F and 29.92 (in Hg) barometer. This is substantially equivalent to $0.075 \frac{\text{lb}}{\text{ft}^3}$. Specific heat of dry air = 0.24 Btu/lb/°F.

Turn-down ratio: The degree to which the operating performance of a system can be reduced to satisfy part-load conditions. Usually expressed as a ratio; for example, 30:1 means the minimum operation point is 1/30th of full load.

Source: From ACGIH®, *Industrial Ventilation: A Manual of Recommended Practice for Design*, 28th ed., © 2013.

Ventilation Abbreviations

Abbreviation	Definition
AR	Aspect ratio
A_s	Slot area
B	Barometric pressure
C_c	Hood flow coefficient
CLR	Centerline radius
df	Overall density factor
df_e	Elevation density factor
df_m	Moisture density factor
df_p	Pressure density factor
df_t	Temperature density factor
F'_d	Loss per unit length (duct)
F_{el}	Elbow loss coefficient
F_{en}	Entry loss coefficient
F_h	Hood entry-loss coefficient
F_s	Slot loss coefficient
gr	Grains
H	Height
h_d	Loss in straight duct run
h_e	Overall hood entry loss
h_{el}	Elbow loss
h_{en}	Entry loss
h_h	Hood entry loss
h_s	Slot or opening entry loss
HEPA	High-efficiency particulate air filter
HV	Humid volume, in $\frac{\text{ft}^3 \text{ mix}}{\text{lb dry air}}$

Abbreviation	Definition
HVAC	Heating, ventilation, and air conditioning
"wg	Inches water gauge
L	Length
$\dot{m}$	Mass flow rate
ME	Mechanical efficiency
mm wg	Millimeters water gauge
MRT	Mean radiant temperature
Q	Flow rate, in cfm
sfpm	Surface feet per minute
SP	Static pressure
SP_{gov}	Higher static pressure at junction of 2 ducts
SP_h	Hood static pressure
SP_s	SP, system handling standard air
TP	Total pressure
V	Velocity, in fpm
V_d	Duct velocity
VP	Velocity pressure
VP_d	Duct velocity pressure
VP_r	Resultant velocity pressure
VP_s	Slot velocity pressure
V_s	Slot velocity
V_t	Duct transport velocity
ω	Moisture content, in $\frac{\text{lbm H}_2\text{O}}{\text{lbm dry air}}$
z	Elevation, in feet above sea level

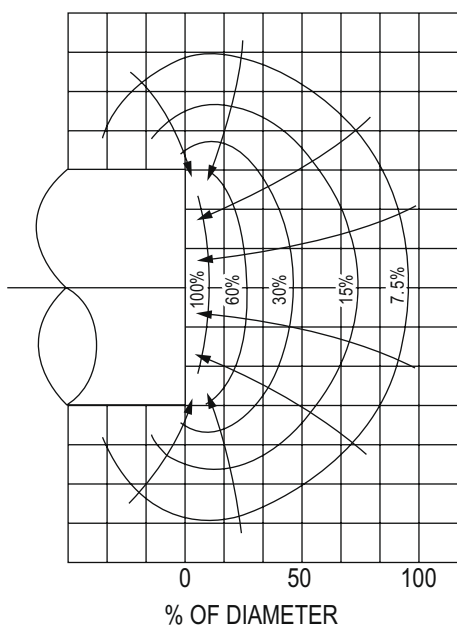
Source: From ACGIH®, *Industrial Ventilation: A Manual of Recommended Practice for Design*, 28th ed., © 2013.

Ventilation Equations

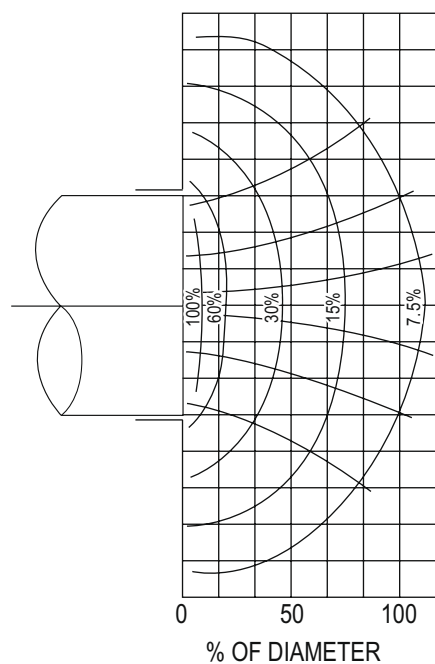
Description	Equation	Units
Velocity pressure (VP)	$VP = \frac{\rho V^2}{2g_c} = \left(\frac{V}{4005}\right)^2 \text{ df}$	V in fpm VP in "wg
Total pressure (TP)	$TP = SP + VP$	"wg
Hood entry loss (h_h)	$h_h = F_h(VP_d)$	"wg Values of F_h can be found in the Hood Loss Coefficients table later in this chapter.
Hood static pressure (SP_h)	$SP_h = -(VP_d + h_h)$	"wg

Velocity Contours

Plain Circular Opening—% of Opening Velocity

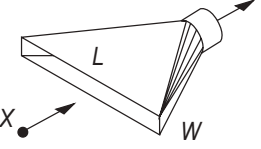
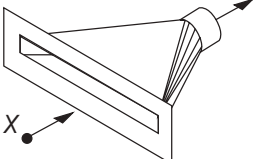
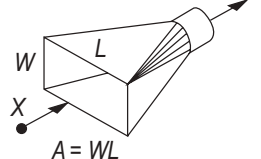
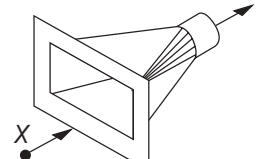
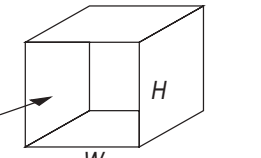
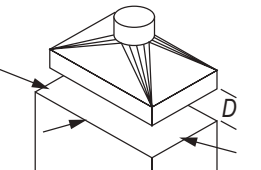
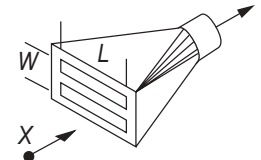
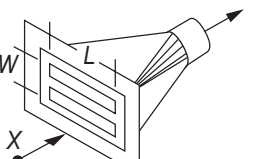


Flanged Circular Opening—% of Opening Velocity



Source: From ACGIH®, *Industrial Ventilation: A Manual of Recommended Practice for Design*, 28th ed., © 2013.

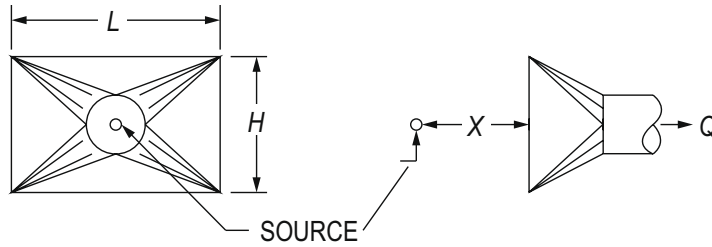
Summary of Hood Airflow Equations

Hood Type	Description	Aspect Ratio, W/L	Airflow
	Slot	0.2 or less	$Q = 3.7 LV_x X$
	Flanged Slot	0.2 or less	$Q = 2.6 LV_x X$
	Plain opening	0.2 or greater and round	$Q = V_x(10X^2 + A)$
	Flanged opening	0.2 or greater and round	$Q = 0.75V_x(10X^2 + A)$
	Booth	To suit work	$Q = VA = VWH$
	Canopy	To suit work	$Q = 1.4 PVD$ $P = \text{Perimeter}$ $D = \text{Height above work}$
	Plain multiple-slot opening, 2 or more slots	0.2 or greater	$Q = V_x(10X^2 + A)$
	Flanged multiple-slot opening, 2 or more slots	0.2 or greater	$Q = 0.75V_x(10X^2 + A)$

Source: From ACGIH®, *Industrial Ventilation: A Manual of Recommended Practice for Design*, 28th ed., © 2013.

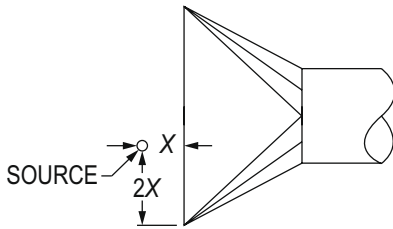
8.2.4.9 Flow-Capture Velocity of Suspended Hoods (Small Side-Draft Hoods)

Freely Suspended Hood



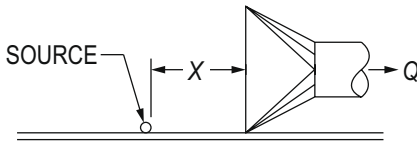
$$Q = V_X(10X^2 + A)$$

Large Hood



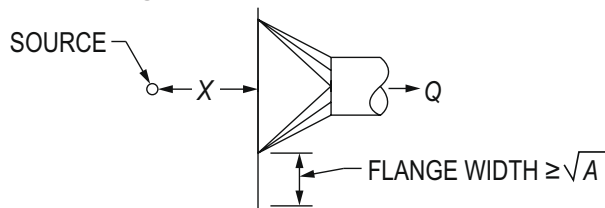
For a large hood with small X , measure X perpendicular to the hood face and not less than $2X$ from the edge of the opening.

Hood on Bench or Floor



$$Q = V_X(5X^2 + A)$$

Hood with Wide Flange



$$Q = 0.75V_X(10X^2 + A)$$

where

Q = required exhaust airflow, in acfm or $\frac{\text{m}^3}{\text{s}}$

X = distance from hood face to farthest point of contamination, in ft or m

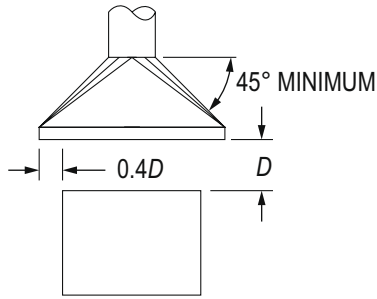
A = hood face area, in ft^2 or m^2

V_X = capture velocity at distance X , in fpm or $\frac{\text{m}}{\text{s}}$, at distance X

Note: Airflow rate must increase as the square of distance of the source from the hood. Baffling by flanging or by placing on bench, floor, etc. has a beneficial effect.

Source: Hood illustrations in this section are from ACGIH®, *Industrial Ventilation: A Manual of Recommended Practice for Design*, 28th ed., © 2013.

8.2.4.10 Flow-Capture Velocity of Canopy Hood



$$Q = 1.4 PDV$$

where P = Perimeter of tank, in ft or m

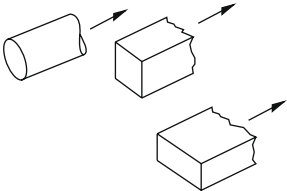
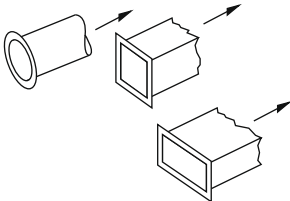
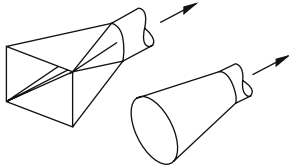
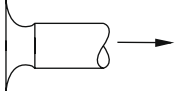
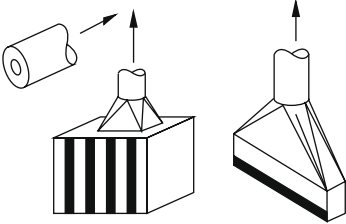
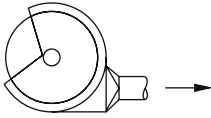
Not recommended if workers must bend over source. V ranges from 50 to 500 fpm or 0.25 to 2.50 $\frac{m}{s}$, depending on crossdrafts. Side curtains on two or three sides to create a semi-booth or booth are desirable.

Recommended Capture Velocities

Energy of Dispersion	Examples	V_x	
		$\frac{ft}{min}$	$\frac{m}{s}$
Little motion	Evaporation from tanks, degreasing	75–100	0.38–0.51
Average motion	Intermittent container filling, low-speed conveyor transfers, welding, plating, pickling	100–200	0.51–1.02
High	Barrel filling, conveyor loading, crushers	200–500	1.02–2.54
Very high	Grinding, abrasive blasting, tumbling	500–2000	2.54–10.2
Factors affecting choices within ranges: • Strength of cross-drafts due to makeup air, traffic, etc. • Need for effectiveness in collection: - Toxicity of contaminants produced by the source - Exposures from other sources, which reduce acceptable exposure from this source - Quantity of air contaminants generated: production rate, volatility, time generated			

Source: From ACGIH®, *Industrial Ventilation: A Manual of Recommended Practice for Design*, 28th ed., © 2013.

Hood Type Efficiency

Hood Loss Coefficients		
Hood Type	Description	Hood Entry Loss (F_h) Coefficient
	Plain opening	0.93
	Flanged opening	0.49
	Taper or cone hood	0.15–0.4
	Bell mouth inlet	0.04
	Orifice	0.55 when duct velocity = slot velocity
	Typical grinding hood	Straight takeoff: 0.65
		Tapered takeoff: 0.40

Source: From ACGIH®, *Industrial Ventilation: A Manual of Recommended Practice for Design*, 28th ed., © 2013.

8.2.4.11 Concentrations of Vaporized Liquids

Vaporization rate (Q_m , mass/time) from a liquid surface:

$$Q_m = \left[\frac{MK A_s P^{\text{sat}}}{R_g T_L} \right]$$

where

M = molecular weight of volatile substance

K = mass transfer coefficient

A_s = area of liquid surface

P^{sat} = saturation vapor pressure of the pure liquid at T_L

R_g = ideal gas constant

T_L = absolute temperature of the liquid

Mass flow rate of liquid from a hole in the wall of a process unit:

$$Q_m = A_H C_0 (2\rho g_c P_g)^{1/2}$$

where

A_H = area of hole

C_0 = discharge coefficient

ρ = density of the liquid

P_g = gauge pressure within the process unit

Concentration (C_{ppm}) of vaporized liquid in ventilated space:

$$C_{\text{ppm}} = \left[\frac{Q_m R_g T \times 10^6}{(k Q_V P M)} \right]$$

where

T = absolute ambient temperature

k = nonideal mixing factor

Q_V = ventilation rate

P = absolute ambient pressure

Sweep-through concentration change in a vessel:

$$Q_V t = V \ln \left[\frac{C_1 - C_0}{C_2 - C_0} \right]$$

where

Q_V = volumetric flow rate

t = time

V = vessel volume

C_0 = inlet concentration

C_1 = initial concentration

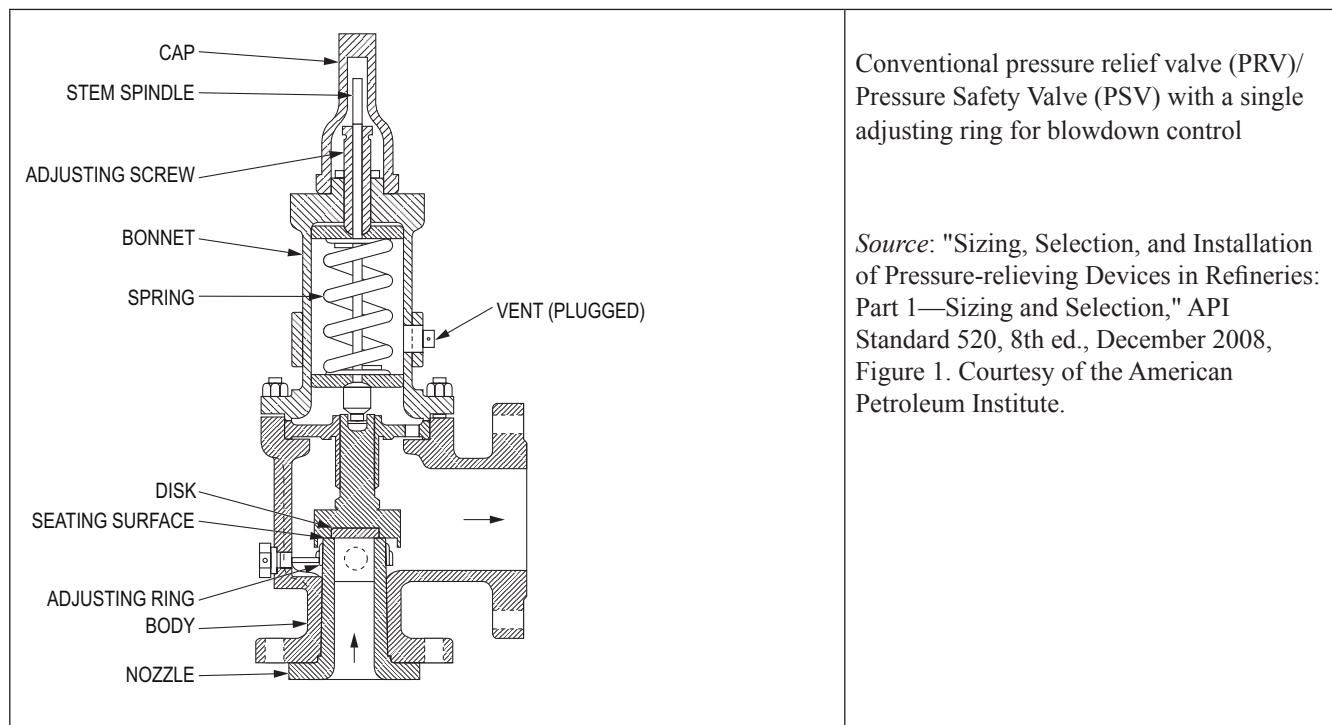
C_2 = final concentration

8.3 Protective Systems

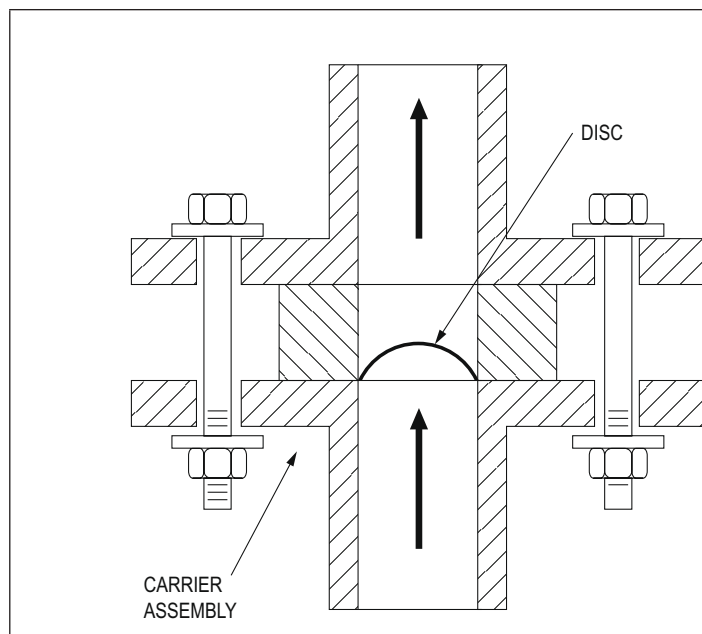
8.3.1 Overpressure Protection/Pressure Relief

8.3.1.1 Major Types of Relief Devices

Relief Devices



	Balance-Bellows PRV/PSV <i>Source:</i> "Sizing, Selection, and Installation of Pressure-relieving Devices in Refineries: Part 1—Sizing and Selection," API Standard 520, 8th ed., December 2008, Figure 2. Courtesy of the American Petroleum Institute.
	Pop-Action Pilot-Operated Valve (Flowing Type) <i>Source:</i> "Sizing, Selection, and Installation of Pressure-relieving Devices in Refineries: Part 1—Sizing and Selection," API Standard 520, 8th ed., December 2008, Figure 10. Courtesy of the American Petroleum Institute.

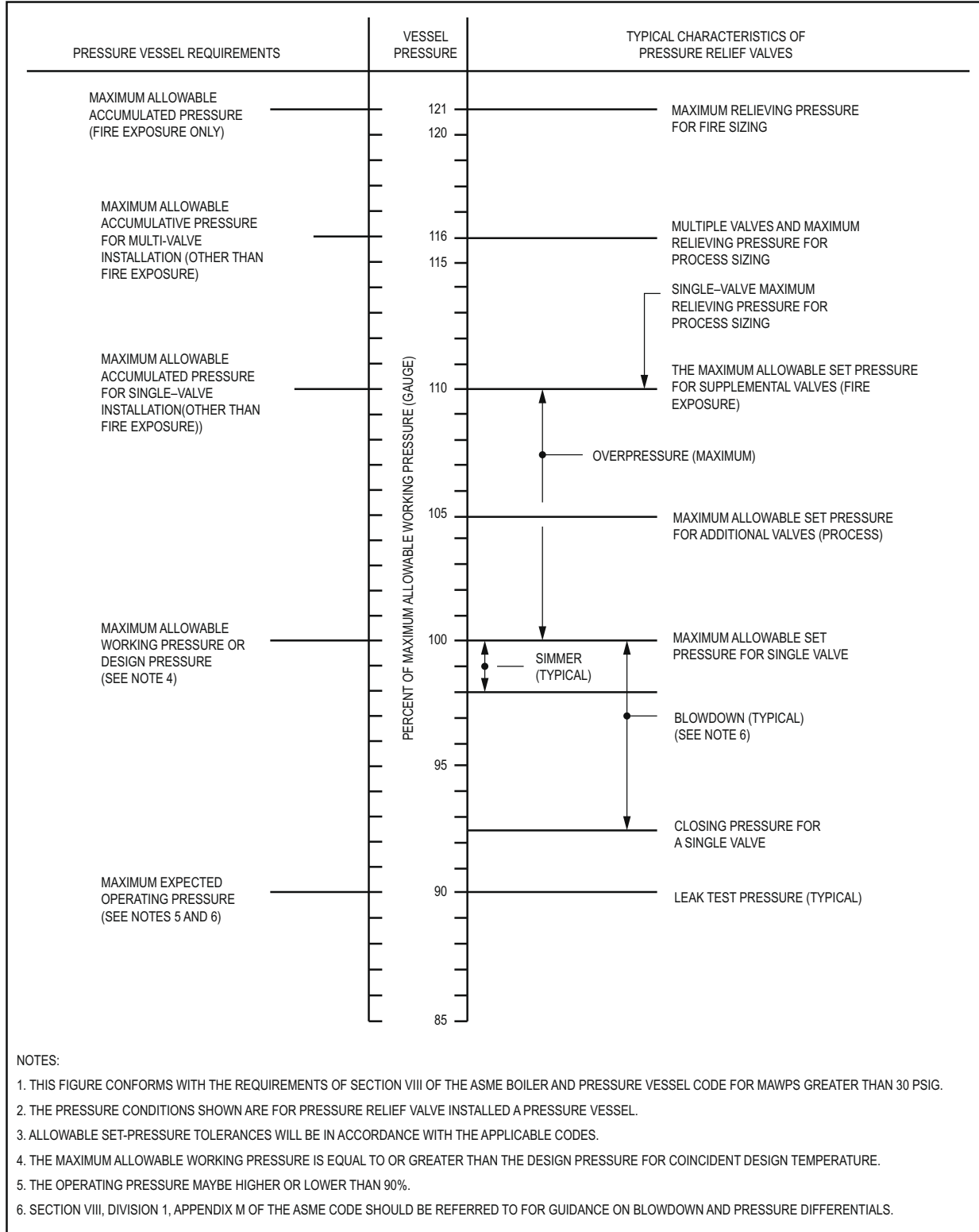


Rupture Disc Assembly

Source: Crowl, Daniel A., and Louvar, Joseph F., *Chemical Process Safety: Fundamentals with Applications*, 2nd ed., New York: Pearson Education 2002. With permission.

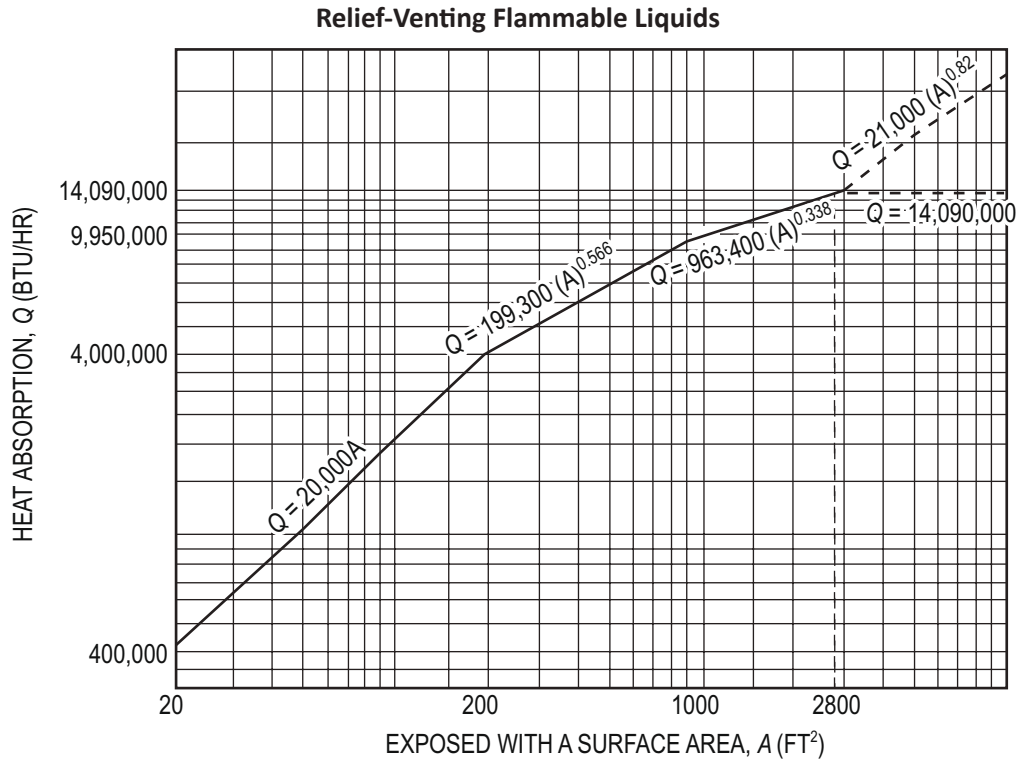
8.3.1.2 Pressure-Level Relationships for Pressure Relief Valves

Pressure-Level Relationships for PRVs



Source: "Sizing, Selection, and Installation of Pressure-relieving Devices in Refineries: Part 1—Sizing and Selection," API Standard 520, 8th ed., December 2008. Courtesy of the American Petroleum Institute.

8.3.1.3 Relief Vent Sizing



Source: Reproduced with permission from NFPA 30, *Flammable and Combustible Liquid Code*, © 2015, National Fire Protection Association. This is not the complete and official position of the NFPA on the referenced subject, which is represented only by the standard in its entirety.

Estimation of Emergency Relief Venting for Specific Liquids

$$CFH = \frac{70.5Q}{L\sqrt{M}}$$

where

CFH = cubic feet of free air per hour

70.5 = factor for converting pounds of gas to ft^3 of air

Q = total heat input per hour (Btu)

L = latent heat of vaporization $\left(\frac{\text{Btu}}{\text{lb}}\right)$

M = molecular weight

8.3.1.4 Pressure Relief Variables and Constants

Pressure Relief Variables and Constants

Symbol	Description	Units (U.S.)	Units (metric)
A	Required effective discharge area of the device	in ²	mm ²
C	A function of the ratio of the ideal gas-specific heats $\left(k = \frac{C_p}{C_v}\right)$ of the gas or vapor at inlet-relieving temperature	$\frac{\sqrt{\text{lbm} \cdot \text{lb mole} \cdot ^\circ\text{R}}}{\text{lb} \cdot \text{hr}}$	$\frac{\sqrt{\text{kg} \cdot \text{kg mol} \cdot \text{K}}}{\text{mm}^2 \cdot \text{hr} \cdot \text{K Pa}}$
C_p	Specific heat at constant pressure	$\frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{F}}$	$\frac{\text{KJ}}{\text{kg K}}$
C_v	Specific heat at constant volume	$\frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{F}}$	$\frac{\text{KJ}}{\text{kg K}}$
F_2	Coefficient of subcritical flow		
G_1	Specific gravity of a liquid at flowing temperature referred to water at standard conditions		
k	Ratio of the specific heats $\left(\frac{C_p}{C_v}\right)$ for an ideal gas at relieving temperature. The ideal-gas to specific-heat ratio is independent of pressure.	dimensionless	
K_b	Capacity correction factor due to back pressure; can be obtained from manufacturer's literature or estimated for preliminary sizing. The back-pressure correction factor applies to balanced-bellows valves only. For conventional and pilot-operated valves, use a value for K_b equal to 1.0.	dimensionless	
K_c	Combination correction factor for installations with a rupture disc upstream of the pressure relief valve. Equals 1.0 when a rupture disc is not installed; equals 0.9 when a rupture disc is installed in combination with a PRV and the combination does not have a certified value.	dimensionless	
K_d for liquid	Rated coefficient of discharge that should be obtained from the valve manufacturer. For preliminary sizing, an effective discharge coefficient can be used as follows: 0.65 when a PRV is installed with or without a rupture disc in combination 0.62 when a PRV is not installed and sizing is for a rupture disc with minimum net flow area	dimensionless	

Pressure Relief Variables and Constants (cont'd)

Symbol	Description	Units (U.S.)	Units (metric)
K_d for gas, vapor, steam	Effective coefficient of discharge. For preliminary sizing, use the following values: 0.975 when a PRV is installed with or without a rupture disc in combination 0.62 when a PRV is not installed and sizing is for a rupture disc with minimum net flow area	dimensionless	
K_N	Correction factor for the <i>Napier equation</i> ($K_N = 1.0$)	dimensionless	
K_{SH}	Superheat correction factor; can be obtained from the "Superheat Correction Factors" table in this section. For saturated steam at any pressure, $K_{SH} = 1.0$. For temperatures above 1200°F, use the critical vapor sizing equations.	dimensionless	
K_v	Correction factor due to viscosity	dimensionless	
K_w	Correction factor due to back pressure. If the back pressure is atmospheric, use a value for K_w of 1.0. Balanced-bellows valves in back-pressure service require the correction determined from the figure "Capacity Correction Factor, K_w , Due to Back Pressure on Balanced-Bellows PRVs in Liquid Service." Conventional and pilot-operated valves require no special correction.	dimensionless	
M	Molecular weight of the gas or vapor at inlet-relieving conditions. Various handbooks carry tables of molecular weights of materials; however, the composition of the flowing gas or vapor is seldom the same as that listed in such tables. This composition should be obtained from the process data.		
P_1	Upstream relieving pressure; set pressure plus allowable overpressure plus atmospheric pressure	psia	kPa
P_2	Back pressure	psia	kPa
Q	Flow rate	$\frac{\text{U.S. gal}}{\text{min}}$	$\frac{\text{L}}{\text{min}}$
r	Ratio of back pressure to upstream relieving pressure, $\frac{P_2}{P_1}$	dimensionless	
Re	Reynolds number	dimensionless	
T	Relieving temperature of the inlet gas or vapor	°R (°F + 460)	K (°C + 273)
μ	Absolute viscosity at the flowing temperature	cP	
U	Viscosity at the flowing temperature	Saybolt universal seconds	
V	Required flow through the device	scfm at 14.7 psia and 60°F	$\frac{\text{normal m}^3}{\text{min}}$ at 0°C and 101.325 kPa
W	Required flow through the device.	$\frac{\text{lb}}{\text{h}}$	$\frac{\text{kg}}{\text{h}}$
Z	Compressibility factor for the deviation of the actual gas from a perfect gas, evaluated at inlet-relieving conditions.	dimensionless	

Pressure Relief Equations

Description	Units (U.S.) (units per previous table)	Units (metric) (units per previous table)
Coefficient C	$C = 520 \sqrt{k \left(\frac{2}{k+1} \right)^{\frac{(k+1)}{(k-1)}}}$	$C = 0.03948 \sqrt{k \left(\frac{2}{k+1} \right)^{\frac{(k+1)}{(k-1)}}}$
Correction Factor K_N	$K_N = 1.0$ where $P_1 \leq 1,500$ psia	$K_N = 1.0$ where $P_1 \leq 10,339$ kPa
	$K_N = \frac{0.1906 P_1 - 1,000}{0.2292 P_1 - 1,061}$ where $P_1 > 1,500$ psia and $\leq 3,200$ psia	$K_N = \frac{0.02764 P_1 - 1,000}{0.03324 P_1 - 1,061}$ where $P_1 > 10,339$ kPa and $\leq 22,057$ kPa
Coefficient F_2	$F_2 = \sqrt{\left(\frac{k}{k-1} \right) r^{\left(\frac{2}{k} \right) \left[\frac{1 - r^{\left(\frac{k-1}{k} \right)}}{1 - r} \right]}}$	
Sizing for Gas or Vapor Service at Critical Flow Conditions	$A = \frac{W}{C K_d P_1 K_b K_c} \sqrt{\frac{T Z}{M}}$	
Sizing for Subcritical Flow: Gas or Vapor, Conventional and Pilot-Operated PRVs When the ratio of back pressure to inlet pressure exceeds the critical pressure ratio P_{cf}/P_1 , the flow through the pressure-relief device is subcritical. These equations may be used to calculate the required effective discharge area for a conventional PRV whose spring setting is adjusted to compensate for superimposed back pressure. Equations may also be used for sizing a pilot-operated PRV.	$A = \frac{W}{735 F_2 K_d K_c} \sqrt{\frac{T Z}{M P_1 (P_1 - P_2)}}$	$A = \frac{17.9 W}{F_2 K_d K_c} \sqrt{\frac{T Z}{M P_1 (P_1 - P_2)}}$
Sizing for Steam-Relief Operating at Critical Flow Conditions	$A = \frac{W}{51.5 P_1 K_d K_b K_c K_N K_{SH}}$	$A = \frac{190.5 W}{P_1 K_d K_b K_c K_N K_{SH}}$

Pressure Relief Equations (cont'd)

Description	Units (U.S.)	Units (metric)
Sizing for Liquid Relief: PRVs Requiring Capacity Certification The ASME Code requires that capacity certification be obtained for PRVs designed for liquid service. The procedure for obtaining capacity certification includes testing to determine the rated coefficient of discharge for the liquid PRVs at 10% overpressure. The sizing equations for pressure-relief devices in liquid service provided here assume that the liquid is incompressible (i.e., the density of the liquid does not change as the pressure decreases from the relieving pressure to the total back pressure). Valves in liquid service that are designed in accordance with the ASME Code may be initially sized using these area equations.	$A = \frac{Q}{38 K_d K_w K_c K_v} \sqrt{\frac{G_1}{P_1 - P_2}}$	$A = \frac{11.78 Q}{K_d K_w K_c K_v} \sqrt{\frac{G_1}{P_1 - P_2}}$
K_v: Correction Factor Due to Viscosity	$K_v = \left(0.9935 + \frac{2.878}{Re^{0.5}} + \frac{342.75}{Re^{1.5}} \right)^{-1.0}$	

Pressure Relief Equations (cont'd)

Description	Units (U.S.)	Units (metric)
Re = Reynolds Number When a PRV is sized for viscous liquid service, it should first be sized as if it were for a nonviscous application (i.e., $K_v = 1.0$), so that a preliminary required discharge area A can be obtained from the liquid relief area equations above. From API 526 standard orifice sizes, use the next orifice size larger than A to determine the Reynolds Number, Re, from either of the following relationships: Second equation is not recommended for viscosities less than 100 Saybolt universal seconds (SSU) After determining the Reynolds Number, Re, obtain the factor K_v. Apply K_v in the liquid relief area equations above to correct the preliminary required discharge area. If the corrected area exceeds the chosen standard orifice area, repeat the above calculations using the next larger standard orifice size.	$Re = \frac{Q(2,800 G_1)}{\mu \sqrt{A}}$	$Re = \frac{12,700 Q}{U \sqrt{A}}$

Source: *Sizing, Selection, and Installation of Pressure-Relieving Devices in Refineries:*

Part I—Sizing and Selection, API Standard 520, 8th ed., December 2008. Courtesy of the American Petroleum Institute.

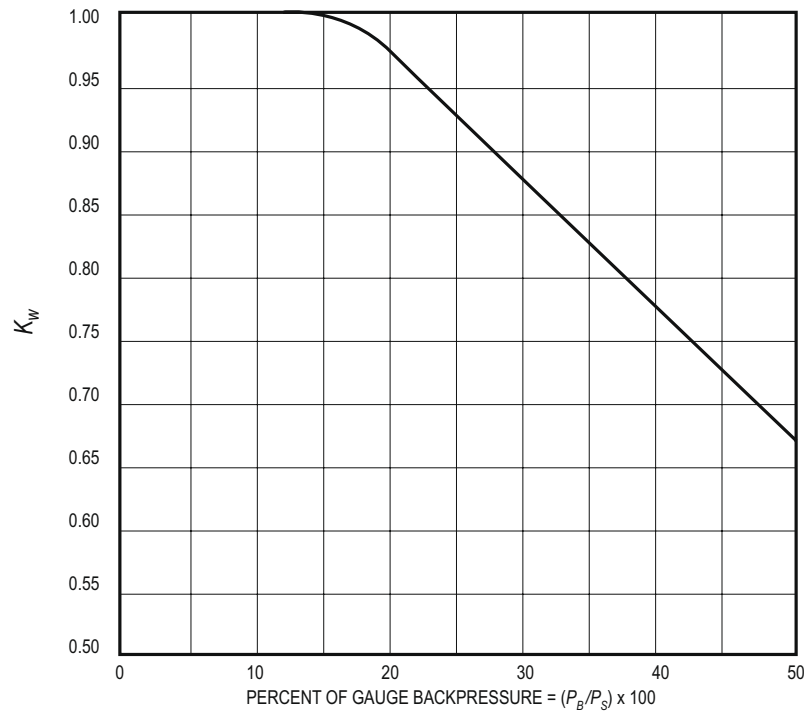
Superheat Correction Factors, K_{SH}

Set Pressure psig (kPag)	Superheat Correction Factors									
	Temperature °F (°C)									
	300 (149)	400 (204)	500 (260)	600 (316)	700 (371)	800 (427)	900 (482)	1000 (538)	1100 (593)	1200 (649)
15 (103)	1.00	0.98	0.93	0.88	0.84	0.80	0.77	0.74	0.72	0.70
20 (138)	1.00	0.98	0.93	0.88	0.84	0.80	0.77	0.74	0.72	0.70
40 (276)	1.00	0.99	0.93	0.88	0.84	0.81	0.77	0.74	0.72	0.70
60 (414)	1.00	0.99	0.93	0.88	0.84	0.81	0.77	0.75	0.72	0.70
80 (551)	1.00	0.99	0.93	0.88	0.84	0.81	0.77	0.75	0.72	0.70
100 (689)	1.00	0.99	0.94	0.89	0.84	0.81	0.77	0.75	0.72	0.70
120 (827)	1.00	0.99	0.94	0.89	0.84	0.81	0.78	0.75	0.72	0.70
140 (965)	1.00	0.99	0.94	0.89	0.85	0.81	0.78	0.75	0.72	0.70
160 (1103)	1.00	0.99	0.94	0.89	0.85	0.81	0.78	0.75	0.72	0.70
180 (1241)	1.00	0.99	0.94	0.89	0.85	0.81	0.78	0.75	0.72	0.70
200 (1379)	1.00	0.99	0.95	0.89	0.85	0.81	0.78	0.75	0.72	0.70
220 (1516)	1.00	0.99	0.95	0.89	0.85	0.81	0.78	0.75	0.72	0.70
240 (1654)	--	1.00	0.95	0.90	0.85	0.81	0.78	0.75	0.72	0.70
260 (1792)	--	1.00	0.95	0.90	0.85	0.81	0.78	0.75	0.72	0.70
280 (1930)	--	1.00	0.96	0.90	0.85	0.81	0.78	0.75	0.72	0.70
300 (2068)	--	1.00	0.96	0.90	0.85	0.81	0.78	0.75	0.72	0.70
350 (2413)	--	1.00	0.96	0.90	0.86	0.82	0.78	0.75	0.72	0.70
400 (2757)	--	1.00	0.96	0.91	0.86	0.82	0.78	0.75	0.72	0.70
500 (3446)	--	1.00	0.96	0.92	0.86	0.82	0.78	0.75	0.73	0.70
600 (4136)	--	1.00	0.97	0.92	0.87	0.82	0.79	0.75	0.73	0.70
800 (5514)	--	--	1.00	0.95	0.88	0.83	0.79	0.76	0.73	0.70
1000 (6893)	--	--	1.00	0.96	0.89	0.84	0.78	0.76	0.73	0.71
1250 (8616)	--	--	1.00	0.97	0.91	0.85	0.80	0.77	0.74	0.71
1500 (10,339)	--	--	--	1.00	0.93	0.86	0.81	0.77	0.74	0.71
1750 (12,063)	--	--	--	1.00	0.94	0.86	0.81	0.77	0.73	0.70
2000 (13,786)	--	--	--	1.00	0.95	0.86	0.80	0.76	0.72	0.69
2500 (17,232)	--	--	--	1.00	0.95	0.85	0.78	0.73	0.69	0.66
3000 (20,679)	--	--	--	--	1.00	0.82	0.74	0.69	0.65	0.62

Source: Sizing, Selection, and Installation of Pressure-Relieving Devices in Refineries:

Part 1—Sizing and Selection, API Standard 520, 8th ed., December 2008. Courtesy of the American Petroleum Institute.

Capacity Correction Factor, K_w , Due to Back Pressure on Balanced-Bellows PRVs in Liquid Service



K_w = CORRECTION FACTOR DUE TO BACK PRESSURE.
 P_B = BACK PRESSURE, IN PSIG.
 P_S = SET PRESSURE, IN PSIG.

NOTE: THE CURVE ABOVE REPRESENTS VALUES RECOMMENDED BY VARIOUS MANUFACTURERS. THIS CURVE MAY BE USED WHEN THE MANUFACTURER IS NOT KNOWN. OTHERWISE, THE MANUFACTURER SHOULD BE CONSULTED FOR THE APPLICABLE CORRECTION FACTOR.

Source: *Sizing, Selection, and Installation of Pressure-Relieving Devices in Refineries: Part 1--Sizing and Selection*, API 520, 8 ed., December 2008. Courtesy of the American Petroleum Institute

8.3.1.5 Discharge Location

The discharge piping connected to the pressure relief device must be no smaller than the discharge opening of the device.

The device must be piped to a point of safe discharge while keeping the run of discharge piping as short as possible.

Discharge piping connected to the device must be supported so as not to impact any loadings on the body of the device.

For multiple devices discharging into a discharge manifold or header, the discharge manifold or header must be sized so the cross-sectional area is equal to or greater than the sum of the discharge cross-sectional areas of all the devices connected to the discharge manifold or header.

8.3.2 Other Protections

8.3.2.1 Inerting and Blanketing

Inerting, or blanketing, is the long-term maintenance of an inert atmosphere in the vapor space of a container or vessel during operation. This practice is used to control the concentration of oxygen, thereby reducing fire and explosion hazards.

8.3.2.2 Secondary Containment

Secondary containment systems prevent migration of wastes or accumulated liquid out of the system to the soil, groundwater, or surface water during the use of the tank system.

Federal hazardous waste storage regulations require the secondary containment system to have sufficient capacity to contain at least 10% of the total volume of the primary containers or 100% of the volume of the largest container, whichever is greater.

8.3.2.3 Safety Instrumented Systems

Safety Instrumented Systems (SIS) are used where mechanical protection devices are not effective or not economically attractive. The SIS includes all the Safety Instrumented Functions in a process area.

A Safety Instrumented Function (SIF) is a single shutdown loop with a sensor, a logic solver, and a final acting element. For example, a high temperature shutdown loop could consist of a temperature indicating transmitter, a programmable logic controller, and an automatically operated isolation valve to shut off a fuel source.

Typically, the SIS will include a logic solver with sufficient reliability to support multiple SIFs that protect against the same undesired event.

Safety Integrity Level (SIL) is a measure of reliability of a SIF. The SIL requirement for each SIF is generally set during Layer of Protection Analysis (LOPA). SIL sets the redundancy and testing requirements.

In detailed design, the specific instrumentation, control device(s), and logic solver are selected for each SIF. Then, the Probability of Failure on Demand (PFD) for the SIF is calculated using data for the failure rate of each component and the proposed testing frequency. The resulting PDF is checked against the SIL required. The SIF design or testing frequency can then be modified to meet requirements if necessary.

8.4 Environmental Considerations

8.4.1 Air Pollution

Concentrations in air can be converted from ppb to $\frac{\mu\text{g}}{\text{m}^3}$ as follows:

$$\frac{\mu\text{g}}{\text{m}^3} = \text{ppb} \frac{P(\text{MW})}{RT}$$

where

ppb = parts per billion

P = pressure, in atm

R = ideal gas law constant = $0.0821 \frac{\text{liter} \cdot \text{atm}}{\text{mol} \cdot \text{K}}$

T = absolute temperature, K = $273.15 + ^\circ\text{C}$

MW = molecular weight, in $\frac{\text{g}}{\text{mol}}$

8.4.1.1 Atmospheric Dispersion Modeling

σ_y and σ_z are functions of downwind distance and stability class:

$$C = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2} \frac{y^2}{\sigma_y^2}\right) \left[\exp\left(-\frac{1}{2} \frac{(z-H)^2}{\sigma_z^2}\right) + \exp\left(-\frac{1}{2} \frac{(z+H)^2}{\sigma_z^2}\right) \right]$$

where

C = steady-state concentration at a point (x, y, z) in $\frac{\mu\text{g}}{\text{m}^3}$

Q = emissions rate in $\frac{\mu\text{g}}{\text{s}}$

σ_y = horizontal dispersion parameter, in meters

σ_z = vertical dispersion parameter, in meters

u = average wind speed at stack height in $\frac{\text{m}}{\text{s}}$

x = downwind distance along plume center line, in meters

y = horizontal distance from plume center line, in meters

z = vertical distance from ground level, in meters

H = effective stack height (m) = $h + \Delta h$

where h = physical stack height

Δh = plume rise

Maximum concentration at ground level and directly downwind from an elevated source:

$$C_{\max} = \frac{Q}{\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2} \frac{H^2}{\sigma_z^2}\right)$$

where variables are as above except for

$C_{\max}$ = maximum ground-level concentration

$\sigma_z = \frac{H}{\sqrt{2}}$ for neutral atmospheric conditions

Atmospheric Stability Under Various Conditions

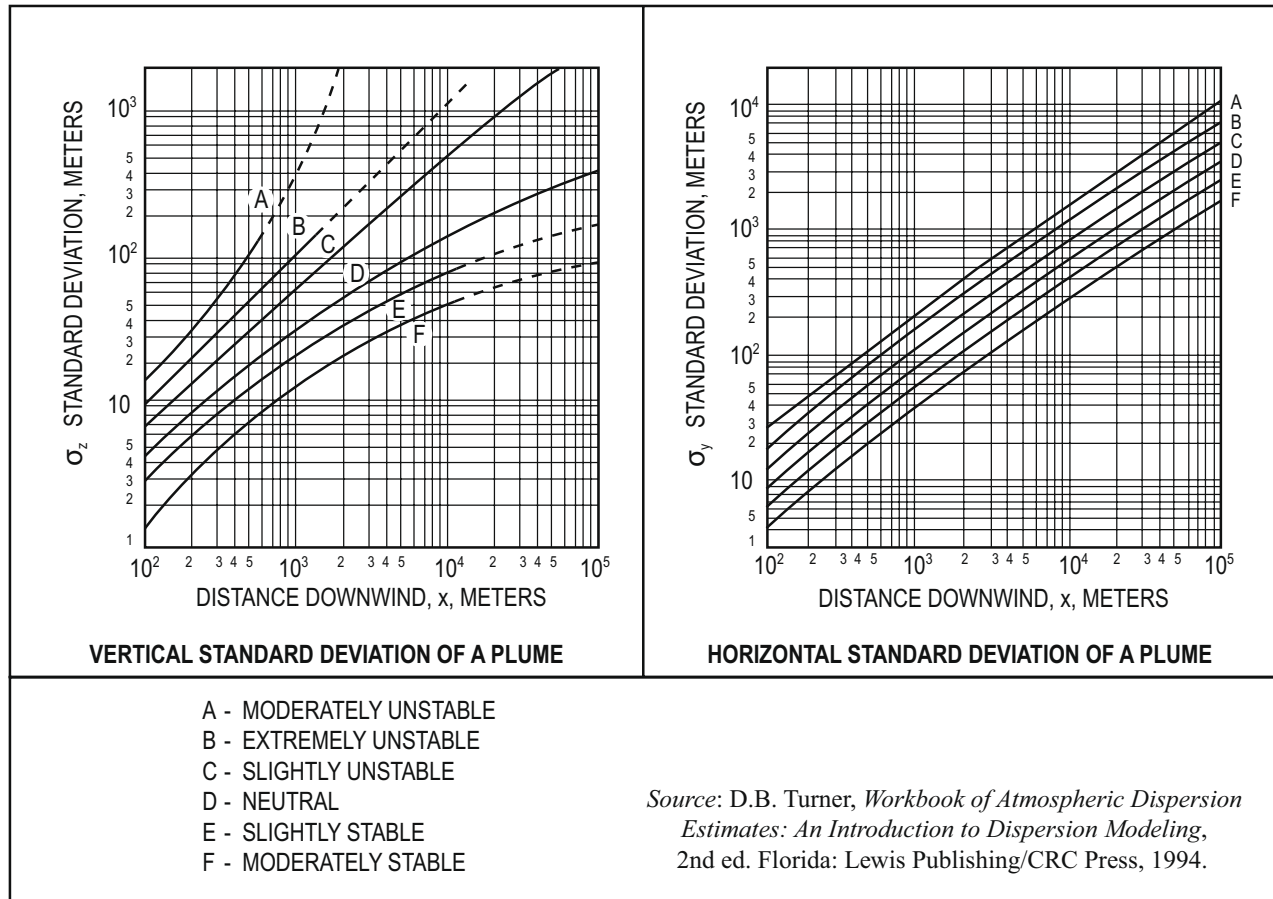
Surface Wind Speed ^a in $\frac{\text{m}}{\text{s}}$	Day: Solar Insulation			Night: Cloudiness ^e	
	Strong ^b	Moderate ^c	Slight ^d	Cloudy ($\leq 4/8$)	Clear ($\leq 3/8$)
<2	A	A–B ^f	B	E	F
2–3	A–B	B	C	E	F
3–5	B	B–C	C	D	E
5–6	C	C–D	D	D	D
> 6	C	D	D	D	D

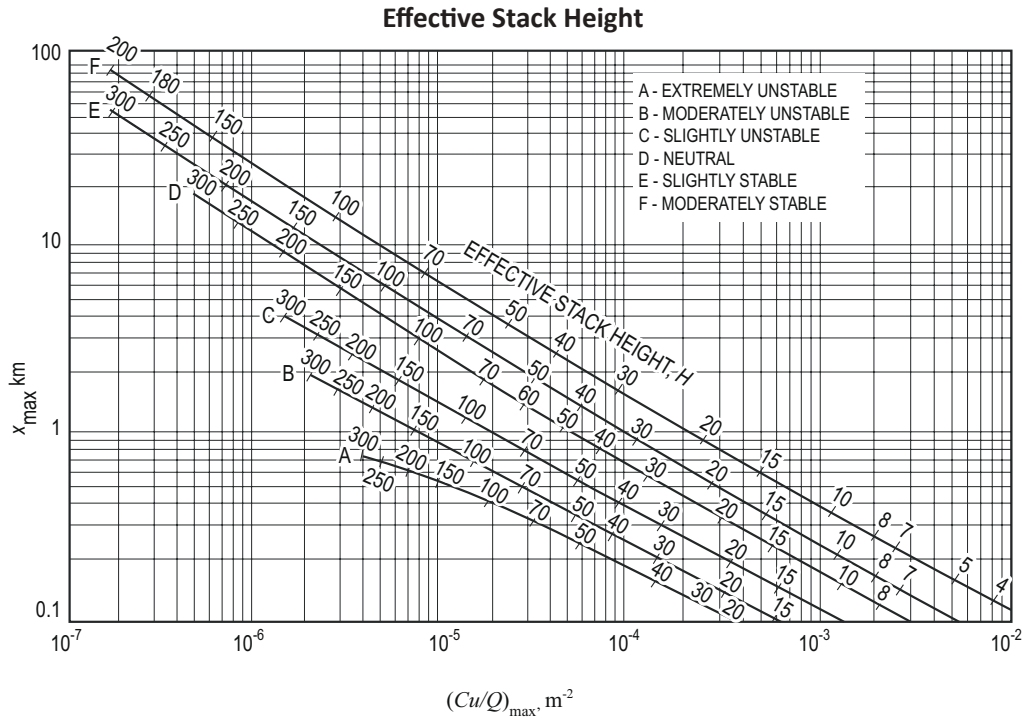
Source: Turner, D.B., *Workbook of Atmospheric Dispersion Estimates: An Introduction to Dispersion Modeling*, 2nd ed., Florida: Lewis Publishing/CRC Press, 1994.

- Surface wind speed is measured at 10 m above the ground.
- Corresponds to a clear summer day with sun higher than 60° above the horizon.
- Corresponds to a summer day with a few broken clouds, or clear day with sun 35–60° above the horizon.
- Corresponds to a fall afternoon or a cloudy summer day with the sun 15–35°.
- Cloudiness is defined as the fraction of sky covered by the clouds.
- For A – B, B – C, or C – D conditions, average the values obtained for each.

A = Very unstable B = Moderately unstable C = Slightly unstable D = Neutral E = Slightly stable F = Stable
Regardless of wind speed, Class D should be assumed for overcast conditions, day or night.

Standard Deviations of a Plume





Source: Turner, D.B., *Workbook of Atmospheric Dispersion Estimates: An Introduction to Dispersion Modeling*, 2nd ed., CRC Press (Lewis Publishing), 1994.

Effective stack height is shown on curves numerically.

$x_{\max}$ = distance along plume center line to the point of maximum concentration

$$(Cu/Q)_{\max} = e^{[a+b(\ln H)+c(\ln H)^2+d(\ln H)^3]}$$

H = effective stack height, stack height + plume rise, in meters

Values of Curve-Fit Constants for Estimating $(Cu/Q)_{\max}$ from H as a Function of Atmospheric Stability

Stability	Constants			
	a	b	c	d
A	-1.0563	-2.7153	0.1261	0
B	-1.8060	-2.1912	0.0389	0
C	-1.9748	-1.9980	0	0
D	-2.5302	-1.5610	-0.0934	0
E	-1.4496	-2.5910	0.2181	-0.0343
F	-1.0488	-3.2252	0.4977	-0.0765

Source: Table 1, Ranchoux, R.J.P., "Determination of Maximum Ground Level Concentration," *Journal of the Air Pollution Control Association*, vol. 26, no. 11, Lexington: Taylor & Francis Ltd, 1976, p. 1089, reprinted by permission of the Air & Waste Management Association, www.awma.org, and Taylor & Francis Ltd, <http://www.tandfonline.com>.
Journal's website can be found at <http://informaworld.com>.

8.4.2 Mitigation

8.4.2.1 Incineration

$$DRE = \frac{W_{in} - W_{out}}{W_{in}} \times 100\%$$

where

DRE = destruction and removal efficiency (%)

W_{in} = mass feed rate of a particular POHC*, in $\frac{kg}{hr}$ or $\frac{lb}{hr}$

W_{out} = mass emission rate of the same POHC*, in $\frac{kg}{hr}$ or $\frac{lb}{hr}$

*POHC = principal organic hazardous contaminant

$$CE = \frac{CO_2}{CO_2 + CO} \times 100\%$$

where

CO_2 = volume concentration (dry) of CO_2 , in parts per million (volume: ppm_v)

CO = volume concentration (dry) of CO , in ppm_v

CE = combustion efficiency

8.4.2.2 Kiln Formula

$$t = \frac{2.28 \frac{L}{D}}{S N}$$

where

t = mean residence time, in minutes

L/D = internal length-to-diameter ratio

S = kiln rake slope, in $\frac{in.}{ft}$ of length

N = rotational speed, in $\frac{rev}{min}$

Energy Content of Waste

Typical Waste Values	Moisture (%)	Energy $\left(\frac{Btu}{lb}\right)$
Food waste	70	2000
Paper	6	7200
Cardboard	5	7000
Plastics	2	14,000
Wood	20	8000
Glass	2	60
Bimetallic cans	3	300

8.5 Design

8.5.1 Process Design

The symbology used in this reference is intended to be used for the PE Chemical exam. It does not necessarily correspond to a particular standard.

8.5.2 Process Diagrams

8.5.2.1 Process Flow Diagrams

The process flow diagram (PFD) or flowsheet is the key document in process design. It shows the arrangement of the equipment selected to carry out the process, the stream connection, stream flow rates and compositions, and the operating conditions. It is used as the primary means of transmitting and recording process information.

A full PFD should include all of the process vessels and equipment and show all the process and utility flow lines. A full heat and material balance of the process showing the composition, flow rate, and the temperature of every stream is usually included in or with a PFD. The PFD also indicates the location of every control valve. A piping and instrumentation diagram (P&ID) is a more detailed version of the PFD that also includes information on ancillary instruments and valves, sampling and drain lines, start-up and shut-down systems, and pipe sizes and metallurgy. The P&ID is used in detailed design and safety analysis.

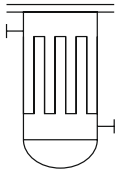
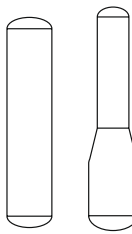
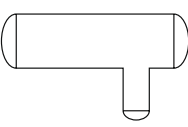
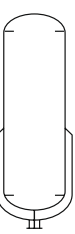
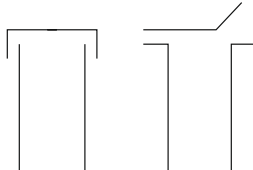
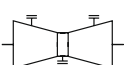
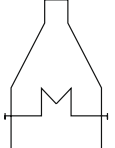
Source: Reprinted from *Chemical Engineering Design*, 2nd ed., Gavin Towler and Ray Sinnott, "Process Flow Development," pp. 33–34, ©2013, with permission from Elsevier, www.elsevier.com.

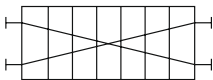
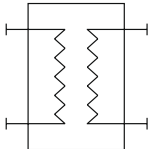
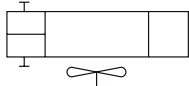
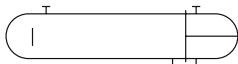
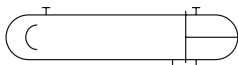
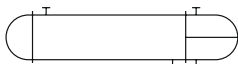
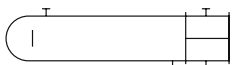
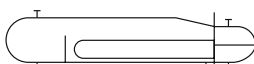
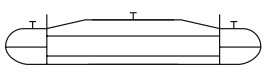
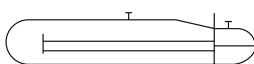
8.5.2.2 Piping and Instrumentation Diagram (P&ID) Equipment Tag Nomenclature

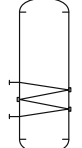
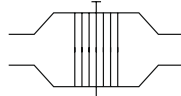
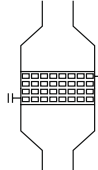
Mechanical Function Codes

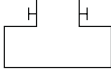
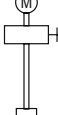
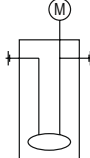
Code	Equipment or Function
AG	Agitator
AX	Packaged unit
BL	Blower
BX	Boiler
CL	Column
CM	Compressor
CR	Crane and winch
CV	Conveyor
DR	Dryer
EX	Expander
FA	Fan
FI	Filter
FL	Flare
FX	Fired furnace, heater
GN	Generator
HV	HVAC
HX	Unfired heat-transfer equipment, e.g., heat exchanger, condenser, cooler, reboiler
MI	Mixer, stirrer, mixing nozzle, inductor, ejector
PM	Pump
RX	Reactor
TB	Turbine
TK	Tank
TX	Thermal oxidizer, incinerator
VS	Vessel, pig receiver/launcher
XX	Material handling equipment, lift

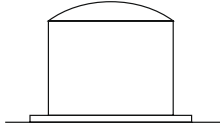
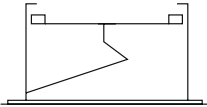
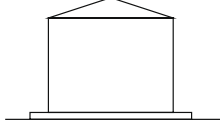
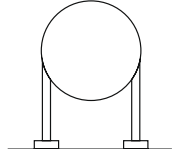
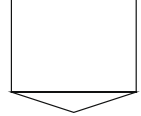
8.5.2.3 P&ID Equipment Symbols

FILTER  CARTRIDGE TYPE FILTER	VESSELS  VERTICAL 1 AND 2 DIAMETERS  HORIZONTAL  HORIZONTAL WITH BOOT  FLAT-TOP VESSEL  VESSEL WITH HEATING OR COOLING JACKET  VESSEL WITH BODY FLANGE  NON-PRESSURE VESSEL
DRIVERS  LIQUID EXPANDER HYDRAULIC TURBINE  GAS EXPANDER/STEAM TURBINE  GAS TURBINE  GEAR BOX  DIESEL ENGINE  MOTOR  GENERATOR	
FIRED HEATER  FURNACE	
OTHERS  EJECTOR/EDUCTOR  MIXER/AGITATOR	

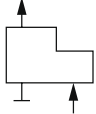
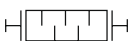
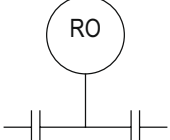
HEAT EXCHANGERS	
	PLATE-FRAME-TYPE HEAT EXCHANGER
	BRAZED PLATE-FIN HEAT EXCHANGER
	AIR-COOLED HEAT EXCHANGER
	FLOATING HEAD
	U-TUBE
	FIXED TUBESHEET
	FLOATING HEAD AND COVER PLATE
	KETTLE-TYPE U-TUBE
	KETTLE-TYPE FIXED TUBESHEET
	KETTLE-TYPE FLOATING HEAD

HEAT EXCHANGERS	
	VERTICAL HEAT EXCHANGER FIXED TUBESHEET
	VERTICAL HEAT EXCHANGER FLOATING HEAD
	OUTER COIL FOR EQUIPMENT
	HEAT RECOVERY COILS VERTICAL TUBES
	HEAT RECOVERY COILS HORIZONTAL TUBES

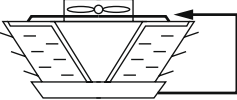
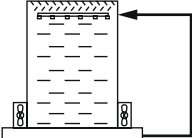
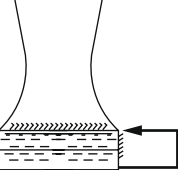
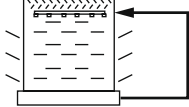
PUMP AND COMPRESSORS	
	CENTRIFUGAL COMPRESSOR
	AXIAL COMPRESSOR
	RECIPROCATING COMPRESSOR
	SCREW COMPRESSOR
	BLOWER/FAN
	ROTARY (GEAR SCREW) PUMP
	PROPORTIONING (METERING) PUMP
	DIAPHRAGM PUMP
	VACUUM PUMP
	SUMP PUMP
	VERTICAL PUMP
	CENTRIFUGAL PUMP

TANKS	
	DOME ROOF TANK
	FLOATING ROOF TANK
	CONE ROOF TANK
	SPHERICAL TANK
	CONICAL BOTTOM TANK

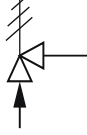
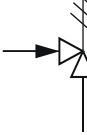
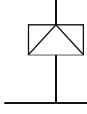
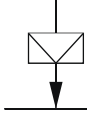
LINE SYMBOLS	
	PRIMARY LINE
	SECONDARY LINE
	UNDERGROUND LINE
	TRACED LINE
	JACKETED LINE
	FLEXIBLE HOSE (FLANGED)
	FLEXIBLE HOSE (COUPLING)

GENERAL PIPING SYMBOLS	
	SIGHT GLASS
	DRAIN TRAP
	FLAME ARRESTOR
	SPARK ARRESTOR
	REDUCER
	CONSERVATION VENT
	VENT (WITH HOOD)
	OPEN VENT WITH GOOSENECK
	SIPHON DRAIN
	Y TYPE STRAINER
	BASKET-TYPE STRAINER
	IN-LINE STATIC MIXER
	RESTRICTION ORIFICE
	RUPTURE DISK

GENERAL PIPING SYMBOLS	
	FLANGE
	BLIND FLANGE
	WELDED END CAP
	SCREWED CAP
	SPECTACLE BLIND (OPEN)
	SPECTACLE BLIND (CLOSED)
	HOSE CONNECTION
	QUICK COUPLING

COOLING TOWERS	
	INDUCED-DRAFT COOLING TOWER
	FORCED-DRAFT COOLING TOWER
	HYPERBOLIC COOLING TOWER
	NATURAL-DRAFT COOLING TOWER

VALVE AND CONTROL SYMBOLS	
	GATE VALVE
	GLOBE VALVE
	BALL VALVE
	PLUG VALVE
	BUTTERFLY VALVE
	NEEDLE VALVE
	DIAPHRAGM VALVE
	PINCH VALVE
	CHECK VALVE
	3-WAY VALVE
	4-WAY VALVE
	DAMPER

VALVE AND CONTROL SYMBOLS	
	PRESSURE SAFETY RELIEF VALVE
	VACUUM SAFETY RELIEF VALVE
	PILOT-ACTUATED RELIEF OR SAFETY VALVE ACTUATOR
	SPRING- OR WEIGHT-ACTUATED RELIEF OR SAFETY VALVE ACTUATOR
	RUPTURE DISC, PRESSURE RELIEF
	RUPTURE DISC, VACUUM RELIEF

8.5.2.4 Instrumentation Tag Identifiers and Symbols

The following identifiers are used in P&ID instrument tags.

Identification Letters					
First 4 Letters			Succeeding 3 Letters		
	Measured or Initiating Variable	Modifier	Readout or Passive Function	Output Function	Modifier
A	Analysis		Alarm		
B	Burner, combustion				
C				Control	
D		Differential			
E	Voltage		Sensor (primary element)		
F	Flow rate	Ratio (fraction)			
G			Glass, viewing device		
H	Hand				High
I	Current (electrical)		Indicate		
J	Power	Scan			
K	Time, time schedule	Time rate of change		Control station	
L	Level		Light		Low
M		Momentary			Middle, intermediate
O			Orifice, restriction		
P	Pressure, vacuum		Point (test) connection		
Q	Quantity	Integrate, totalize			
R	Radiation		Record		
S	Speed, frequency	Safety		Switch	
T	Temperature			Transmit	
U	Multivariable		Multifunction	Multifunction	Multifunction
V	Vibration, mechanical analysis			Valve, damper, louver	
W	Weight, force		Well		
X	Unclassified	X axis	Unclassified	Unclassified	Unclassified
Y	Event, state, or presence	Y axis		Relay, compute, convert	
Z	Position, dimension	Z axis		Driver, actuator, unclassified final control element	

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General Instrument or Function Symbols

	PRIMARY LOCATION **NORMALLY ACCESSIBLE TO OPERATOR	FIELD MOUNTED	AUXILIARY LOCATION **NORMALLY ACCESSIBLE TO OPERATOR
DISCRETE INSTRUMENTS	 *IP1		
SHARED DISPLAY, SHARED CONTROL			
COMPUTER FUNCTION			
PROGRAMMABLE LOGIC CONTROL			

* Abbreviations of the user's choice—such as IP1 (Instrument Panel #1), IC2 (Instrument Console #2), CC3 (Computer Console #3), etc.—may be used when it is necessary to specify instrument or function location.

** Normally inaccessible or behind-the-panel devices or functions may be depicted by using the same symbol but with dashed horizontal lines, as in:

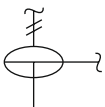
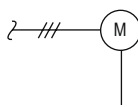
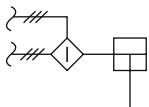
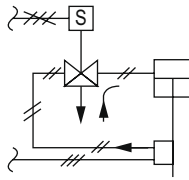
Additional General Instrument or Function Symbols

		 *
PILOT LIGHT	DIAPHRAGM SEAL	INTERLOCK LOGIC



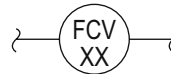
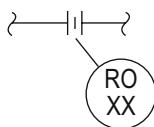
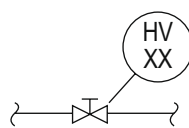
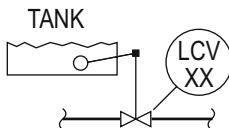
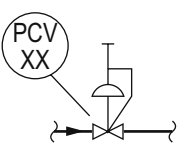
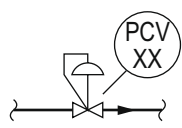
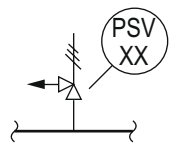
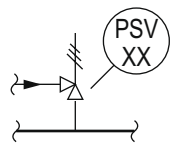
* This diamond is approximately half the size of the larger symbols.

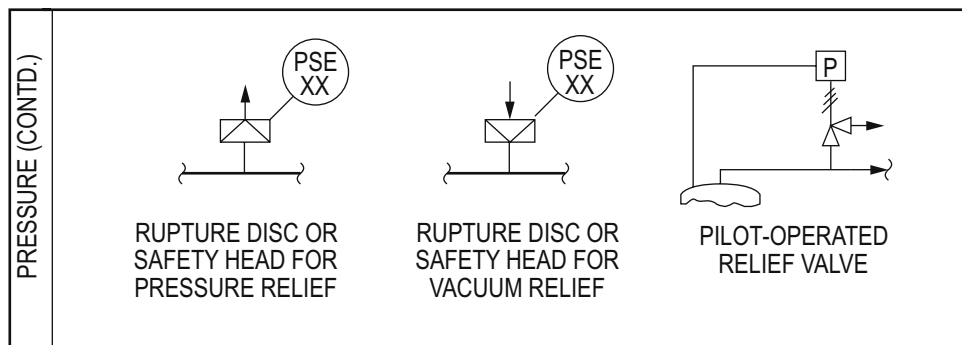
Actuator Symbols

 WITH OR WITHOUT POSITIONER OR OTHER PILOT	 PREFERRED FOR DIAGRAM ASSEMBLED WITH PILOT*, ASSEMBLY IS ACTUATED BY ONE INPUT (SHOWN TYPICALLY WITH ELECTRIC INPUT)	 DIAPHRAGM PRESSURE-BALANCED	 ROTARY MOTOR (SHOWN TYPICALLY WITH ELECTRIC SIGNAL, MAY BE HYDRAULIC OR PNEUMATIC)
 SPRING-OPPOSED SINGLE-ACTING	 DOUBLE-ACTING	 PREFERRED FOR ANY CYLINDER THAT IS ASSEMBLED WITH A PILOT* SO THAT ASSEMBLY IS ACTUATED BY ONE CONTROLLED INPUT	
 SOLENOID	 PREFERRED ALTERNATIVE. A BUBBLE WITH INSTRUMENT TAGGING, E.G. TY-I MAY BE USED INSTEAD OF THE INTERLOCK SYMBOL 		 SINGLE-ACTING CYLINDER (IMPLIED I/P)
 FOR PRESSURE RELIEF OR SAFETY VALVES ONLY. SPRING WEIGHT DENOTES A SPRING WEIGHT OR INTEGRAL PILOT		 HAND ACTUATOR OR HANDWHEEL	

* Pilot may be positioned solenoid valve signal converter, etc.

Symbols for Self-Actuated Regulators, Valves, and Other Devices

FLOW	 AUTOMATIC REGULATOR WITH INTEGRAL FLOW INDICATION	 AUTOMATIC REGULATOR WITHOUT INDICATION		
	 RESTRICTION ORIFICE (ORIFICE PLATE, CAPILLARY TUBE OR MULTI-STAGE TYPE. ETC.) IN PROCESS LINE	 FLOW SIGHT GLASS, PLAIN OR WITH PADDLE WHEEL FLAPPER, ETC.		
HAND	 HAND CONTROL VALVE IN PROCESS LINE			
LEVEL	 LEVEL REGULATOR WITH MECHANICAL LINKAGE			
PRESSURE	 PRESSURE-REDUCING REGULATOR, SELF- CONTAINED, WITH HANDWHEEL ADJUST- ABLE SETPOINT	 BACKPRESSURE REGULATOR SELF-CONTAINED	 PRESSURE RELIEF OR SAFETY VALVE, GENERAL SYMBOL	 VACUUM RELIEF VALVE, GENERAL SYMBOL



8.5.3 Layout and Siting Considerations

8.5.3.1 Fixed Facilities

Building Siting Evaluation: The procedures used to evaluate the hazards and establish the design criteria for new buildings and the suitability of existing buildings at their specific locations.

Facility: The physical location where the management system activity is performed. In early life-cycle stages, a facility may be the company's central research laboratory or the engineering offices of a technology vendor. In later stages, the facility may be a typical chemical plant, storage terminal, distribution center, or corporate office. *Site* is used synonymously with *facility* when describing to Risk Management Plan (RMP) audit criteria.

Fixed Facility: A portion of or a complete plant, unit, site, complex, or any combination thereof that is generally not moveable. In contrast, mobile facilities, such as ships (e.g., transport vessels, floating platform storage and offloading vessels, drilling platforms), trucks, and trains, are designed to be moveable.

Siting: The process of locating a complex, site, plant, or unit.

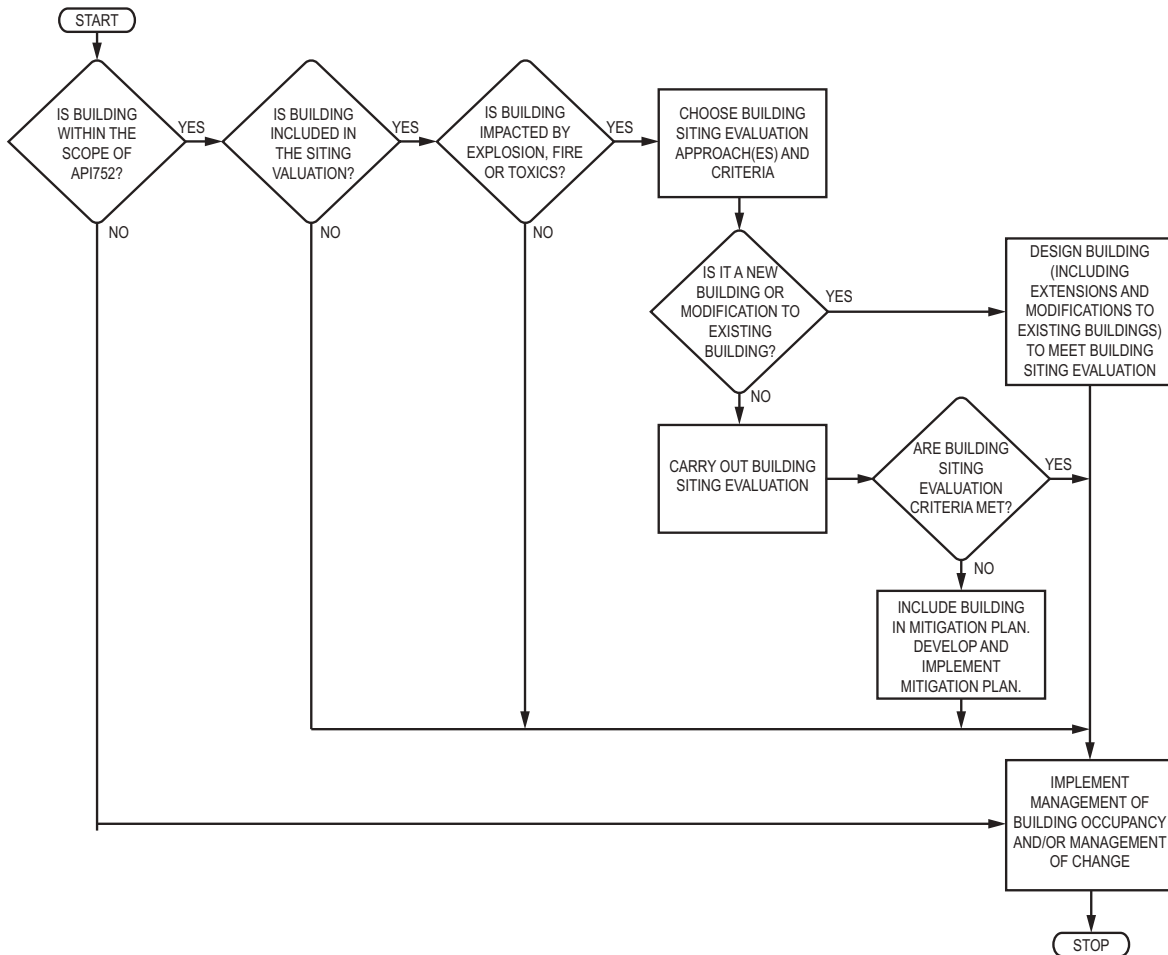
Guiding Principles for Location of Fixed Facility

API Recommended Practice 752 is based on the following guiding principles:

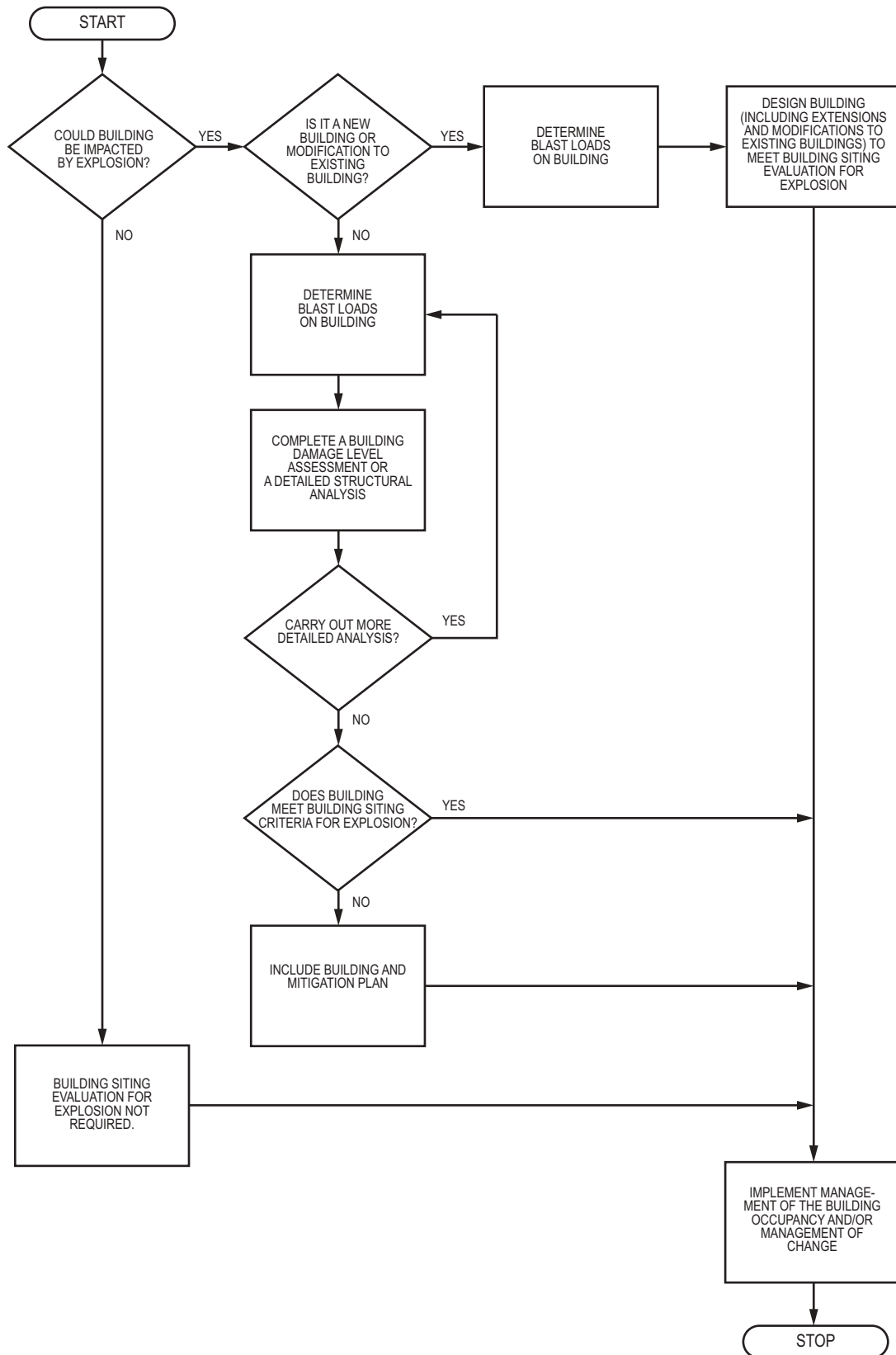
- Locate personnel away from process areas consistent with safe and effective operations.
- Minimize the use of buildings intended for occupancy in close proximity to process areas.
- Manage the occupancy of buildings in close proximity to process areas.
- Design, construct, install, modify, and maintain buildings intended for occupancy to protect occupants against explosion, fire, and toxic material releases.
- Manage the use of buildings intended for occupancy as an integral part of the design, construction, maintenance, and operation of a facility.

Source: API Recommended Practice 752: *Management of Hazards Associated with Location of Process Plant Buildings*, 3rd ed.: American Petroleum Institute, 2009, p. 1. Reproduced courtesy of the American Petroleum Institute.

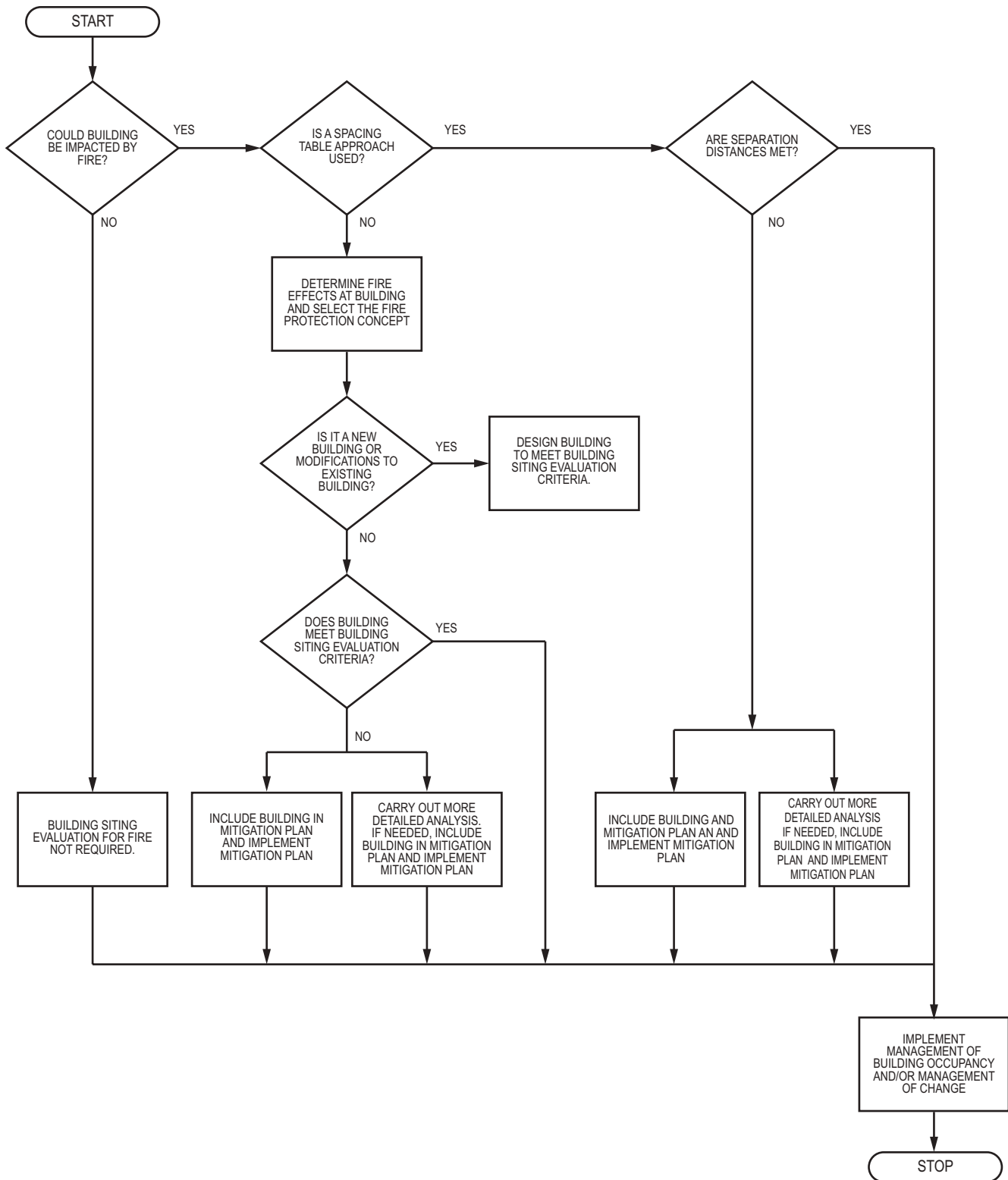
Overall Building Siting Evaluation Flow Chart



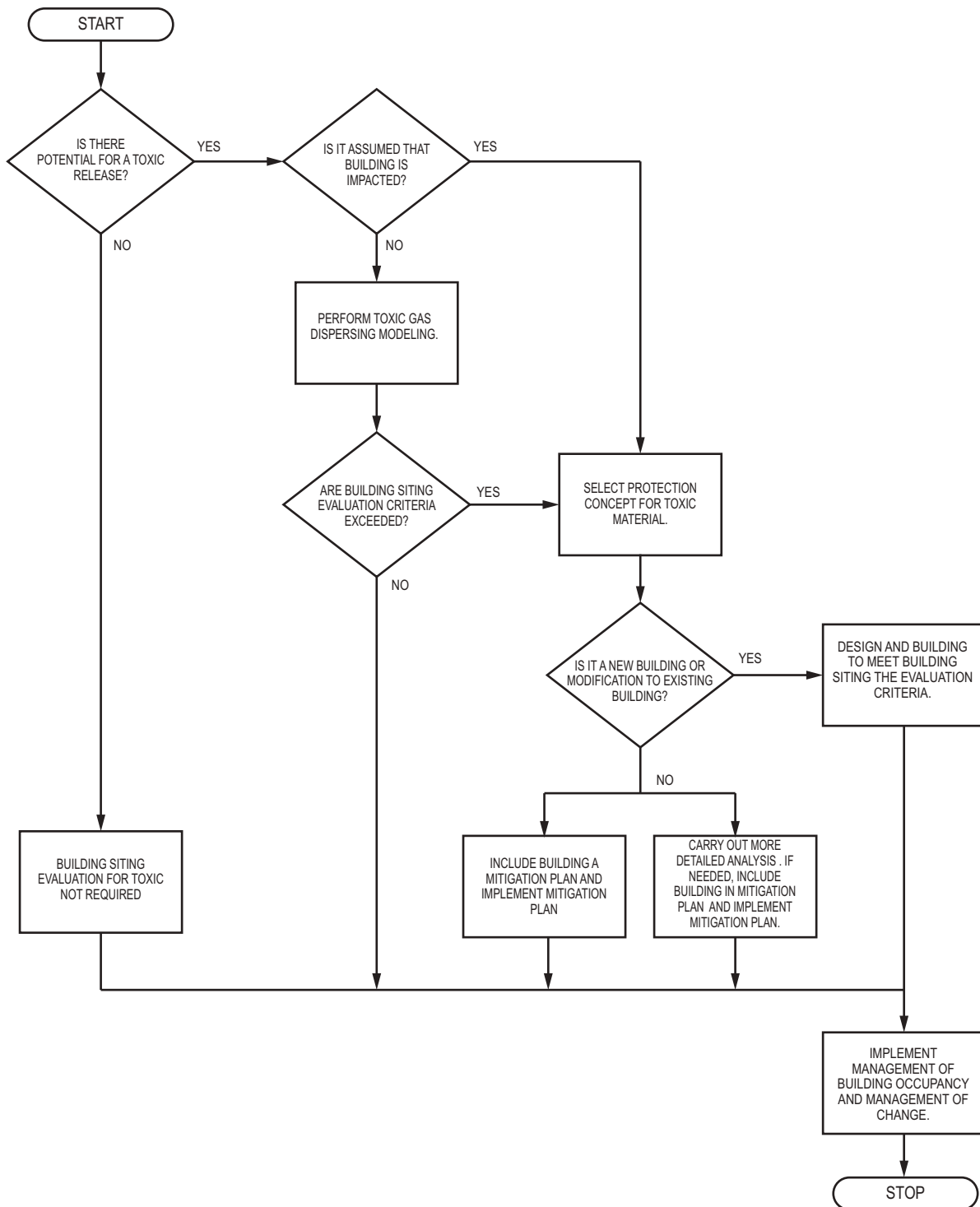
Building Siting Evaluation for Explosions



Building Siting Evaluation for Fire



Building Siting Evaluation for Toxic Material Release



8.5.3.2 Portable Buildings

Portable Building: Any rigid structure that can be moved easily to another location within the facility, regardless of the length of time it is kept at the site. Examples of portable buildings include wood framed trailers (single- and double-wide), container boxes, semi-trailers, and portable structures designed to be blast resistant. Lightweight fabric enclosures, such as tents, are excluded.

Guiding Principles for Siting Portable Buildings

API Recommended Practice 753 is based on the following guiding principles:

- a. Locate personnel away from covered process areas consistent with safe and effective operations.
- b. Minimize the use of occupied portable buildings in close proximity to covered process areas.
- c. Manage the occupancy of portable buildings, especially during periods of increased risk including start-up or planned shut-down operations.
- d. Design, construct, install, and maintain occupied portable buildings to protect occupants against potential hazards.
- e. Manage the use of portable buildings as an integral part of the design, construction, maintenance, and operation of a facility.

Source: API Recommended Practice 751: *Management of Hazards Associated with Location of Process Plant Portable Buildings*: American Petroleum Institute, 2007. Reproduced courtesy of the American Petroleum Institute.

8.5.3.3 Building Occupancy

IBC Table 307.1 (1)

Maximum Allowable Quantity per Control Area of Hazardous Materials Posing a Physical Hazard^{a,j,m,n,p}

Material	Class	Group When the Maximum Allowable Quantity Is Exceeded	Storage ^b			Used in Closed Systems ^b			Used in Open Systems ^b	
			Solid Pounds (Cubic Feet)	Liquid Gallons (Pounds)	Gas (Cubic Feet at NTP)	Solid Pounds (Cubic Feet)	Liquid Gallons (Pounds)	Gas (Cubic Feet at NTP)	Solid Pounds (Cubic Feet)	Liquid Gallons (Pounds)
Combustible dust	N/A	H-2	q	N/A	N/A	q	N/A	N/A	q	N/A
Combustible liquid ^{c,i}	II IIIA IIIB	H-2 or H-3 H-2 or H-3 N/A	N/A	120 ^{d,e} 330 ^{d,e} 13,200 ^{e,f}	N/A	N/A	120 ^d 330 ^d 13,200 ^f	N/A	N/A	30 ^d 80 ^d 3300 ^f
Combustible fiber	Loose Baled ^o	H-3	(100) (1000)	N/A	N/A	(100) (1000)	N/A	N/A	(20) (200)	N/A
Consumer fireworks	1.4G	H-3	125 ^{d,e,l}	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Cryogenics, flammable	N/A	H-2	N/A	45 ^d	N/A	N/A	45 ^d	N/A	N/A	10 ^d
Cryogenics, inert	N/A	N/A	N/A	N/A	NL	N/A	N/A	NL	N/A	N/A
Cryogenics, oxidizing	N/A	H-3	N/A	45 ^d	N/A	N/A	45 ^d	N/A	N/A	10 ^d
Explosives	Division 1.1	H-1	1 ^{e,g}	(1) ^{e,g}	N/A	0.25 ^g	(0.25) ^g	N/A	0.25 ^g	(0.25) ^g
	Division 1.2	H-1	1 ^{e,g}	(1) ^{e,g}	N/A	0.25 ^g	(0.25) ^g	N/A	0.25 ^g	(0.25) ^g
	Division 1.3	H-1 or H-2	5 ^{e,g}	(5) ^{e,g}	N/A	1 ^g	(1) ^g	N/A	1 ^g	(1) ^g
	Division 1.4	H-3	50 ^{e,g}	(50) ^{e,g}	N/A	50 ^g	(50) ^g	N/A	N/A	N/A
	Division 1.4G	H-3	125 ^{d,e,l}	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	Division 1.5	H-1	1 ^{e,g}	(1) ^{e,g}	N/A	0.25 ^g	(0.25) ^g	N/A	0.25 ^g	(0.25) ^g
	Division 1.6	H-1	1 ^{d,e,g}	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Flammable gas	Gaseous Liquefied	H-2	N/A	N/A (150) ^{d,e}	1000 ^{d,e} N/A	N/A	N/A (150) ^{d,e}	1000 ^{d,e} N/A	N/A	N/A
Flammable liquid ^c	1A 1B and 1C	H-2 or H-3	N/A	30 ^{d,e} 120 ^{d,e}	N/A	N/A	30 ^{d,e} 120 ^{d,e}	N/A	N/A	10 ^d 30 ^d
Flammable liquid, combination (1A, 1B, 1C)	N/A	H-2 or H-3	N/A	120 ^{d,e,h}	N/A	N/A	120 ^{d,h}	N/A	N/A	30 ^{d,h}
Flammable solid	N/A	H-3	125 ^{d,e}	N/A	N/A	125 ^d	N/A	N/A	25 ^d	N/A
Inert gas	Gaseous Liquefied	N/A	N/A	N/A	NL	N/A	N/A	NL	N/A	N/A
		N/A	N/A	N/A	NL	N/A	N/A	NL	N/A	N/A

Maximum Allowable Quantity per Control Area of Hazardous Materials Posing a Physical Hazard^{a,j,m,n,p} (cont'd)

Material	Class	Group When the Maximum Allowable Quantity Is Exceeded	Storage ^b			Used in Closed Systems ^b			Used in Open Systems ^b	
			Solid Pounds (Cubic Feet)	Liquid Gallons (Pounds)	Gas (Cubic Feet at NTP)	Solid Pounds (Cubic Feet)	Liquid Gallons (Pounds)	Gas (Cubic Feet at NTP)	Solid Pounds (Cubic Feet)	Liquid Gallons (Pounds)
Organic peroxide	UD	H-1	1 ^{e,g}	(1) ^{e,g}	N/A	0.25 ^g	(0.25) ^g	N/A	0.25 ^g	(0.25) ^g
	I	H-2	5 ^{d,e}	(5) ^{d,e}	N/A	1 ^d	(1) ^d	N/A	1 ^d	(1) ^d
	II	H-3	50 ^{d,e}	(50) ^{d,e}	N/A	50 ^d	(50) ^d	N/A	10 ^d	(10) ^d
	III	H-3	125 ^{d,e}	(125) ^{d,e}	N/A	125 ^d	(125) ^d	N/A	25 ^d	(25) ^d
	IV	N/A	NL	NL	N/A	NL	NL	N/A	NL	NL
	V	N/A	NL	NL	N/A	NL	NL	N/A	NL	NL
Oxidizer	4	H-1	1 ^{e,g}	(1) ^{e,g}	N/A	0.25 ^g	(0.25) ^g	N/A	0.25 ^g	(0.25) ^g
	3 ^k	H-2 or H-3	10 ^{d,e}	(10) ^{d,e}	N/A	2 ^d	(2) ^d	N/A	2 ^d	(2) ^d
	2	H-3	250 ^{d,e}	(250) ^{d,e}	N/A	250 ^d	(250) ^d	N/A	50 ^d	(50) ^d
	1	N/A	4000 ^{e,f}	(4000) ^{e,f}	N/A	4000 ^f	(4000) ^f	N/A	1000 ^f	(1000) ^f
Oxidizing gas	Gaseous Liquefied	H-3	N/A	N/A	1500 ^{d,e}	N/A	N/A	1500 ^{d,e}	N/A	N/A
			N/A	(150) ^{d,e}	N/A	N/A	(150) ^{d,e}	N/A	N/A	N/A
Pyrophoric material	N/A	H-2	4 ^{e,g}	(4) ^{e,g}	50 ^{e,g}	1 ^g	(1) ^g	10 ^g	0	0
Unstable (reactive) material	4	H-1	1 ^{e,g}	(1) ^{e,g}	10 ^g	0.25 ^g	(0.25) ^g	2 ^{e,g}	0.25 ^g	(0.25) ^g
	3	H-1 or H-2	5 ^{d,e}	(5) ^{d,e}	50 ^{d,e}	1 ^d	(1) ^d	10 ^{d,e}	1 ^d	(1) ^d
	2	H-3	50 ^{d,e}	(50) ^{d,e}	250 ^{d,e}	50 ^d	(50) ^d	250 ^{d,e}	10 ^d	(10) ^d
	1	N/A	NL	NL	NL	NL	NL	NL	NL	NL
Water-reactive material	3	H-2	5 ^{d,e}	(5) ^{d,e}	N/A	5 ^d	(5) ^d	N/A	1 ^d	(1) ^d
	2	H-3	50 ^{d,e}	(50) ^{d,e}	N/A	50 ^d	(50) ^d	N/A	10 ^d	(10) ^d
	1	N/A	NL	NL	N/A	NL	NL	N/A	NL	NL

For SI: 1 cubic foot = 0.028 m³, 1 pound = 0.454 kg, 1 gallon = 3.7785 L

NL = not limited; N/A = not applicable; UD = unclassified detonable

- a. For use of control areas, see Section 414.2.
- b. The aggregate quantity in use and storage shall not exceed the quantity listed for storage.
- c. The quantities of alcoholic beverages in retail and wholesale sales occupancies shall not be limited provided the liquids are packaged in individual containers not exceeding 1.3 gallons. In retail and wholesale sales occupancies, the quantities of medicines, foodstuffs, consumer or industrial products, and cosmetics containing not more than 50 percent by volume of water-miscible liquids, with the remainder of the solutions not being flammable, shall not be limited, provided that such materials are packaged in individual containers not exceeding 1.3 gallons.
- d. Maximum allowable quantities shall be increased 100% in buildings equipped throughout with an *automatic sprinkler system* in accordance with Section 903.3.1.1. Where Note e also applies, the increase for both notes shall be applied accumulatively.
- e. Maximum allowable quantities shall be increased 100% when stored in *approved* storage cabinets, day boxes, gas cabinets, or exhausted enclosures or in *listed* safety cans in accordance with Section 5003.9.10 of the *International Fire Code*. Where Note d also applies, the increase for both notes shall be applied accumulatively.
- f. The permitted quantities shall not be limited in a building equipped throughout with an *automatic sprinkler system* in accordance with Section 903.3.1.1.
- g. Permitted only in buildings equipped throughout with an *automatic sprinkler system* in accordance with Section 903.3.1.1.
- h. Containing not more than the maximum allowable quantity per *control area* of Class IA, IB, or IC flammable liquids.
- i. The maximum allowable quantity shall not apply to fuel oil storage complying with Section 603.3.2 of the *International Fire Code*.
- j. Quantities in parentheses indicate quantity units in parentheses at the head of each column.
- k. A maximum quantity of 200 pounds of solid or 20 gallons of liquid Class 3 oxidizers is allowed when such materials are necessary for maintenance purposes, operation, or sanitation of equipment. Storage containers and the manner of storage shall be *approved*.
- l. Net weight of the pyrotechnic composition of the fireworks. Where the net weight of the pyrotechnic composition of the fireworks is not known, 25% of the gross weight of the fireworks, including packaging, shall be used.
- m. For gallons of liquids, divide the amount in pounds by 10 in accordance with Section 5003.1.2 of the *International Fire Code*.
- n. For storage and display quantities in Group M and storage quantities in Group S occupancies complying with Section 414.2.5, see Tables 414.2.5(1) and 414.2.5(2).
- o. Densely packed baled cotton that complies with the packing requirements of ISO 8115 shall not be included in this material class.
- p. The following shall not be included in determining the maximum allowable quantities:
 - 1. Liquid or gaseous fuel in fuel tanks on vehicles
 - 2. Liquid or gaseous fuel in fuel tanks on motorized equipment operated in accordance with this code
 - 3. Gaseous fuels in piping systems and fixed appliances regulated by the *International Fuel Gas Code*
 - 4. Liquid fuels in piping systems and fixed appliances regulated by the *International Mechanical Code*
- q. Where manufactured, generated, or used in such a manner that the concentration and conditions create a fire or explosion hazard based on information prepared in accordance with Section 414.1.3.

Source: 2012 International Building Code, Country Club Hills, Illinois: International Code Council.
All footnote references are to the IBC.

IBC Table 307.1 (2)
Maximum Allowable Quantity Per Control Area of Hazardous Material Posing a Health Hazard^{a,b,c,i}

Material	Liquefied (150) ^h Storage ^d			Used in Closed Systems ^d			Used in Open Systems ^d	
	Solid Pounds (Cubic Feet)	Liquid Gallons (Pounds) ^{e,f}	Gas (Cubic Feet at NTP) ^e	Solid Pounds ^e	Liquid Gallons (Pounds) ^e	Gas (Cubic Feet at NTP) ^e	Solid Pounds ^e	Liquid Gallons (Pounds) ^e
Corrosive	5000	500	Gaseous 810 ^f Liquefied (150) ^h	5000	500	Gaseous 810 ^f Liquefied (150) ^h	1000	100
Highly toxic	10	(10) ^h	Gaseous 20 ^g Liquefied (4) ^{g,h}	10	(10) ⁱ	Gaseous 20 ^g Liquefied (4) ^{g,h}	3	(3) ⁱ
Toxic	500	(500) ^h	Gaseous 810 ^f Liquefied (150) ^{f,h}	500	(500) ⁱ	Gaseous 810 ^f Liquefied (150) ^{f,h}	125	(125)

For SI: 1 cubic foot = 0.028 m³, 1 pound = 0.454 kg, 1 gallon = 3.785 L

- For use of control areas, see Section 414.2.
- In retail and wholesale occupancies, the quantities of medicines, foodstuffs, consumer or industrial products, and cosmetics containing not more than 50% by volume of water-miscible liquids—with the remainder of the solutions not being flammable—shall not be limited, provided that such materials are packaged in individual containers not exceeding 1.3 gallons.
- For storage and display quantities in Group M and storage quantities in Group S, occupancies complying with Section 414.2.5, see Tables 414.2.5(1) and 414.2.5(2).
- The aggregate quantity in use and storage shall not exceed the quantity listed for storage.
- Maximum allowable quantities shall be increased 100% in buildings equipped throughout with an *approved automatic sprinkler system* in accordance with Section 903.3.1.1. Where Note f below also applies, the increase for both notes shall be applied cumulatively.
- Maximum allowable quantities shall be increased 100% when stored in *approved* storage cabinets, gas cabinets, or exhausted enclosures as specified in the *International Fire Code*. Where Note e above also applies, the increase for both notes shall be applied cumulatively.
- Allowed only when stored in approved exhausted gas cabinets or exhausted enclosures as specified in the *International Fire Code*.
- Quantities in parentheses indicate quantity units in parentheses at the head of each column.
- For gallons of liquids, divide the amount in pounds by 10 in accordance with Section 5003.1.2 of the *International Fire Code*.

Source: 2012 *International Building Code*, Country Club Hills, Illinois: International Code Council. All footnote references are to the IBC.

International Building Code Area Classification Descriptions

Occupancy Class	Description
A	Assembly Group A occupancy includes, among others, the use of a building or structure, or a portion thereof, for the gathering of persons for purposes such as civic, social or religious functions; recreation, food or drink consumption or awaiting transportation.
B	Business Group B occupancy includes, among others, the use of a building or structure, or a portion thereof, for office, professional or service-type transactions, including storage of records and accounts. Business occupancies shall include, but not be limited to, the following: - Airport traffic control towers - Ambulatory care facilities - Animal hospitals, kennels, and pounds - Banks - Barber and beauty shops - Car wash - Civic administration - Clinic, outpatient - Dry cleaning and laundries: pick-up and delivery stations and self-service - Educational occupancies for students above the 12th grade - Electronic data processing - Laboratories: testing and research - Motor vehicle showrooms - Post offices - Print shops - Professional services (architects, attorneys, dentists, physicians, engineers, etc.) - Radio and television stations - Telephone exchanges - Training and skill development not within a school or academic program
F	Factory Industrial Group F occupancy includes, among others, the use of a building or structure, or a portion thereof, for assembling, disassembling, fabricating, finishing, manufacturing, packaging, repair, or processing operations that are not classified as a Group H hazardous or Group S storage occupancy.

International Building Code Area Classification Descriptions (cont'd)

Occupancy Class	Description
F-1	Factory Industrial uses which are not classified as Factory Industrial F-2 Low Hazard. Examples include: Aircraft (manufacturing, not to include repair) Appliances Athletic equipment Automobiles and other motor vehicles Bakeries Beverages over 16-percent alcohol content Bicycles Boats Brooms or brushes Business machines Cameras and photo equipment Canvas or similar fabric Carpets and rugs (including cleaning) Clothing Construction and agricultural machinery Disinfectants Dry cleaning and dyeing Electric generation plants Electronics Engines (including rebuilding) Food processing and commercial kitchens not associated with restaurants, cafeterias and similar dining facilities Furniture Hemp products Jute products Laundries Leather products Machinery Metals Millwork (sash and door) Motion pictures and television filming (without spectators) Musical instruments Optical goods Paper mills or products Photographic film Plastic products Printing or publishing Recreational vehicles Refuse incineration Shoes Soaps and detergents Textiles Tobacco Trailers Wood: distillation Woodworking (cabinet) Upholstering

International Building Code Area Classification Descriptions (cont'd)

Occupancy Class	Description
F-2	Factory industrial uses that involve the fabrication or manufacturing of noncombustible materials which during finishing, packing or processing do not involve a significant fire hazard. Examples include: Beverages up to and including 16-percent alcohol content Brick and masonry Ceramic products Foundries Glass products Gypsum Ice Metal products (fabrication and assembly)
H	High-Hazard Group H occupancy includes, among others, the use of a building or structure, or a portion thereof, that involves the manufacturing, processing, generation or storage of materials that constitute a physical or health hazard in quantities in excess of those allowed in <i>control areas</i> complying with Section 414, based on the maximum allowable quantity limits for <i>control areas</i> set forth in Tables 307.1(1) and 307.1(2). Hazardous occupancies are classified in Groups H-1, H-2, H-3, H-4 and H-5 and shall be in accordance with this section, with the requirements of Section 415 and the <i>International Fire Code</i>. Hazardous materials stored, or used on top of roofs or canopies shall be classified as outdoor storage or use and shall comply with the <i>International Fire Code</i>.
H-1	Buildings and structures containing materials that pose a detonation hazard. Examples include: Deton-able pyrophoric materials, explosives, organic peroxides (unclassified detonable), Class 4 oxidizers, Class 3 detonable and Class 4 unstable (reactive) materials.
H-2	Buildings and structures containing materials that pose a deflagration hazard or a hazard from accelerated burning. Examples include: Class I, II, or IIIA flammable or combustible liquids which are used or stored in normally open containers or systems, or in closed containers or systems pressurized at more than 15 psi (103.4 kPa) gauge Combustible dusts where manufactured, generated or used in such a manner that the concentration and conditions create a fire or explosion hazard based on information prepared in accordance with Section 414.1.3 Cryogenic fluids, flammable Flammable gases Organic peroxides, Class I Oxidizers, Class 3, that are used or stored in normally open containers or systems or in closed containers or systems pressurized at more than 15 psi (103.4 kPa) gauge Pyrophoric liquids, solids, and gases, nondetonable Unstable (reactive) materials, Class 3, nondetonable Water-reactive materials, Class 3

International Building Code Area Classification Descriptions (cont'd)

Occupancy Class	Description
H-3	Buildings and structures containing materials that readily support combustion or that pose a physical hazard. Examples include: Class I, II, or IIIA flammable or combustible liquids that are used or stored in normally closed containers or systems pressurized at 15 pounds psi (103.4 kPa) gauge or less Combustible fibers, other than densely packed baled cotton Consumer fireworks, 1.4G (Class C, Common) Cryogenic fluids, oxidizing Flammable solids Organic peroxides, Class II and III Oxidizers, Class 2 Oxidizers, Class 3, that are used or stored in normally closed containers or systems pressurized at 15 pounds psi (103.4 kPa) gauge or less Oxidizing gases Unstable (reactive) materials, Class 2 Water-reactive materials, Class 2
H-4	Buildings and structures that contain materials that are health hazards. Examples include: Corrosives Highly toxic materials Toxic materials
H-5	Semiconductor fabrication facilities and comparable research and development areas in which hazardous production materials (HPM) are used and the aggregate quantity of materials is in excess of those listed in Tables 307.1(1) and 307.1(2).
I	Institutional Group I occupancy includes, among others, the use of a building or structure, or a portion thereof, in which care or supervision is provided to persons who are or are not capable of self-preservation without physical assistance or in which persons are detained for penal or correctional purposes or in which the liberty of the occupants is restricted.
M	Mercantile Group M occupancy includes, among others, the use of a building or structure, or a portion thereof, for the display and sale of merchandise and involves stocks of goods, wares, or merchandise incidental to such purposes and accessible to the public. Mercantile occupancy shall include, but not be limited to, the following: Department stores Drug stores Markets Motor fuel-dispensing facilities Retail or wholesale stores Sales rooms
R	Residential Group R includes among others, the use of a building or structure, or a portion thereof, for sleeping purposes when not classified as an Institution Group I or when not regulated by the <i>International Residential Code</i> .
S	Storage Group S occupancy includes among others, the use of a building or structure, or a portion thereof, for storage that is not classified as a hazardous occupancy.

International Building Code Area Classification Descriptions (cont'd)

Occupancy Class	Description
S-1	Moderate hazard Storage Group S-1. Buildings occupied for storage uses that are not classifies as Group S-2, including, but not limited to, storage of the following: Aerosols, levels 2 and 3 Aircraft hangars (storage and repair) Bags: cloth, burlap and paper Bamboos and rattan Baskets Belting: canvas and leather Books and paper in rolls or packs Boots and shoes Buttons, including cloth covered, pearl or bone Cardboard and cardboard boxes Clothing, woolen wearing apparel Cordage Dry boats (indoor) Furniture Furs Glues, mucilage, pastes and size Grains Horns and combs, other than celluloid Leather Linoleum Lumber Motor vehicle repair garages complying with the maximum allowable quantities of hazardous materials listed in Table 307.1(1) (see Section 406.8) Photo engravings Resilient flooring Silks Soaps Sugar Tires, bulk storage of Tobacco, cigars, cigarettes and snuff Upholstery and mattresses Wax candles

International Building Code Area Classification Descriptions (cont'd)

Occupancy Class	Description
S-2	Low-hazard storage, Group S-2. Includes among others, buildings used for the storage of noncombustible materials such as products on wood pallets or in paper cartons with or without single thickness divisions; or in paper wrappings. Such products are permitted to have a negligible amount of plastic <i>trim</i> such as knobs, handles or film wrapping. Group S-2 storage shall include, but not be limited to, storage of the following: Asbestos Beverages up to and including 16-percent alcohol in metal, glass or ceramic containers Cement in bags Chalk and crayons Dairy products in nonwaxed coated paper containers Dry cell batteries Electrical coils Electrical motors Empty cans Food products Foods in noncombustible containers Fresh fruits and vegetables in nonplastic trays or containers Frozen foods Glass Glass bottles, empty or filled with noncombustible liquids Gypsum board Inert pigments Ivory Meats Metal cabinets Metal desks with plastic tops and <i>trim</i> Metal parts Metals Mirrors Oil-filled and other types of distribution transformers Parking garages, open or enclosed Porcelain and pottery Stoves Talc and soapstones Washers and dryers

Chapter 8: Plant Design and Operations

International Building Code Area Classification Descriptions (cont'd)

Occupancy Class	Description
U	General. Buildings and structures of an accessory character and miscellaneous structures not classified in any specific occupancy shall be constructed, equipped and maintained to conform to the requirements of this code commensurate with the fire and life hazard incidental to their occupancy. Group U shall include, but not be limited to, the following: Agricultural buildings Aircraft hangars, accessory to a one- or two-family residence (see Section 412.5) Barns Carports Fences more than 6 feet (1829 mm) in height Grain silos, accessory to a residential occupancy Greenhouses Livestock shelters Private garages Retaining wall Sheds Stables Tanks Towers

Source: 2012 International Building Code, Country Club Hills, Illinois: International Code Council.

8.5.3.4 Area Separation Requirements

Required Separation of Occupancies (Hours)

Occupancy	A, E		I-1, I-3, I-4		I-2		R		F-2, S-2 ^b , U		B, F-1, M, S-1		H-1		H-2		H-3, H-4		H-5	
	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS
A, E	N	N	1	2	2	NP	1	2	N	1	1	2	NP	NP	3	4	2	3	2	NP
I-1, I-3, I-4	—	—	N	N	2	NP	1	NP	1	2	1	2	NP	NP	3	NP	2	NP	2	NP
I-2	—	—	—	—	N	N	2	NP	2	NP	2	NP	NP	NP	3	NP	2	NP	2	NP
R	—	—	—	—	—	—	N	N	1	2	1	2	NP	NP	3	NP	2	NP	2	NP
F-2, S-2 ^b , U	—	—	—	—	—	—	—	—	N	N	1	2	NP	NP	3	4	2	3	2	NP
B, F-1, M, S-1	—	—	—	—	—	—	—	—	—	—	N	N	NP	NP	2	3	1	2	1	NP
H-1	—	—	—	—	—	—	—	—	—	—	—	—	N	NP	NP	NP	NP	NP	NP	NP
H-2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	N	NP	1	NP	1	NP
H-3, H-4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1 ^d	NP	1	NP
H-5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	N	NP

S = Buildings equipped throughout with an automatic sprinkler system installed in accordance with Section 903.3.1.1

NS = Buildings not equipped throughout with an automatic sprinkler system installed in accordance with Section 903.3.1.1

N = No separation requirement

NP = Not permitted

a. See Section 420.

b. The required separation from areas used only for private or pleasure vehicles shall be reduced by 1 hour but to not less than 1 hour.

c. See Section 406.3.4.

d. Separation is not required between occupancies of the same classification.

Source: 2012 International Building Code, Country Club Hills, Illinois: International Code Council. All footnote references are to the IBC.

8.5.3.5 Wind Direction

Prevailing winds should be considered in both plant siting and layout:

- a. For siting, it is undesirable to locate a plant where prevailing winds would carry any fugitive emissions into nearby residential areas.
- b. In laying out a plant, safety considerations dictate that process units be located such that:
 1. Prevailing winds would not carry potentially flammable releases to an area of the plant where there could be a source of ignition.
 2. Prevailing winds would not carry potentially hazardous or toxic releases to an area of the plant where workers are in enclosed areas, e.g., offices, control rooms, or enclosed process buildings.

8.5.3.6 Designs for Preventing Fires and Explosions

Designs for Fire- and Explosion-Prevention

Feature	Explanation
Maintenance programs	The best way to prevent fires and explosions is to stop the release of flammable materials. Preventive maintenance programs are designed to upgrade systems before failures occur.
Fireproofing	Insulate vessels, pipes, and structures to minimize damage resulting from fires. Add deluge systems and design to withstand some damage from fires and explosions, e.g., use multiple deluge systems with separate shutoffs.
Control rooms	Design control rooms to withstand explosions.
Water supplies	Provide supply for maximum demand. Consider many deluge systems running simultaneously. Diesel-engine pumps are recommended.
Control valves for deluge	Place shutoffs well away from process areas.
Manual fire protection	Install hydrants, monitors, and deluge systems. Add good drainage.
Separate units	Separate (space) plants on a site, and separate units within plants. Provide access from two sides.
Utilities	Design steam, water, electricity, and air supplies to be available during emergencies. Place substations away from process areas.
Personnel areas	Locate personnel areas away from hazardous process and storage areas.
Group units	Group units in rows. Design for safe operation and maintenance. Create islands of risk by concentrating hazardous process units in one area. Space units so hot work can be performed on one group while another is operating.
Isolation valves	Install isolation valves for safe shutdowns. Install in safe and accessible locations at edge of unit or group.
Railroads and flares	Process equipment should be separated from flares and railroads.
Compressors	Place gas compressors downwind and separated from fired heaters.
Dikes	Locate flammable storage vessels at edge of unit. Dike vessels to contain and carry away spills.
Block valves	Place automated block valves to stop and/or control flows during emergencies. Consider the ability to transfer hazardous materials from one area to another.
Online analyzers	Add appropriate online analyzers to (1) monitor the status of the process, (2) detect problems at their incipient stage, and (3) take appropriate action to minimize effects of problems while still in initial phase of development.
Fail-safe designs	Design all controls to fail safely. Add safeguards for automated and safe shutdowns during emergencies.
Safety-instrumented systems (SIS)	Use SIS to automatically bring process to a safe state upon detection of potentially hazardous conditions.

Source: Davenport, John A., "Prevent Vapor Cloud Explosions," *Hydrocarbon Processing*: 1977, pp. 205–214.

8.5.4 Economics

Nomenclature

Abbreviation	Definition
A	Uniform amount per interest period
BV	Book value
C	Cost, present worth
D_j	Depreciation in year j
F	Future worth, value, or amount
G	Uniform gradient amount per interest period
i	Interest rate per interest period
m	Number of compounding periods per interest period
n	Number of interest periods, or the expected life of an asset
P	Present worth, value, or amount
r	Nominal annual interest rate
S_n	Expected salvage value in year n
<i>Subscripts</i>	
e	Effective
j	At time j
n	At time n

8.5.4.1 Cost Estimation and Project Evaluations

Basic Equations

Factor Name	Converts	Symbol	Formula
Single payment compound amount	to F given P	$(F/P, i\%, n)$	$F = P(1 + i)^n$
Single payment present worth	to P given F	$(P/F, i\%, n)$	$P = F(1 + i)^{-n}$
Uniform series sinking fund	to A given F	$(A/F, i\%, n)$	$A = F\left(\frac{i}{(1 + i)^n - 1}\right)$
Capital recovery	to A given P	$(A/P, i\%, n)$	$A = P\left(\frac{i(1 + i)^n}{(1 + i)^n - 1}\right)$
Uniform series compound amount	to F given A	$(F/A, i\%, n)$	$F = A\left(\frac{(1 + i)^n - 1}{i}\right)$
Uniform series present worth	to P given A	$(P/A, i\%, n)$	$P = A\left(\frac{(1 + i)^n - 1}{i(1 + i)^n}\right)$
Uniform gradient present worth	to P given G	$(P/G, i\%, n)$	$P = G\left(\frac{(1 + i)^n - 1}{i^2(1 + i)^n} - \frac{n}{i(1 + i)^n}\right)$
Uniform gradient* future worth	to F given G	$(F/G, i\%, n)$	$F = G\left(\frac{(1 + i)^n - 1}{i^2} - \frac{n}{i}\right)$
Uniform gradient uniform series	to A given G	$(A/G, i\%, n)$	$A = G\left(\frac{1}{i} - \frac{n}{(1 + i)^n - 1}\right)$
Interest rate	to i_e given r, m		$i_e = \left(1 + \frac{r}{m}\right)^m - 1$
Book value			$BV = \text{initial cost} - \sum D_j$

$$* \frac{F}{G} = \frac{\frac{F}{A} - n}{i} = \frac{F}{A} \times \frac{A}{G}$$

Depreciation Methods

Method	Description	Formula	Stipulations
Straight line	Annual depreciation cost	$d = \frac{V - V_s}{n}$	where d = annual depreciation, in \$ per year V = original value of the property at start the service-life period, completely installed and ready for use, in \$ V_s = salvage value of property at end of service life, in \$ n = service life, in years V_a = asset or book value a = number of years in actual use i = annual interest rate expressed as a fraction R = uniform annual payments made at end of each year (annual depreciation cost), in \$ $V - V_s$ = total amount of the annuity accumulated in an estimated service life of n years (original value of property minus salvage value at end of service life), in \$
	Book value	$V_a = V - ad$	
Declining balance (or fixed percentage)	Fixed percentage factor (f)	$f = 1 - \left(\frac{V_s}{V}\right)^{\frac{1}{n}}$	
	Book value	$V_a = V(1 - f)^a$	
Sinking fund	Uniform annual depreciation cost (R)	$R = (V - V_s) \frac{i}{(1 + i)^n - 1}$	
	Book value	$V_a = V - (V - V_s) \frac{(1 + i)^a - 1}{(1 + i)^n - 1}$	
Modified Accelerated Cost Recovery System	MACRS	$D_j = (\text{factor}) C$	factor—see table

Source: Peters, Max S., and Klaus D. Timmerhaus, *Plant Design and Economics for Chemical Engineers*, 4th ed., New York: McGraw-Hill, Inc., 1991, pp. 278, 280, 283, and 284.

MACRS FACTORS				
Year	Recovery Period (Years)			
	3	5	7	10
	Recovery Rate (Percent)			
1	33.33	20.00	14.29	10.00
2	44.45	32.00	24.49	18.00
3	14.81	19.20	17.49	14.40
4	7.41	11.52	12.49	11.52
5		11.52	8.93	9.22
6		5.76	8.92	7.37
7			8.93	6.55
8			4.46	6.55
9				6.56
10				6.55
11				3.28

8.5.4.2 Cost Indices

Cost indices are used to update historical cost data to the present. If a purchase cost is available for an item of equipment in year M , the equivalent current cost can be found using:

$$\text{Current \$} = (\text{Cost in year } M) \left(\frac{\text{Current Index}}{\text{Index in year } M} \right)$$

Cost Index

Year	Equipment Index	Labor Index	Material Index
0	341	100	100
1	344	107	106
2	352	116	112
3	360	128	113
4	368	139	120
5	383	147	127
6	401	155	139
7	421	164	161
8	432	176	174
9	444	187	188
10	503	197	205
11	552	210	228
12	579	218	241
13	514	223	248
14	554	231	251
15	569	236	255

8.5.4.3 Scaling Equipment Costs

The cost of Unit A at one capacity related to the cost of a similar Unit B with X times the capacity of Unit A is approximately X^n times the cost of Unit B, or:

$$\text{Cost of Unit A} = \text{Cost of Unit B} \left(\frac{\text{Capacity of Unit A}}{\text{Capacity of Unit B}} \right)^n$$

Typical Scaling Factors (n) for Equipment Cost vs. Capacity

Equipment	Size Range	Exponent
Agitator, propeller		0.50
Agitator, turbine		0.30
Boiler, industrial, all sizes		0.50
Boiler, package		0.72
Centrifuge, horizontal basket		1.72
Centrifuge, solid bowl		1.00
Conveyor, belt		0.65
Conveyor, bucket		0.77
Conveyor, screw		0.76
Conveyor, vibrating		0.87
Compressor, reciprocating, air-cooled, two-stage, 150 psig discharge	10 to 400 $\frac{\text{ft}^3}{\text{min}}$	0.69

Typical Scaling Factors (*n*) for Equipment Cost vs. Capacity (cont'd)

Equipment	Size Range	Exponent
Compressor, rotary, single-stage, sliding vane, 150 psig discharge	100 to 1000 $\frac{\text{ft}^3}{\text{min}}$	0.79
Crystallizer, growth		0.65
Crystallizer, forced circulation		0.55
Crystallizer, batch		0.70
Dryer, drum, single vacuum	10 to 10 ² ft ²	0.76
Dryer, drum, single atmospheric	10 to 10 ² ft ²	0.40
Dust collector, cyclone		0.80
Dust collector, cloth filter		0.68
Dust collector, precipitator		0.75
Evaporator, forced circulation		0.70
Evaporator, vertical and horizontal tube		0.53
Fan, centrifugal	10 ³ to 10 ⁴ $\frac{\text{ft}^3}{\text{min}}$	0.44
Fan, centrifugal	2 × 10 ⁴ to 7 × 10 ⁴ $\frac{\text{ft}^3}{\text{min}}$	1.17
Filter, plate and press		0.58
Filter, pressure leaf		0.55
Heat exchanger, shell and tube, floating head, carbon steel	100 to 400 ft ²	0.60
Heat exchanger, shell and tube, fixed sheet, carbon steel	100 to 400 ft ²	0.44
Mill, ball and roller		0.65
Mill, hammer		0.85
Motor, squirrel cage, induction, 440 volts, explosion proof	5 to 10 hp	0.69
Motor, squirrel cage, induction, 440 volts, explosion proof	20 to 200 hp	0.99
Pump, centrifugal, carbon steel		0.67
Pump, centrifugal, stainless steel		0.70
Pump, reciprocating, cast iron, horizontal, including motor	2 to 100 gpm	0.34
Reactor, stainless steel, 300 psi	100 to 1000 gal	0.56
Tanks and vessels, pressure, carbon steel		0.60
Tanks and vessels, horizontal, carbon steel		0.50
Tanks and vessels, stainless steel		0.68
Tray, bubble cap, carbon steel	3- to 10-ft diameter	1.20
Tray, sieve, carbon steel	3- to 10-ft diameter	0.86

Source: Guthrie, K.M., "Data and Techniques for Preliminary Capital Cost Estimating," *Chemical Engineering*, New York: Chemical Engineering, 1969.

8.5.4.4 Return on Investment (ROI)

Return on Investment

$$\text{ROI} = \text{rate of return (after taxes)} = \frac{\text{net income per year}}{\text{total investment}}$$

Payout

$$\text{Payout period (years)} = \frac{\text{total plant investment less working capital}}{\text{cash flow}}$$

where

cash flow = net yearly income after taxes plus depreciation

Taxation

Income taxes are paid at a specific rate on taxable income. Taxable income is total income less depreciation and ordinary expenses. Expenses do not include capital items, which should be depreciated.

Inflation

To account for inflation, the dollars are deflated by the general inflation rate per interest period f , and then they are shifted over the time scale using the interest rate per interest period i . Use an inflation adjusted interest rate per interest period d for computing present worth values P .

The formula for d is $d = i + f + (i \times f)$

Capitalized costs

Capitalized costs are present worth values using an assumed perpetual period of time.

$$\text{Capitalized Costs} = P = \frac{A}{i}$$

Bonds

Bond value equals the present worth of the payments the purchaser (or holder of the bond) receives during the life of the bond at some interest rate i .

Bond yield equals the computed interest rate of the bond value when compared with the bond cost.

Rate-of-return

The minimum acceptable rate-of-return (MARR) is that interest rate that one is willing to accept, or the rate one desires to earn on investments. The rate-of-return on an investment is the interest rate that makes the benefits and costs equal.

Benefit-cost analysis

In a benefit-cost analysis, the benefits B of a project should exceed the estimated costs C .

$$B - C \geq 0, \text{ or } B/C \geq 1$$

8.6 Materials of Construction

8.6.1 Material Selection

8.6.1.1 Thermoplastics

Typical Thermoplastic Properties

Property	Unit	PP		PVC	CPVC	PVDF		ECTFE	ETFE	FEP	TFE	PFA
		Homo-polymer	Copolymer			Homo-polymer	Copolymer					
Density	$\frac{\text{g}}{\text{cm}^3}$	0.91	0.88–0.91	1.38	1.5	1.75–1.79	1.76–1.79	1.68	1.70	2.12–2.17	2.2–2.3	2.12–2.17
Melting point (crystalline)	°C	160–175	150–175	—	—	160–170	141–160	220–245	270	275	327	310
	°F	320–347	302–347	—	—	320–340	285–320	460	518	527	621	590
<i>Physical Properties</i>												
Break strength; ASTM D 638	kpsi	4.5–6.0	4.0–5.3	6.0–7.5	—	4.5–7.0	3.5–6.0	6.6–7.8	6.5	2.7–3.1	2.0–2.7	4.0–4.5
Modulus flex @ 73°F; ASTM D 790	MPa	1135–1550	345–1035	—	—	—	—	—	—	—	—	—
	kpsi	165–225	50–150	—	—	165–325	90–180	180–260	200	80–95	190–235	120
Yield strength; ASTM D 638	kpsi	4.5–5.4	1.6–4.0	—	—	5.0–8.0	2.9–5.5	—	7.1	—	—	—
<i>Thermal Properties</i>												
HTD at 0.46 MPa (66 psi); ASTM D 648	°C	107–121	75–89	57	—	132–150	93–110	90	104	70	221	75
	°F	225–250	167–192	135	—	270–300	200–230	194	220	158	250	166
Linear coefficient of expansion; ASTM D 696	$\frac{\text{in.}}{\text{in.} \cdot ^\circ\text{C}} \times 10^{-5}$	10	7–9.5	4.4	3.9	7.2–14.4	14.0	8	6	8–11	10	12
Conductivity; ASTM C 177	$\frac{\text{W}}{\text{m} \cdot \text{K}}$	0.1	0.16	—	—	0.17–0.19	0.16	—	—	—	—	—
	$\frac{\text{Btu}}{\text{ft}^2 \cdot \text{hr} \cdot \frac{^\circ\text{F}}{\text{in.}}}$	0.7	1.1	—	—	1.18–1.32	1.11	—	—	—	—	—

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8.6.1.2 Gasket Materials

Important Properties of Gasket Materials*

Material	Max Service Temp °F	Important Properties
Rubber (straight):		
Natural	225	Good mechanical properties. Impervious to water. Fair to good resistance to acids, alkalies. Poor resistance to oils, gasoline. Poor weathering, aging properties.
Styrene-butadiene (SBR)	250	Better water resistance than natural rubber. Fair to good resistance to acids, alkalies. Unsuitable with gasoline, oils, and solvents.
Butyl	300	Very good resistance to water, alkalies, many acids. Poor resistance to oils, gasoline, most solvents (except oxygenated).
Nitrile	300	Very good resistance to water. Excellent resistance to oils, gasoline. Fair to good resistance to acids, alkalies.
Polysulfide	150	Excellent resistance to oils, gasoline, aliphatic, and aromatic hydrocarbon solvents. Very good resistance to water. Good resistance to alkalies. Fair acid resistance. Poor mechanical properties.
Neoprene	250	Excellent mechanical properties. Good resistance to nonaromatic petroleum, fatty oils, solvents (except aromatic, chlorinated, or ketone types). Good water and alkali resistance. Fair acid resistance.
Silicone	600	Excellent heat resistance. Fair water resistance. Poor resistance to steam at high pressures. Fair to good acid, alkali resistance. Poor (except fluorosilicone rubber) resistance to oils, solvents.
Acrylic	450	Good heat resistance but poor cold resistance. Good resistance to oils, aliphatic and aromatic hydrocarbons. Poor resistance to water, alkalies, some acids.
Chlorosulfonated polyethylene (Hypalon)	250	Excellent resistance to oxidizing chemicals, ozone, weathering. Relatively good resistance to oils, grease. Poor resistance to aromatic or chlorinated hydrocarbons. Good mechanical properties.
Fluoroelastomer (Viton, Fluorel 2141, Kel-F)	450	Can be used at high temperatures with many fuels, lubricants, hydraulic fluids, solvents. Highly resistant to ozone, weathering. Good mechanical properties.
Asbestos:		
Compressed asbestos-rubber sheet	To 700	Large number of combinations available; properties vary widely depending on materials used.
Asbestos-rubber woven sheet	To 250	Same as above.
Asbestos-rubber (beater addition process)	400	Same as above.
Asbestos composites	To 1000	Same as above.
Asbestos-TFE	To 500	Combines heat resistance and sealing properties of asbestos with chemical resistance of TFE.
Cork compositions	250	Low cost. Truly compressible materials that permit substantial deflections with negligible side flow. Conform well to irregular surfaces. High resistance to oils. Good resistance to water, many chemicals. Should not be used with inorganic acids, alkalies, oxidizing solutions, live steam.

Important Properties of Gasket Materials* (cont'd)

Material	Max Service Temp °F	Important Properties
Cork rubber	300	Controlled compressibility properties. Good conformability, fatigue resistance. Chemical resistance depends on kind of rubber used.
Plastics:		
TFE (solid) (tetrafluoroethylene, Teflon)	500	Excellent resistance to almost all chemicals and solvents. Good heat resistance; exceptionally good low-temperature properties. Relatively low compressibility and resilience.
TFE (filled)	To 500	Selectively improved mechanical and physical properties. However, fillers may lower resistance to specific chemicals.
TFE composite	To 500	Chemical and heat resistance comparable with solid TFE. Inner gasket material provides better resiliency and deformability.
CFE (chlorotrifluoroethylene, Kel-F)	350	Higher cost than TFE. Better chemical resistance than most other gasket materials, although not quite as good as TFE.
Vinyl	212	Good compressibility, resiliency. Resistant to water, oils, gasoline, and many acids and alkalies. Relatively narrow temperature range.
Polyethylene	150	Resists most solvents. Poor heat resistance.
Plant fiber:		
Neoprene-impregnated wood fiber	175	Nonporous; recommended for glycol, oil, and gasoline to 175°F.
SBR-bonded cotton	230	Good water resistance.
Nitrile rubber-cellulose fiber		Resists oil at high temperatures.
Vegetable fiber, glue binder	212	Resists oil and water to 212°F.
Vulcanized fiber		Low cost. Good mechanical properties. Resists gasoline, oils, greases, waxes, many solvents.
Inorganic fibers	To 2200	Excellent heat resistance. Poor mechanical properties.
Felt:		
Pure felt		Resilient, compressible, and strong, but not impermeable. Resists medium-strength mineral acids and dilute mineral solutions if not intermittently dried. Resists oils, greases, waxes, most solvents. Damaged by alkalies.
TFE-impregnated	300	Good chemical and heat resistance.
Petrolatum- or paraffin-impregnated		High water repellency.
Rubber-impregnated		Many combinations available; properties vary widely depending on materials used.

Important Properties of Gasket Materials* (cont'd)

Material	Max Service Temp °F	Important Properties
Metal:		
Lead	500	Good chemical resistance. Best conformability of metal gaskets.
Tin		Good resistance to neutral solutions. Attacked by acids and alkalies.
Aluminum	800	High corrosion resistance. Slightly attacked by strong acids and alkalies.
Copper, brass		Good corrosion resistance at moderate temperatures.
Nickel	1400	High corrosion resistance.
Monel	1500	High corrosion resistance. Good against most acids and alkalies, but attacked by strong hydrochloric and strong oxidizing acids.
Inconel	2000	Excellent heat, oxidation resistance.
Stainless steel		High corrosion resistance. Properties depend on type used.
Metal composites		Many combinations available; properties vary widely depending on materials used.
Leather	220	Low cost. Limited chemical and heat resistance. Not recommended against pressurized steam, acid, or alkali solutions.
Glass fabric		High strength and heat resistance. Can be impregnated with TFE for high chemical resistance.

Important Properties of Packing and Sealing Materials

Material	Max Service Temp °F	Important Properties
Rubber (straight)	600	See Gasket Materials for properties. Mainly used for ring-type seals, although some types are available as spiral packings.
Rubber composites:		
Cotton-reinforced	350	High strength. Chemical resistance depends on type of rubber used; however, most types are noted for high resistance to water, aqueous solutions.
Asbestos-reinforced	450	High strength combined with good heat resistance.
Asbestos:		
Plain, braided asbestos	500	Heat resistance combined with resistance to water, brine, oil, many chemicals. Can be reinforced with wire.
Impregnated asbestos	To 750	Environmental properties vary widely depending on type of asbestos and impregnant used. Neoprene-cemented type resists hot oils, gasoline, and solvents. Oil-and-wax-impregnated type resists caustics. Wax-impregnated blue asbestos type has high acid resistance. TFE-impregnated has good all-around chemical resistance.
Asbestos composites	To 1200	End properties vary widely depending on secondary material used.

Important Properties of Packing and Sealing Materials

Material	Max Service Temp °F	Important Properties
Metals:		
Copper	To 1500	Properties depend on other construction materials and form of copper used. Packing made of copper foil over asbestos core resists steam and alkalis to 1000°F. Packing of braided copper tinsel resists water, steam, and gases to 1500°F.
Aluminum	To 1000	Resists hot petroleum derivatives, gases, foodstuffs, many organic acids.
Lead	550	Many types are available.
Organic fiber:		
Flax	300	Good water resistance.
Jute	300	Good water resistance.
Ramie	300	Good resistance to water, brine, cold oil.
Cotton	300	Good resistance to water, alcohol, dilute aqueous solutions.
Rayon	300	Good resistance to water, dilute aqueous solutions.
Felt	300	See Gasket Materials.
Leather	To 210	Good mechanical properties for sealing. Resistant to alcohol, gasoline, many oils and solvents, synthetic hydraulic fluids, water.
TFE	To 500	Available in many forms, all of which have high chemical resistance.
Carbon graphite	700	Good bearing and self-lubricating properties. Good resistance to chemicals, heat.

* From *Materials in Engineering Design*, New York: Reinhold, 1959, p. 11-126.

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8.6.2 Corrosion

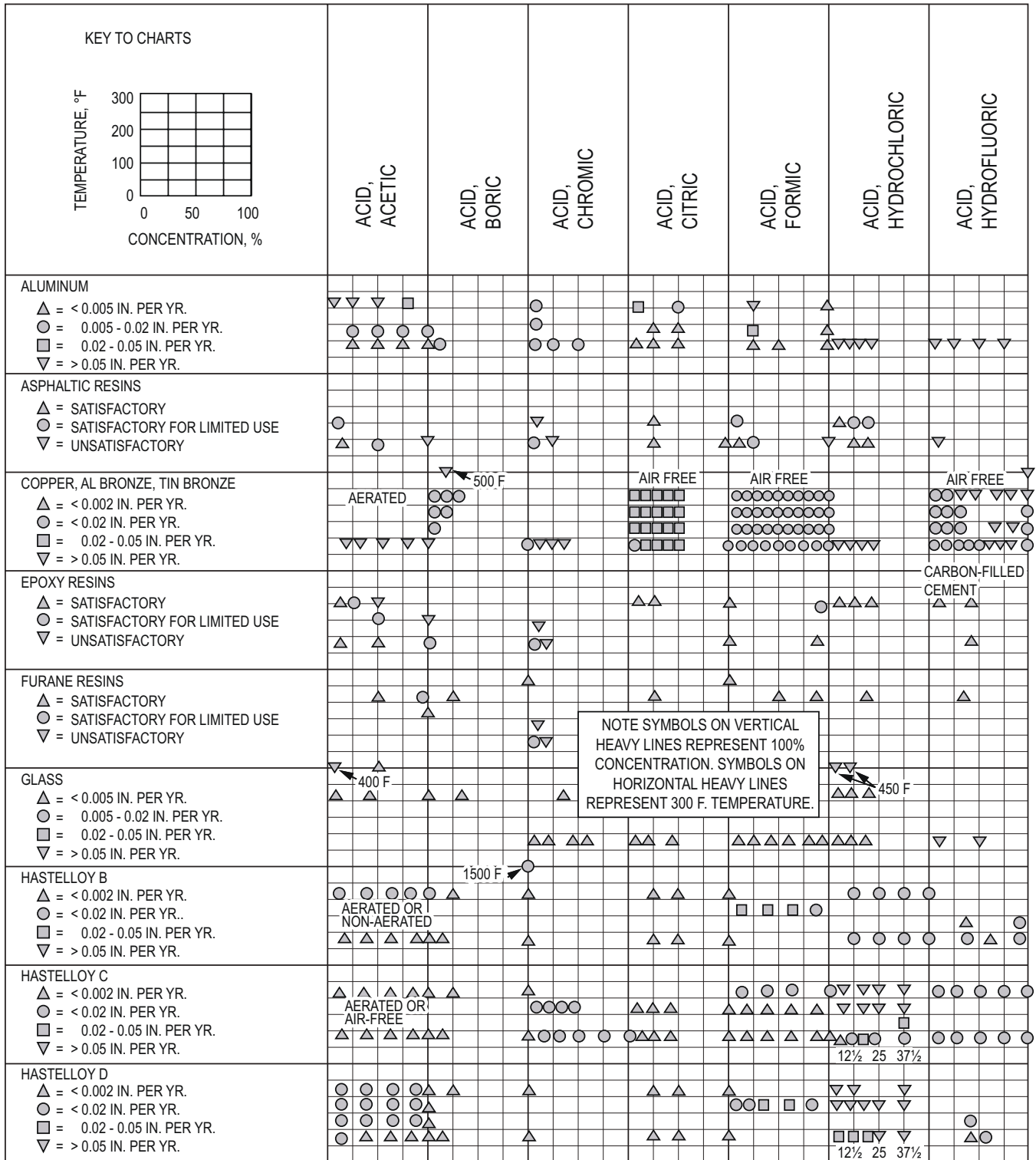
Corrosion is a natural process that converts a refined metal to a more stable form such as its oxide, hydroxide, or sulfide. It is the gradual destruction of a material by chemical reaction with its environment. Corrosion effects must be taken into account during the design of any system, unit, facility, or plant.

Use the following corrosion data charts to assist in narrowing the field of choice of materials. Once the choice has been narrowed, the effects of contaminants, aeration, galvanic coupling, erosion, and so on must be taken into account. Field testing is best for final suitability decisions.

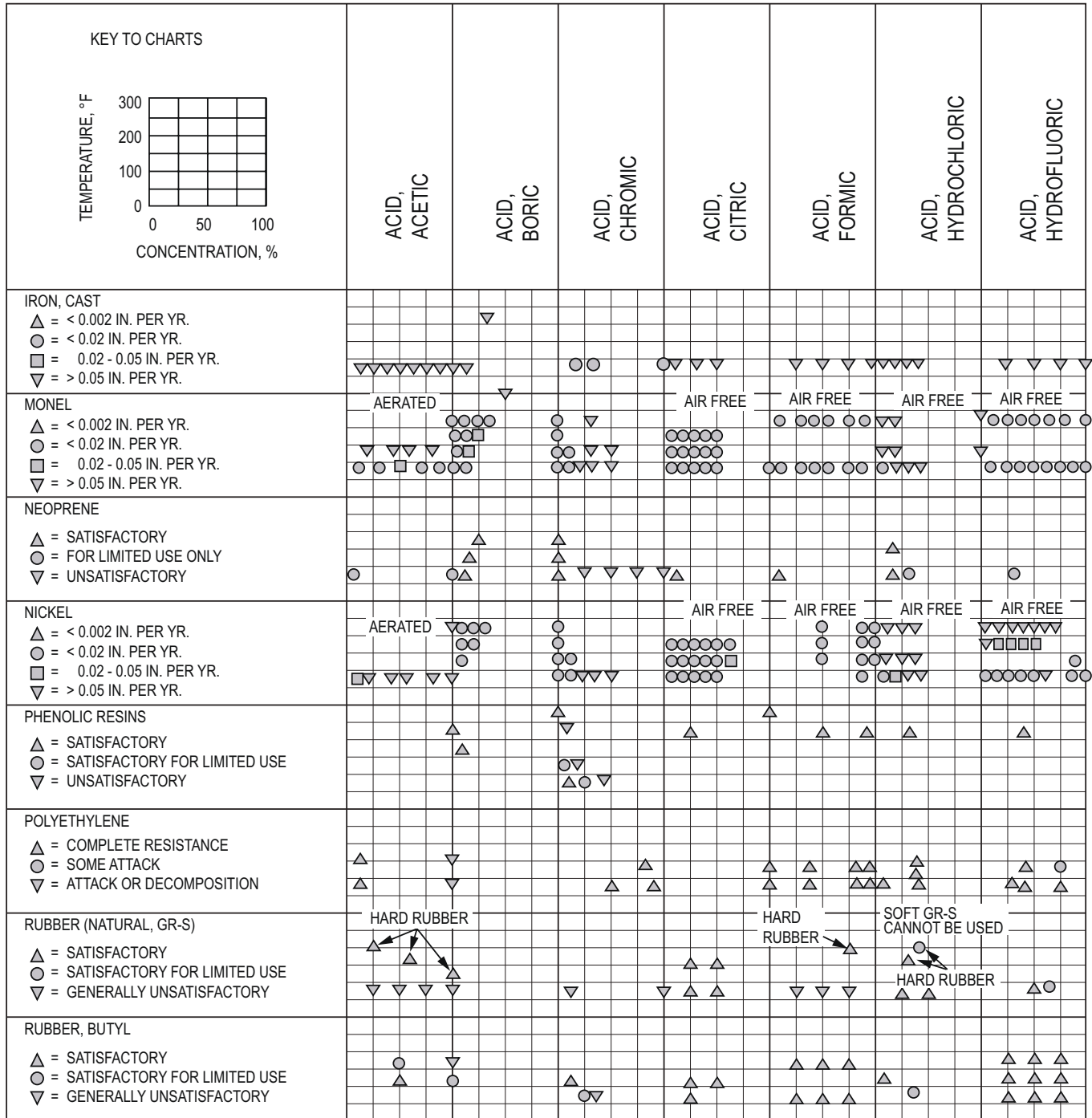
Source: All corrosion data from Perry, John H., and D. Green, *Perry's Chemical Engineers' Handbook*, 6th ed., New York: McGraw-Hill, 1963, pp. 23-13 to 23-30.

Chapter 8: Plant Design and Operations

Detailed Corrosion Data on Construction Materials

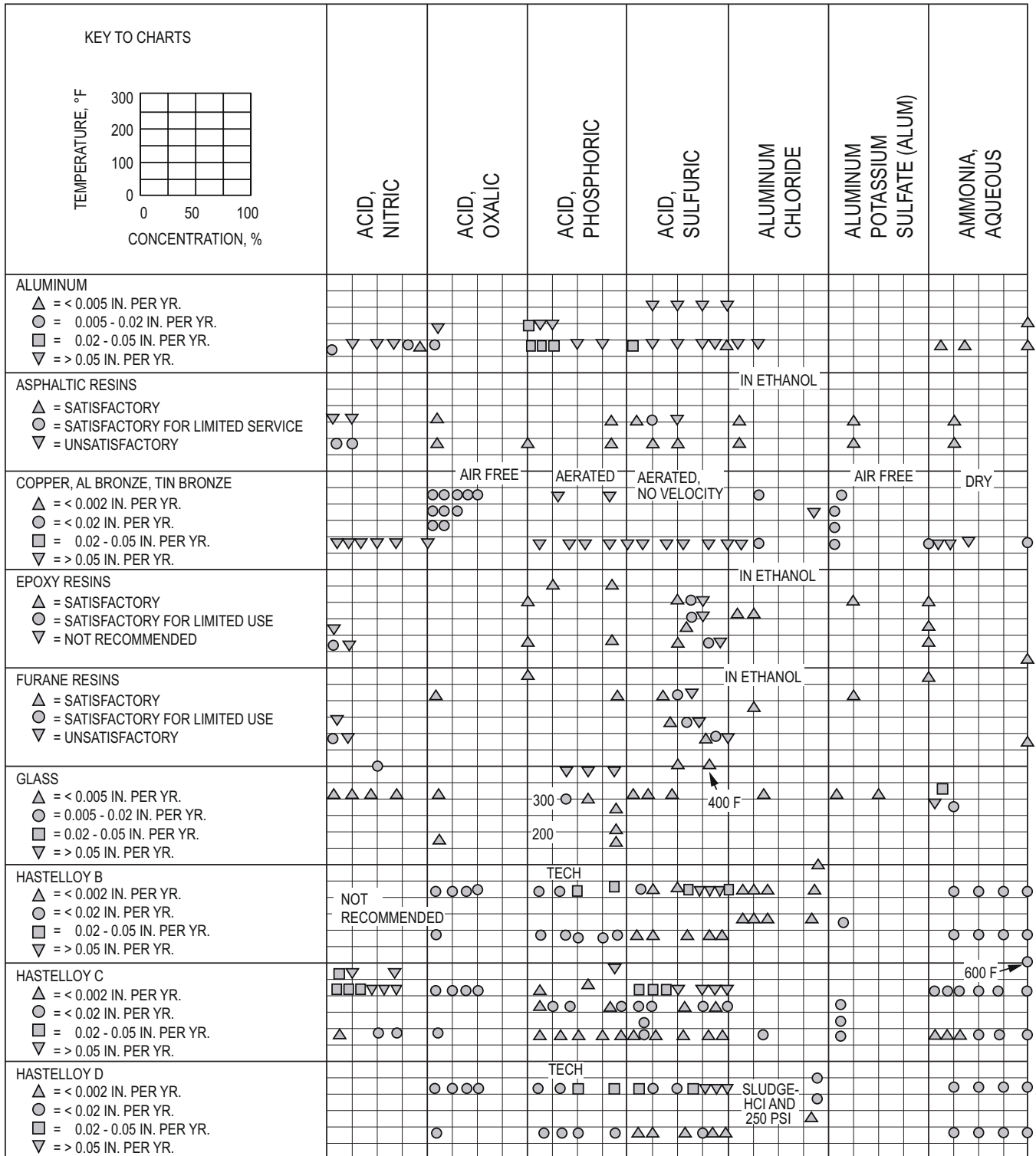


Detailed Corrosion Data on Construction Materials (cont'd)

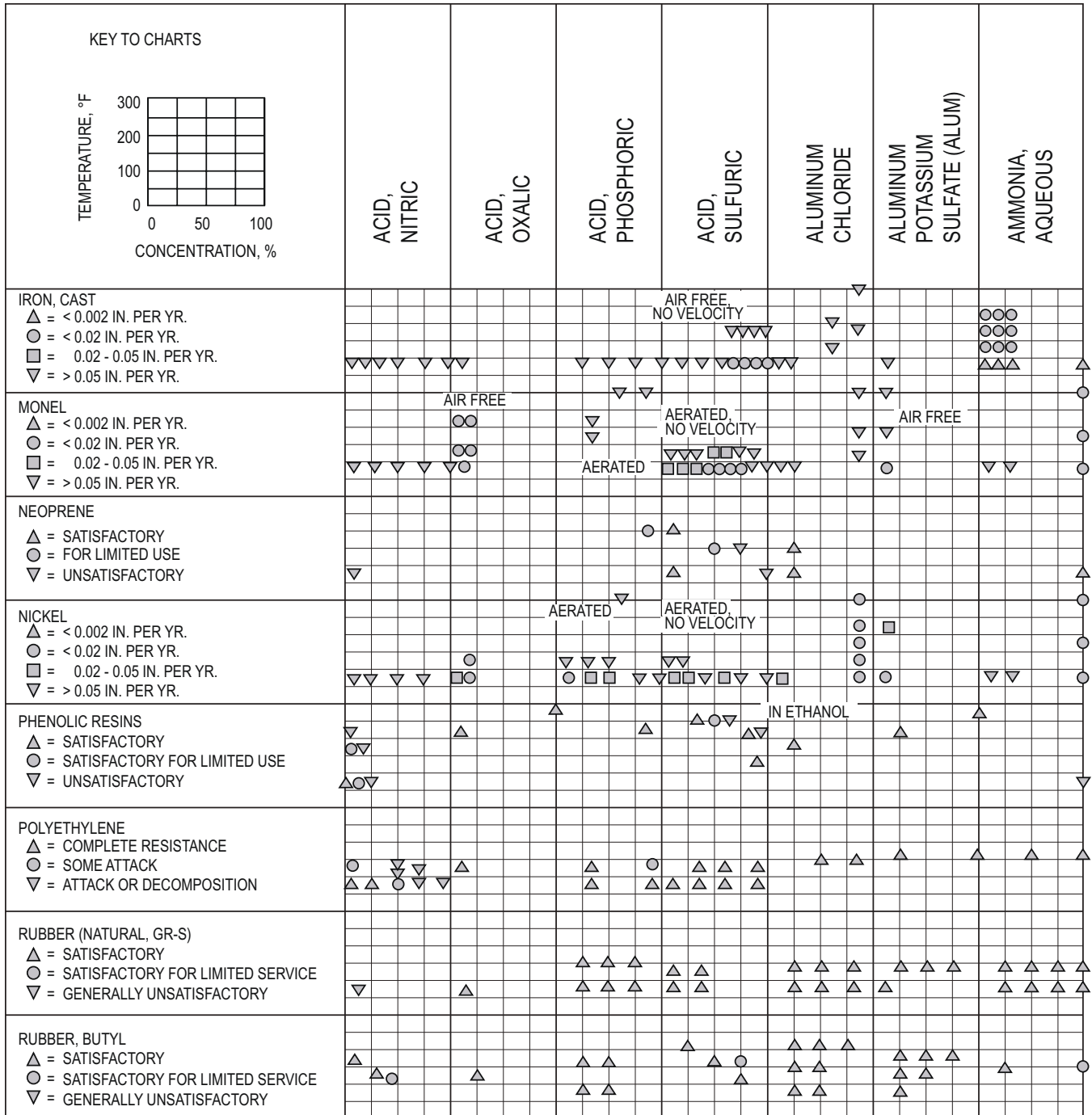




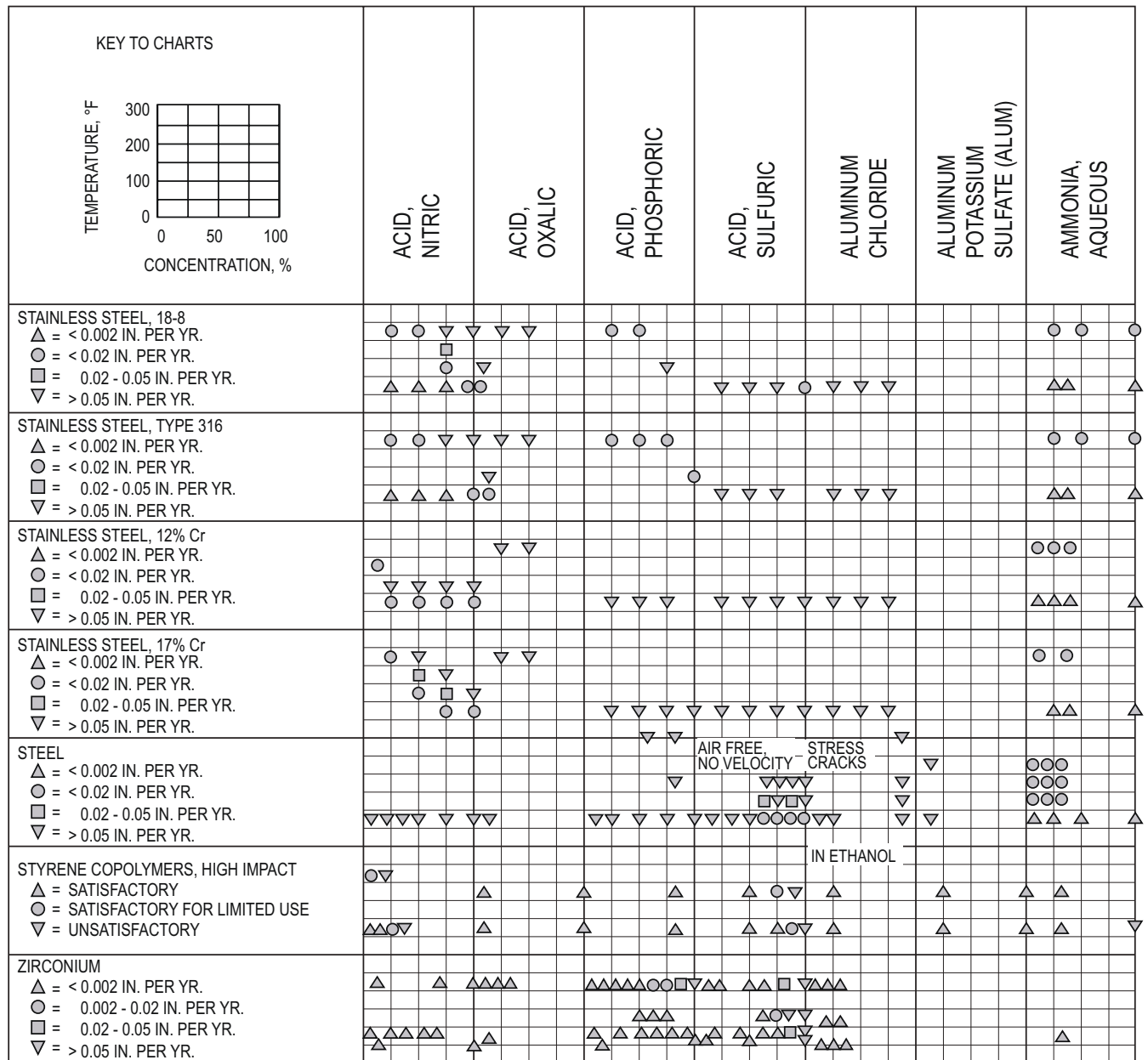
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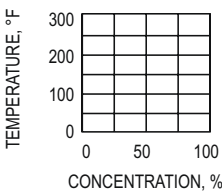
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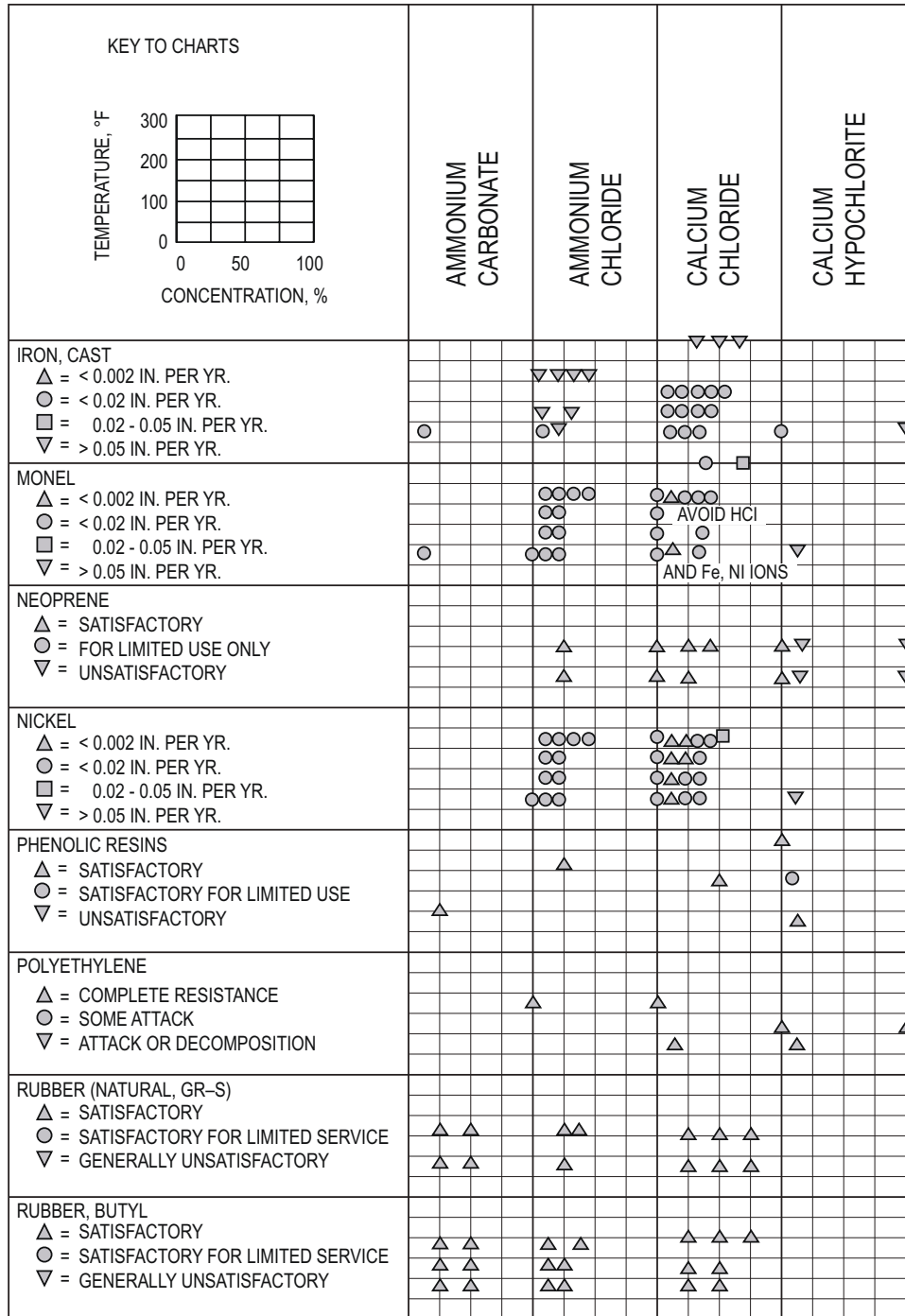
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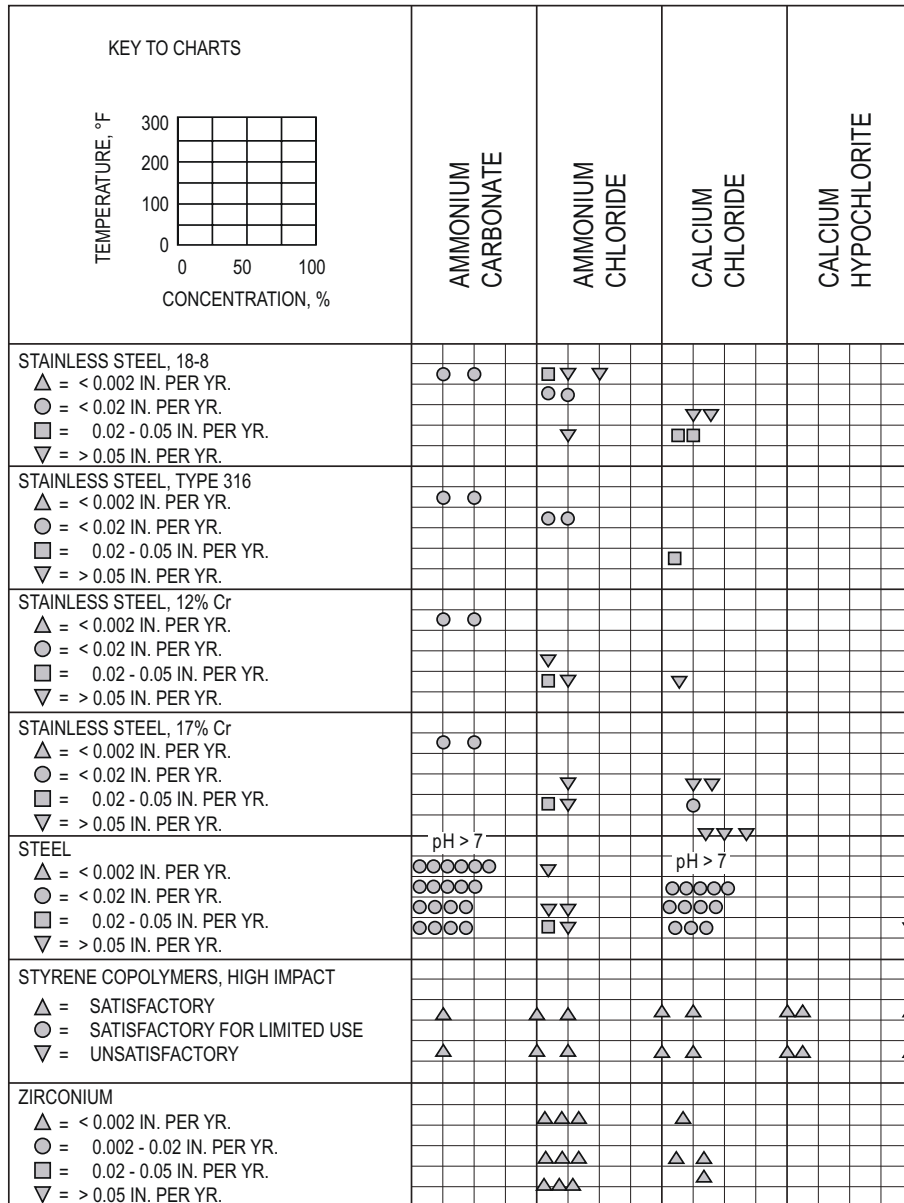
Detailed Corrosion Data on Construction Materials (cont'd)

KEY TO CHARTS 	AMMONIUM CARBONATE	AMMONIUM CHLORIDE	CALCIUM CHLORIDE	CALCIUM HYPOCHLORITE
ALUMINUM △ = < 0.005 IN. PER YR. ○ = 0.005 - 0.02 IN. PER YR. ■ = 0.02 - 0.05 IN. PER YR. ▽ = > 0.05 IN. PER YR.	△	▽		
ASPHALTIC RESINS △ = SATISFACTORY ○ = SATISFACTORY FOR LIMITED USE ▽ = UNSATISFACTORY	△	△	△	△
COPPER, AL BRONZE, TIN BRONZE △ = < 0.002 IN. PER YR. ○ = < 0.02 IN. PER YR. ■ = 0.02 - 0.05 IN. PER YR. ▽ = > 0.05 IN. PER YR.	▽	▽	○	▽
EPOXY RESINS △ = SATISFACTORY ○ = SATISFACTORY FOR LIMITED USE ▽ = NOT RECOMMENDED	△	△	△	△
FURANE RESINS △ = SATISFACTORY ○ = SATISFACTORY FOR LIMITED USE ▽ = UNSATISFACTORY	△	△	△	△
GLASS △ = < 0.005 IN. PER YR. ○ = 0.005 - 0.02 IN. PER YR. ■ = 0.02 - 0.05 IN. PER YR. ▽ = > 0.05 IN. PER YR.		△	△	△
HASTELLOY B △ = < 0.002 IN. PER YR. ○ = < 0.02 IN. PER YR. ■ = 0.02 - 0.05 IN. PER YR. ▽ = > 0.05 IN. PER YR.	○	○	○	○
HASTELLOY C △ = < 0.002 IN. PER YR. ○ = < 0.02 IN. PER YR. ■ = 0.02 - 0.05 IN. PER YR. ▽ = > 0.05 IN. PER YR.	○	○	△	△
HASTELLOY D △ = < 0.002 IN. PER YR. ○ = < 0.02 IN. PER YR. ■ = 0.02 - 0.05 IN. PER YR. ▽ = > 0.05 IN. PER YR.	○	○	○	○

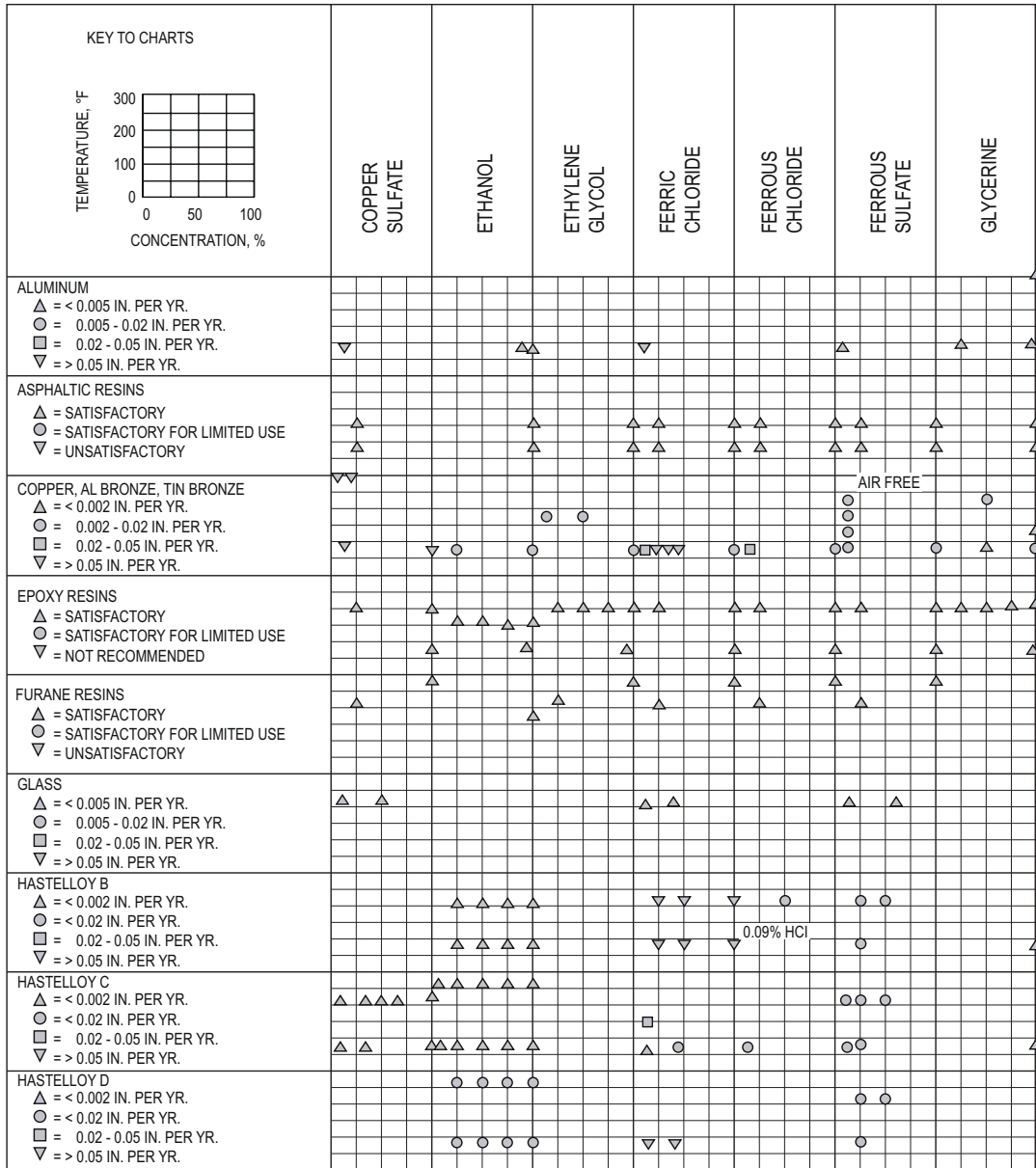
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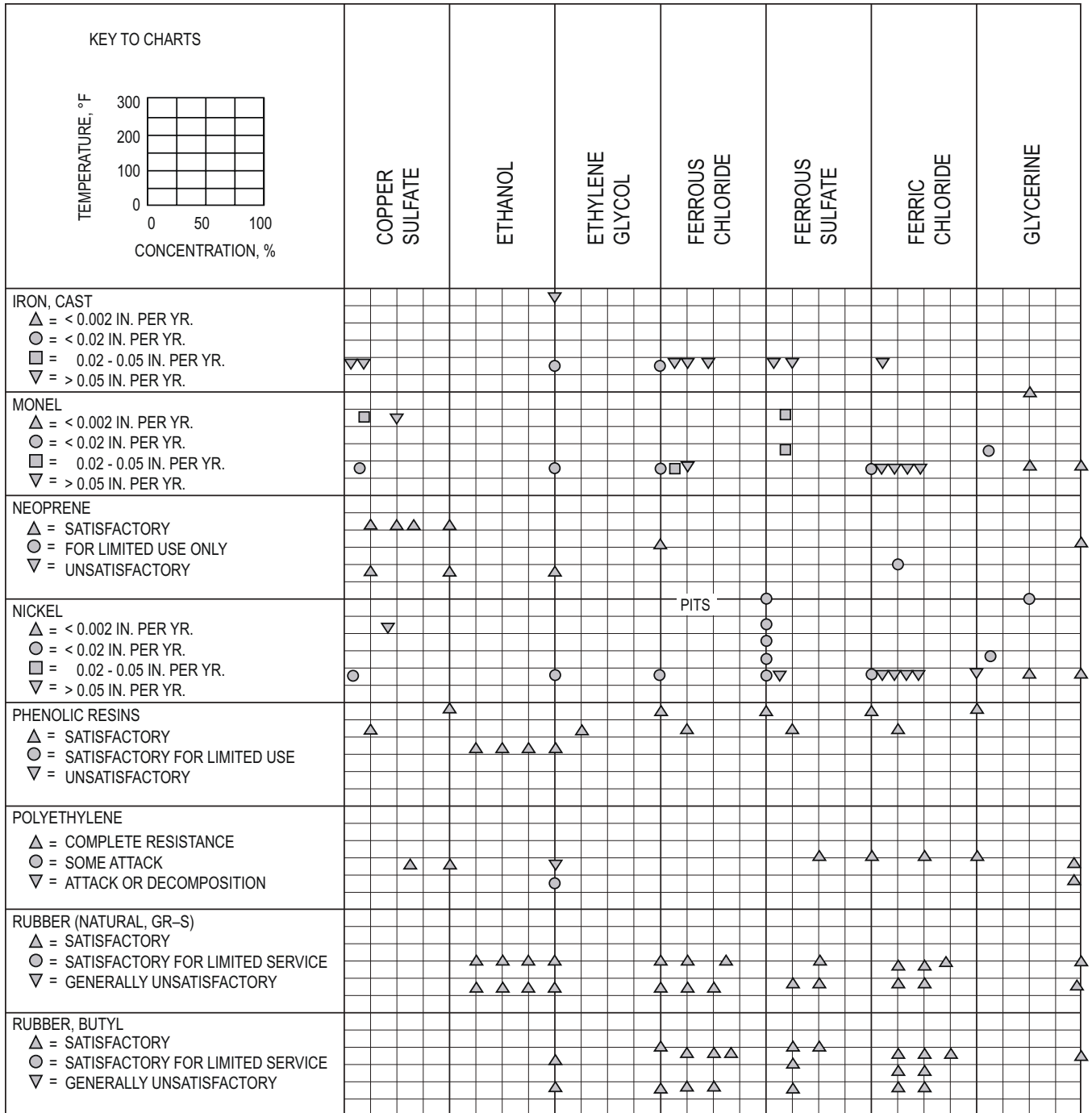
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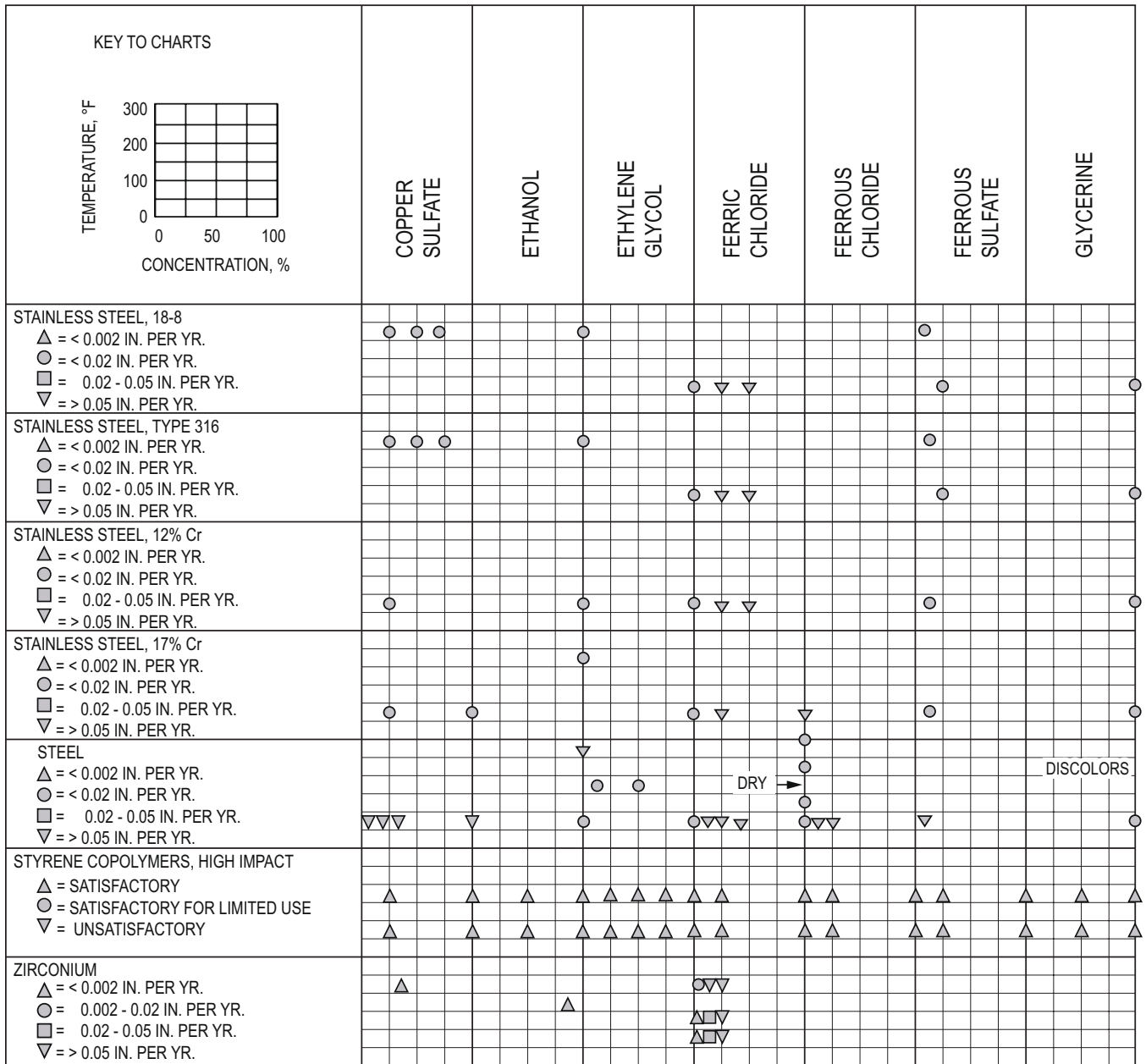
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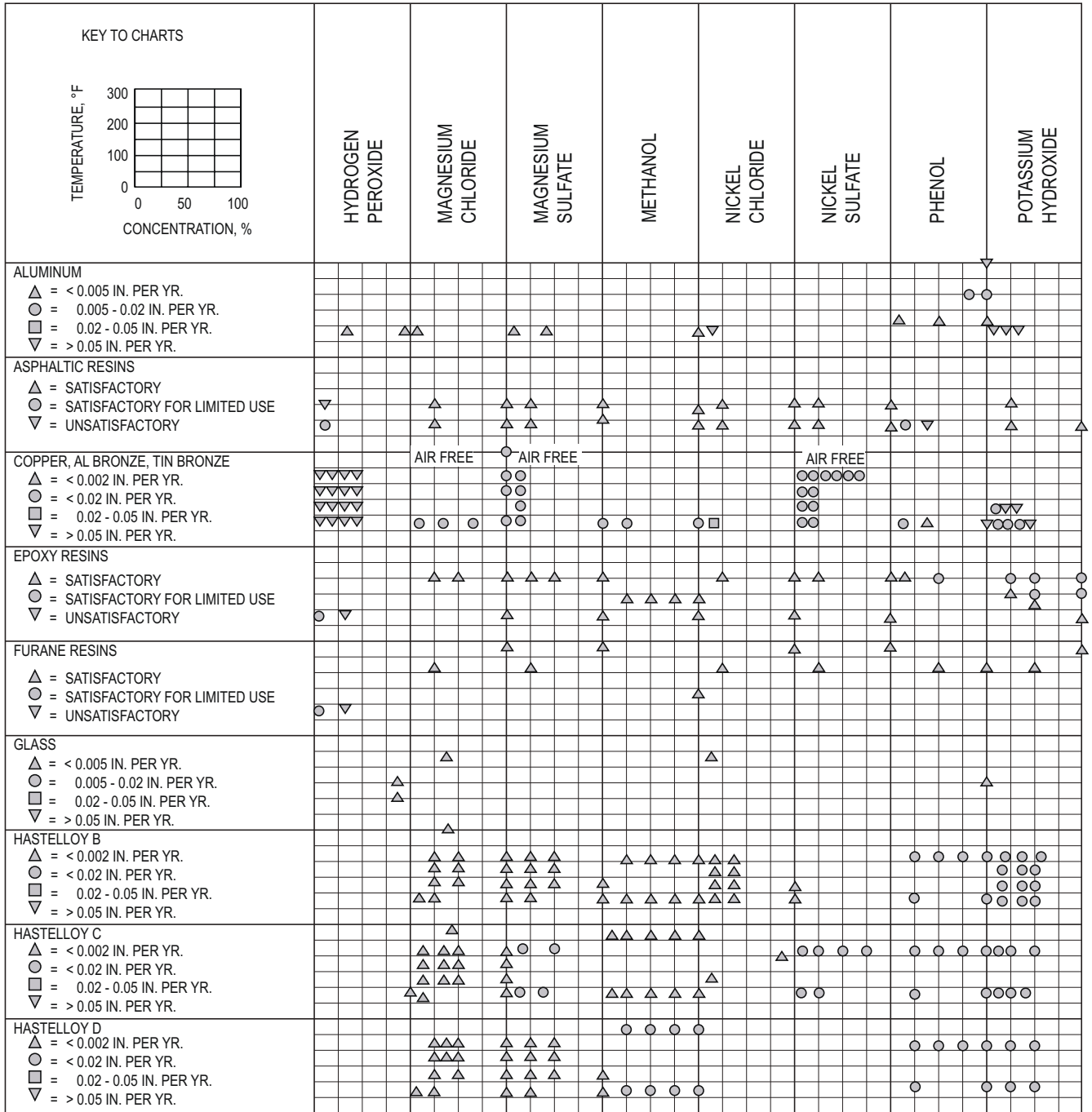
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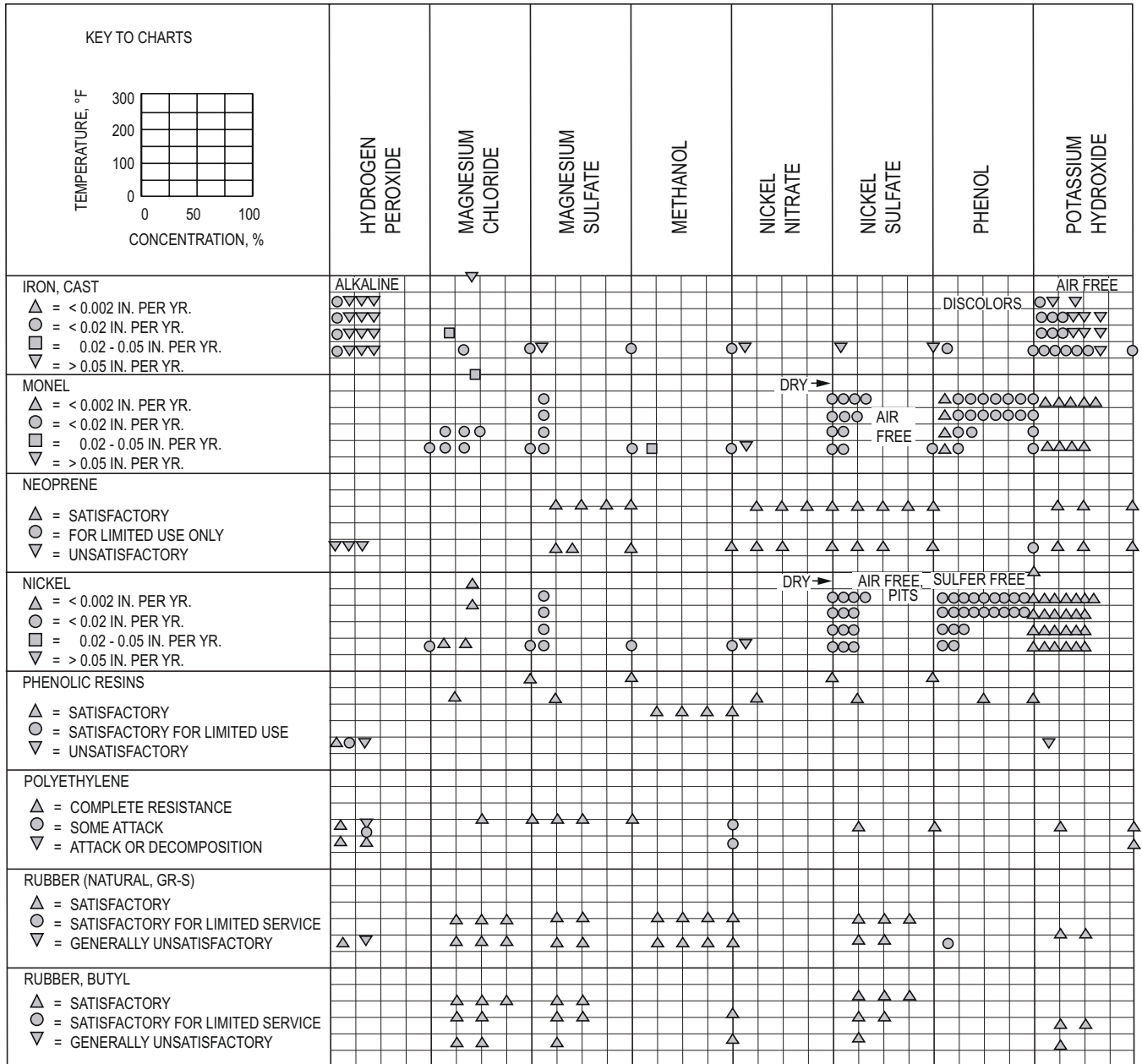
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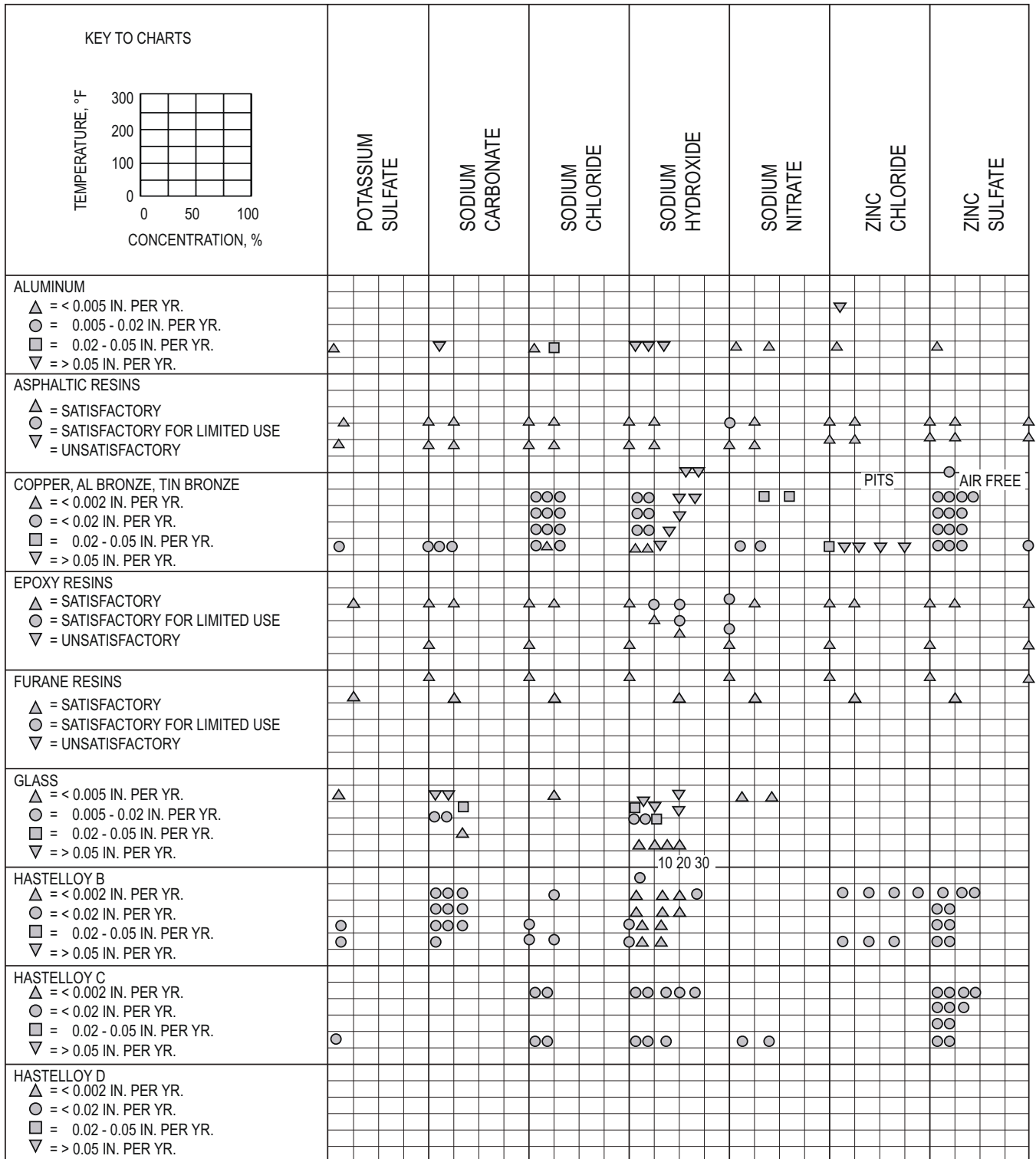


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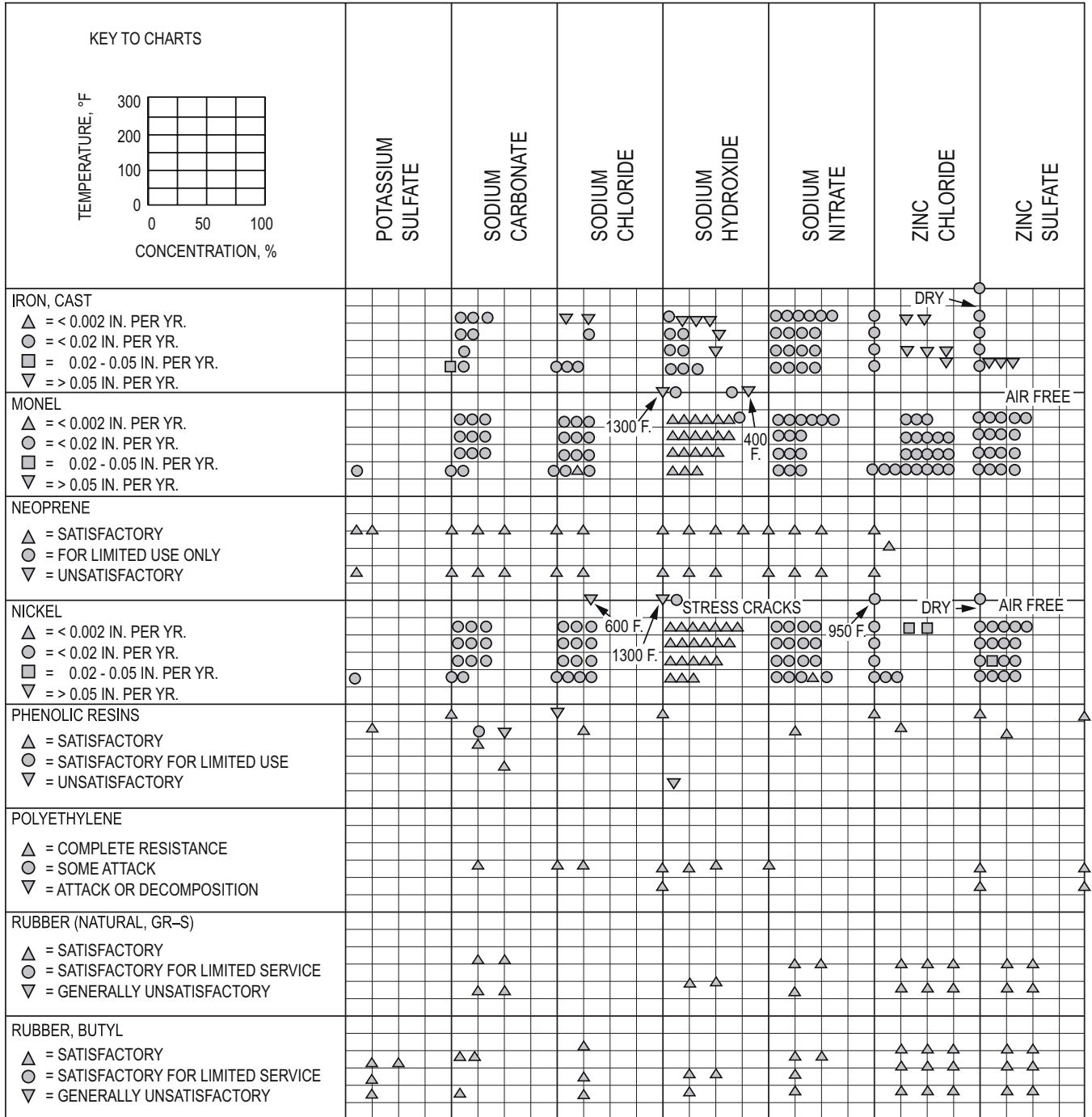




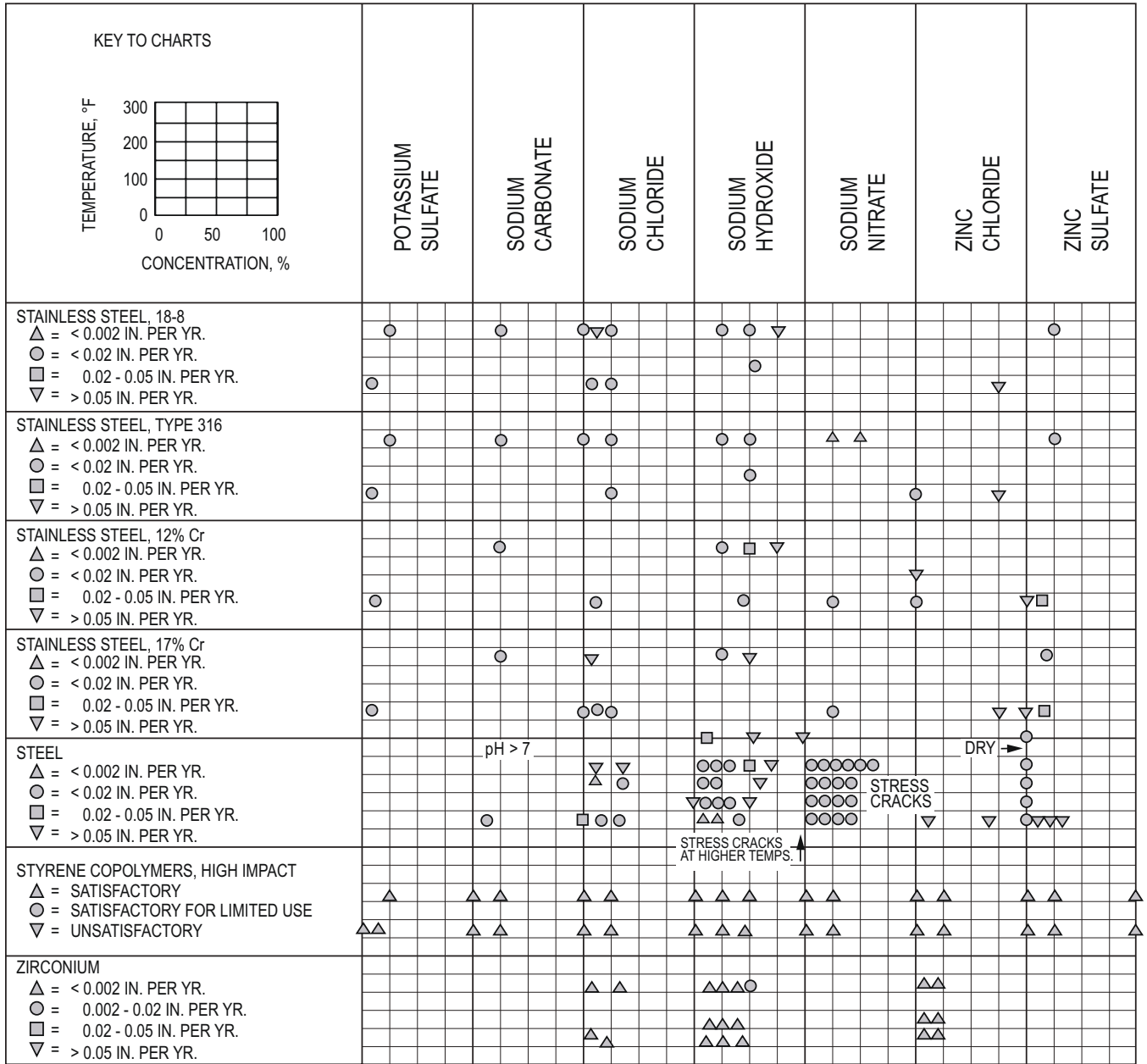
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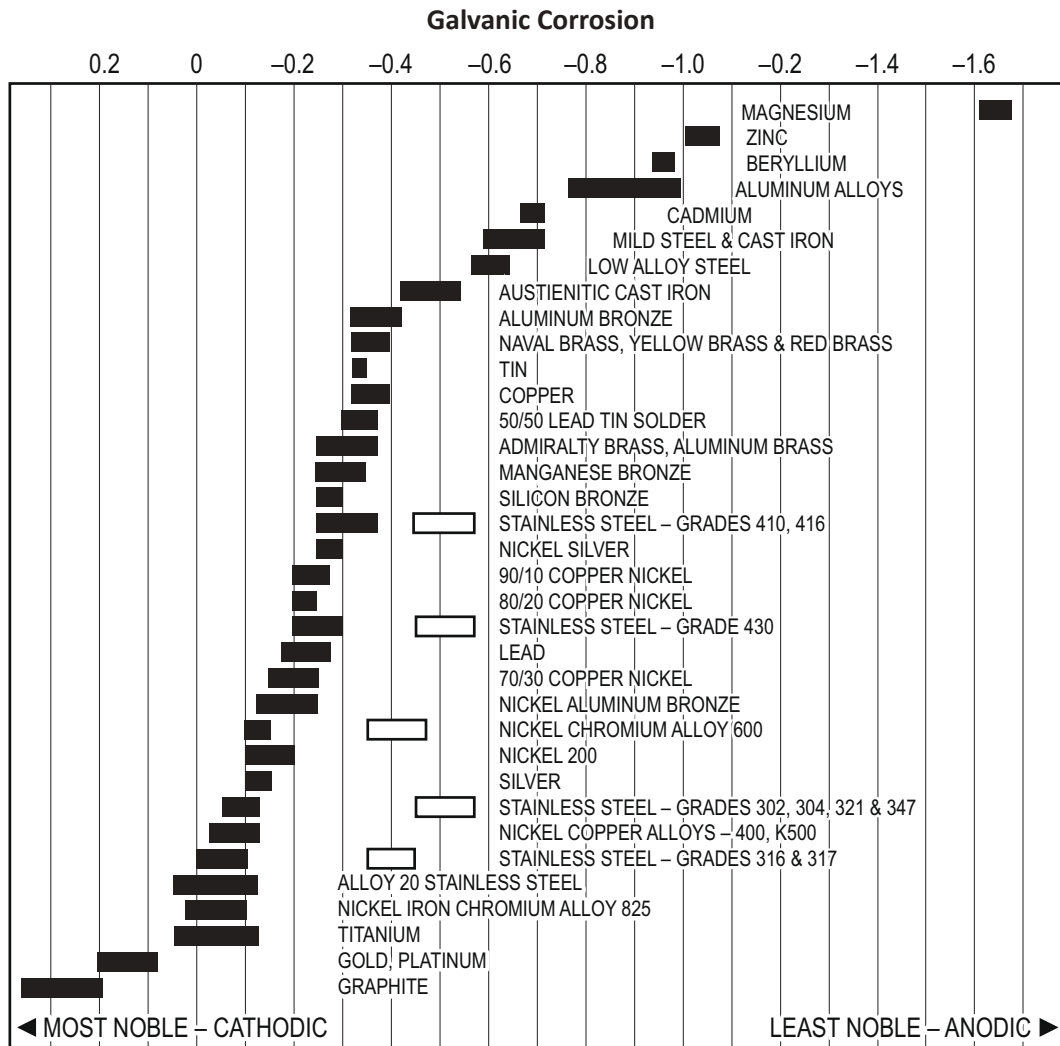
Detailed Corrosion Data on Construction Materials (cont'd)



Detailed Corrosion Data on Construction Materials (cont'd)



8.6.2.1 Galvanic Corrosion



Note: Unshaded symbols show ranges exhibited by stainless steels in acidic water such as may exist in crevices or in stagnant, low-velocity, or poorly aerated water.

Source: Davis, J.R., *ASM Specialty Handbook on Stainless Steel*, 2nd ed.: American Society for Metals, 1996, p. 139. Reprinted with permission of ASM International. All rights reserved. www.asminternational.org.

8.6.2.2 Electrochemistry

Electrochemical Terms

Term	Definition
Cathode	The electrode at which reduction occurs
Anode	The electrode at which oxidation occurs
Oxidation	The loss of electrons
Reduction	The gaining of electrons
Cation	Positive ion
Anion	Negative ion

8.6.2.3 Standard Oxidation Potentials

Standard Oxidation Potentials for Corrosion Reactions*	
Corrosion Reaction	Potential (E_o) Volts vs. Normal Hydrogen Electrode
$\text{Au} \rightarrow \text{Au}^{3+} + 3\text{e}^-$	-1.498
$2\text{H}_2\text{O} \rightarrow \text{O}_2 + 4\text{H}^+ + 4\text{e}^-$	-1.229
$\text{Pt} \rightarrow \text{Pt}^{2+} + 2\text{e}^-$	-1.200
$\text{Pd} \rightarrow \text{Pd}^{2+} + 2\text{e}^-$	-0.987
$\text{Ag} \rightarrow \text{Ag}^+ + \text{e}^-$	-0.799
$2\text{Hg} \rightarrow \text{Hg}_2^{2+} + 2\text{e}^-$	-0.788
$\text{Fe}^{2+} \rightarrow \text{Fe}^{3+} + \text{e}^-$	-0.771
$4(\text{OH})^- \rightarrow \text{O}_2 + 2\text{H}_2\text{O} + 4\text{e}^-$	-0.401
$\text{Cu} \rightarrow \text{Cu}^{2+} + 2\text{e}^-$	-0.337
$\text{Sn}^{2+} \rightarrow \text{Sn}^{4+} + 2\text{e}^-$	-0.150
$\text{H}_2 \rightarrow 2\text{H}^+ + 2\text{e}^-$	+0.000
$\text{Pb} \rightarrow \text{Pb}^{2+} + 2\text{e}^-$	+0.126
$\text{Sn} \rightarrow \text{Sn}^{2+} + 2\text{e}^-$	+0.136
$\text{Ni} \rightarrow \text{Ni}^{2+} + 2\text{e}^-$	+0.250
$\text{Co} \rightarrow \text{Co}^{2+} + 2\text{e}^-$	+0.277
$\text{Cd} \rightarrow \text{Cd}^{2+} + 2\text{e}^-$	+0.403
$\text{Fe} \rightarrow \text{Fe}^{2+} + 2\text{e}^-$	+0.440
$\text{Cr} \rightarrow \text{Cr}^{3+} + 3\text{e}^-$	+0.744
$\text{Zn} \rightarrow \text{Zn}^{2+} + 2\text{e}^-$	+0.763
$\text{Al} \rightarrow \text{Al}^{3+} + 3\text{e}^-$	+1.662
$\text{Mg} \rightarrow \text{Mg}^{2+} + 2\text{e}^-$	+2.363
$\text{Na} \rightarrow \text{Na}^+ + \text{e}^-$	+2.714
$\text{K} \rightarrow \text{K}^+ + \text{e}^-$	+2.925

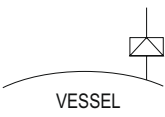
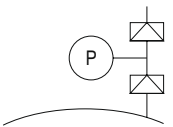
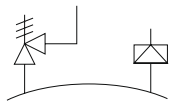
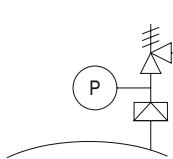
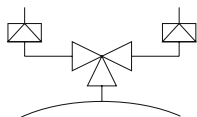
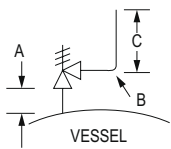
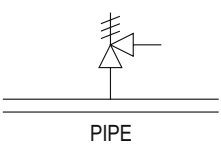
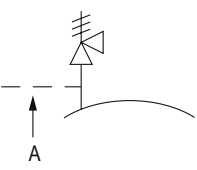
*Measured at 25°C. Reactions are written as anode half-cells. Arrows are reversed for cathode half-cells.

Note: In some chemistry texts, the reactions and the signs of the values (in this table) are reversed; for example, the half-cell potential of zinc is given as -0.763 volt for the reaction $\text{Zn}^{2+} + 2\text{e}^- \rightarrow \text{Zn}$. When the potential E_o is positive, the reaction proceeds spontaneously as written.

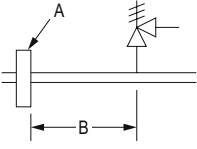
Source: Republished with permission of John Wiley and Sons, from *Engineering Materials and Their Applications*, Richard A. Flinn and Paul K. Trojan, 3rd ed., 1986; permission conveyed through Copyright Clearance Center, Inc.

8.7 Process Equipment Design

Installation Practices for Equipment Relief

System	Recommendations
 Diagram showing a vessel with a single rupture disc installed on top.	For rupture disc in corrosive service, or for highly toxic materials where spring-loaded reliefs may weep.
 Diagram showing a vessel with two rupture discs in series.	For two rupture discs in extremely corrosive service. The first may need to be replaced periodically.
 Diagram showing a vessel with a spring-loaded relief valve and a rupture disc in parallel.	For rupture disc and spring-loaded relief. Normal relief may go through spring-loaded device and rupture disc is backup for larger reliefs.
 Diagram showing a vessel with a spring-loaded relief valve and a rupture disc in series.	For two reliefs in series. The rupture disc protects against toxicity or corrosion. The spring-loaded relief closes and minimizes losses.
 Diagram showing a vessel with two rupture discs and a 3-way valve.	For two rupture discs with 3-way valve that keeps one valve always directly connected to vessel. This design is good for polymerization reactors that require periodic cleaning.
 Diagram showing a vessel with a spring-loaded relief valve. Labels A, B, and C indicate specific piping details.	A. Pressure drop not more than 3% of set pressure. B. Long radius elbow. C. If distance is greater than 10 feet, support weight and reaction forces below the long radius elbow.
 Diagram showing a pipe with a single safety relief valve.	For orifice area of a single safety relief in vapor service; should not exceed 2% of the cross-sectional area of the protected line. May require multiple valves with staggered settings.
 Diagram showing a vessel with a spring-loaded relief valve. Label A indicates process lines.	A. Process lines; should not be connected to safety-valve inlet piping.

Installation Practices for Equipment Relief (cont'd)

System	Recommendations												
 A. Turbulence-causing device. B. Dimension shown below:	<table border="1"> <thead> <tr> <th>Device Causing the Turbulence</th><th>Minimum Number of Straight-Pipe Diameters</th></tr> </thead> <tbody> <tr> <td>Regulator or valve</td><td>25</td></tr> <tr> <td>2 ells or bends not in same plane</td><td>20</td></tr> <tr> <td>2 ells or bends in same plane</td><td>15</td></tr> <tr> <td>1 ell or bend</td><td>10</td></tr> <tr> <td>Pulsation damper</td><td>10</td></tr> </tbody> </table>	Device Causing the Turbulence	Minimum Number of Straight-Pipe Diameters	Regulator or valve	25	2 ells or bends not in same plane	20	2 ells or bends in same plane	15	1 ell or bend	10	Pulsation damper	10
Device Causing the Turbulence	Minimum Number of Straight-Pipe Diameters												
Regulator or valve	25												
2 ells or bends not in same plane	20												
2 ells or bends in same plane	15												
1 ell or bend	10												
Pulsation damper	10												

Source: Jennet, Eric, "Components of Pressure-Relieving Systems," Equipment Relief Installation Practices: *Chemical Engineering Magazine*, 1963, pp. 151–158.

8.8 Instrumentation and Process Control

8.8.1 Sensors

Commonly Used Process Instruments

Measured Variable	Instrument Type	Operating Principle
Pressure (gauge)	Differential pressure (DP) cell	Pressure difference causes displacement of a diaphragm. The displacement can be transmitted mechanically to a bellows to register a pneumatic signal or converted to an electrical signal by a strain gauge or by movement of the diaphragm relative to a static capacitor plate. Gauge pressure is measured relative to atmospheric pressure.
Pressure difference	DP cell	As above. Pressure difference is measured between two points in the process.
Temperature	Thermocouple	Wires of different metals joined together to form a circuit with one joint hotter than the other will develop an EMF through the Seebeck effect. If one joint is at a reference temperature the other temperature can be found from the EMF. The reference temperature is usually ambient temperature, which is determined by measuring the electrical resistance of a platinum wire. Different combinations of metal wire are used depending on the temperature range. See Love (2007) for details of thermocouple types.
Temperature	RTD	Resistance is measured in a length of pure metal wire. The relationship between temperature and resistance is known for the given metal, and the measurement can be used to calculate the temperature.
Volumetric flow	Orifice meter	Flow passes through a restriction orifice. Pressure difference across the orifice is measured with a DP cell. Flow rate is calculated from pressure drop.
Volumetric flow	Venturi meter	Flow passes through a shaped pipe restriction. Pressure difference across the restriction is measured with a DP cell. Flow rate is calculated from pressure drop.
Mass flow	Coriolis meter	Flow through a shaped vibrating pipe loop causes it to twist due to the Coriolis effect. The extent of twist is measured optically. These instruments can be used for multiphase flow, but are expensive, particularly for large flow rates.
Level	DP cell	A DP cell placed between the top and bottom of a vessel can indicate level if there is no internal pressure drop in the vessel.
Level	Capacitance probe	The capacitance between a probe in the center of the vessel and the wall is affected by the dielectric constant of the material between them, and so varies with level.
Level	Radar	The sensor sends a radio wave emission out into the vessel and measures how long it takes for the signal to be reflected back. The return signal can be used to accurately calculate the level.
Interface level	DP cell	A DP cell can determine the interface level between immiscible fluids if they are in a vessel that has an internal weir (so that overall level remains constant).
pH	Glass electrode	The glass electrode and a reference electrode (usually silver/silver chloride) form an electrochemical circuit allowing EMF to be measured.
Composition	Chromatograph	Gas chromatography (GC) can be used to separate simple mixtures and generate a signal through a thermal conductivity detector (TCD) or flame ionization detector (FID). GC methods are difficult to use for online control because the chromatography typically takes a few minutes, but they can be used in cascade control schemes to adjust set points on other controllers.

Source: Reprinted from *Chemical Engineering Design*, 2nd ed., Gavin Towler and Ray Sinnott, "Table 5.2, Instrumentation and Process Control," p. 258, ©2013, with permission from Elsevier, www.elsevier.com.

8.8.2 Controller Actions

In order to determine the action of a controller, it is first necessary to understand the final control element, which is typically a control valve. Control valves are designed to fail in the wide-open position or completely shut. The fail-open (FO) valves require increased air pressure to close, or are air-to-close (AC). The fail-closed (FC) valves require increased air pressure to open (AO). Less commonly, some control valves are fail-last (FL) position. The decision of which type to use depends on the process and safety considerations.

The action of the feedback controller will depend on the action of the transmitter (typically direct), the action of the valve (FO/FC), and the effect of the manipulated variable on the process variable. If the proportional gain of the controller is negative, i.e. the controller output increases when the process variable increases, then it is called a direct-acting controller. If the proportional gain is positive, i.e. the controller output decreases when the process variable increases, then it is called a reverse-acting controller.

Source: Republished with permission of McGraw-Hill, Inc., from *Process Modeling, Simulation and Control for Chemical Engineers*, William L. Luyben, 2nd ed., 1990; permission conveyed through Copyright Clearance Center, Inc.

8.8.2.1 First-Order Control System Models

The transfer function model for a first-order system is

$$\frac{Y(s)}{R(s)} = \frac{K}{\tau s + 1}$$

where

K = steady-state gain

τ = time constant

$$\frac{Y(s)}{R(s)} = \frac{\text{Laplace transforms of output, in deviation form}}{\text{Laplace transforms of input, in deviation form}}$$

The step response of a first-order system to a step input of magnitude M is

$$y(t) = y_0 e^{-t/\tau} + KM(1 - e^{-t/\tau})$$

In the chemical process industry, y_0 is typically taken to be zero, and $y(t)$ is referred to as a deviation variable.

For systems with time delay (dead time or transport lag) θ , the transfer function is

$$\frac{Y(s)}{R(s)} = \frac{K e^{-\theta s}}{\tau s + 1}$$

The step response for $t \geq \theta$ to a step of magnitude M is

$$y(t) = \left[y_0 e^{-(t-\theta)/\tau} + KM(1 - e^{-(t-\theta)/\tau}) \right] u(t - \theta)$$

where

$u(t)$ = unit step function

8.8.2.2 Second-Order Control System Models

One standard second-order control system model is

$$\frac{Y(s)}{R(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where

K = steady-state gain

ζ = the damping ratio

ω_n = the undamped natural ($\zeta = 0$) frequency

$\omega_d = \omega_n \sqrt{1 - \zeta^2}$, the damped natural frequency

$\omega_r = \omega_n \sqrt{1 - 2\zeta^2}$, the damped resonant frequency

If the damping ratio ζ is less than unity, the system is said to be underdamped; if ζ is equal to unity, it is said to be critically damped; and if ζ is greater than unity, the system is said to be overdamped.

For a unit step input to a normalized, underdamped, second-order control system, the time required to reach a peak value t_p and the value of that peak M_p are determined by

$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

$$M_p = 1 + e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

The percent overshoot (%OS) of the response is determined by

$$\%OS = 100e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

For an underdamped, second-order system, the logarithmic decrement is

$$\delta = \frac{1}{m} \ln\left(\frac{x_k}{x_{k+m}}\right) = \frac{2\pi\zeta}{\sqrt{1 - \zeta^2}}$$

where x_k and x_{k+m} are the amplitudes of oscillation at cycles k and $k + m$, respectively.

The period of oscillation τ is related to ω_d by

$$\omega_d \tau = 2\pi$$

The time required for the output of a second-order system to settle to within 2% of its final value (2% settling time) is defined to be

$$T_s = \frac{4}{\zeta\omega_n}$$

An alternative form commonly employed in the chemical process industry is

$$\frac{Y(s)}{R(s)} = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

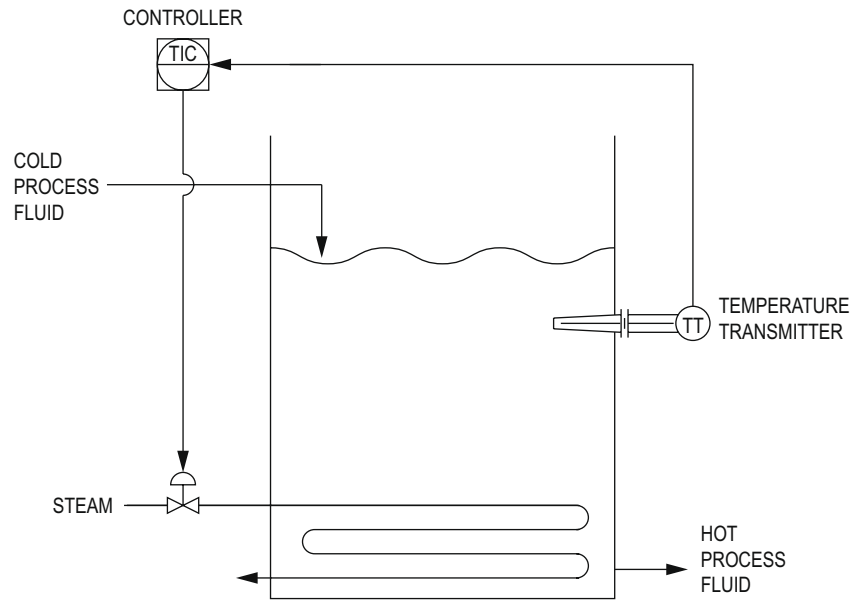
where

K = steady-state gain

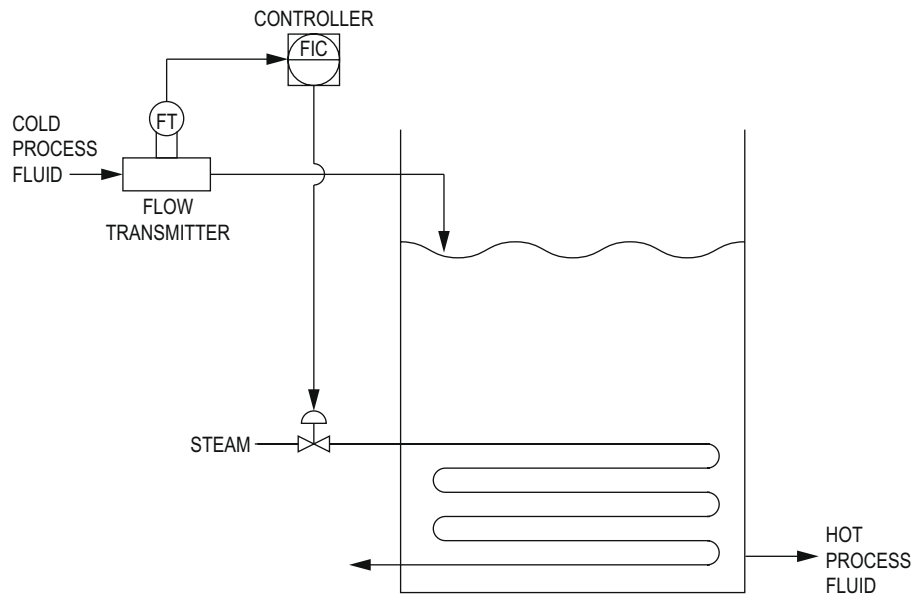
ζ = the damping ratio

τ = the inverse natural frequency

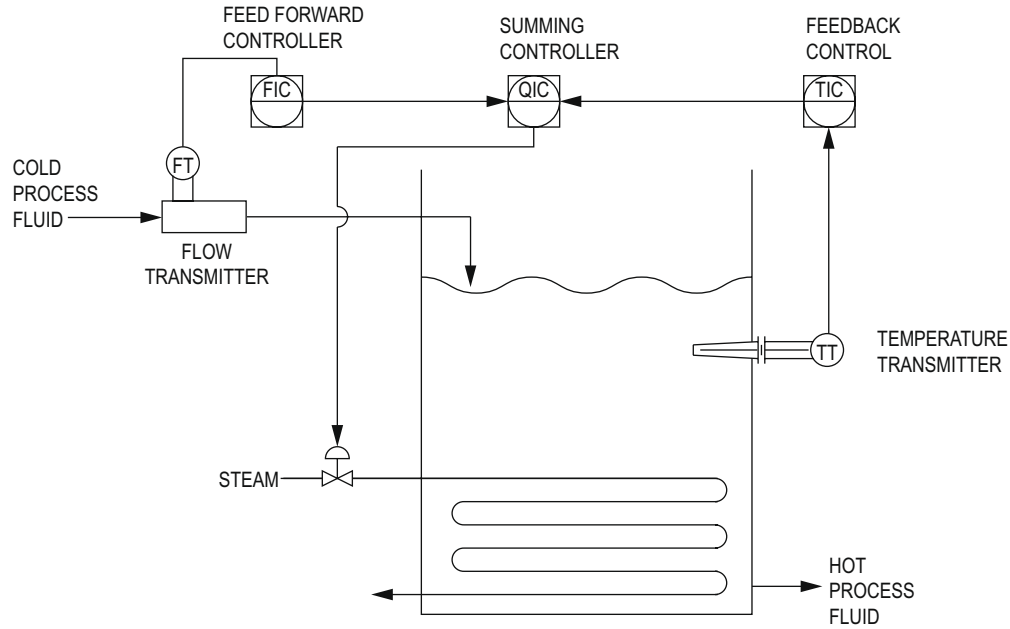
Feedback Control



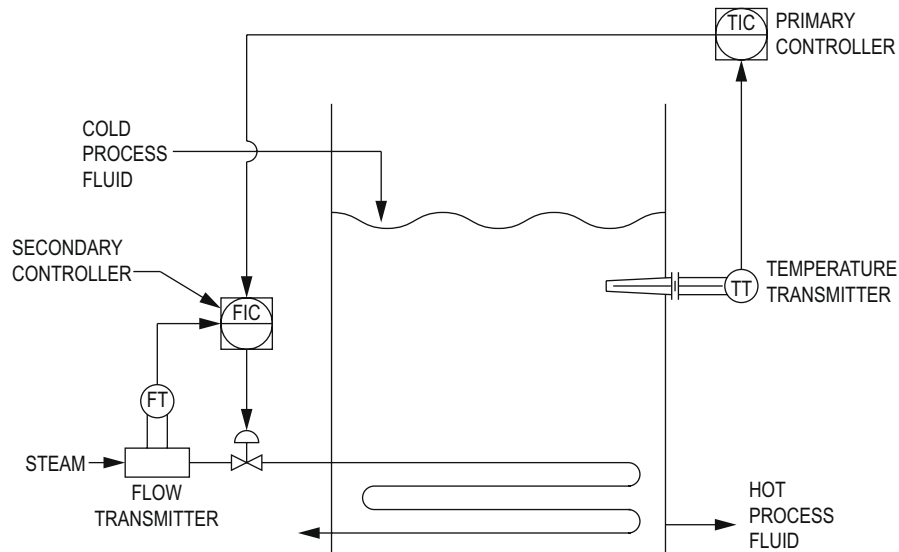
Feed Forward Control



Feed Forward Plus Feedback Control



Cascaded Feedback Control



8.8.3 Alarms

Alarms are tools to help operators quickly identify conditions that are abnormal or threaten process or personal safety. Alarms may be accompanied by audible or visible cues on control panels or SCADA systems. Alarms are often communicated to plant operators to allow rapid response to undesirable conditions. Where a response within the necessary time to prevent a catastrophic failure is either unlikely or impossible, a system should be installed with an automatic trip or interlock to prevent failure.

Alarms are typically generated within a computer or programmable logic device that perform a logical operation based on the system instrumentation and controls. These setpoints are often user configurable and set by the plant operators.

Source: Reprinted from Chemical Engineering Design, 2nd ed., Gavin Towler and Ray Sinnott, "Alarms, Safety Trips and Interlocks," pp. 270–272, and "Layers of Plant Safety," pp. 433–435, ©2013, with permission from Elsevier, www.elsevier.com.

8.8.4 Safety Instrumented Systems

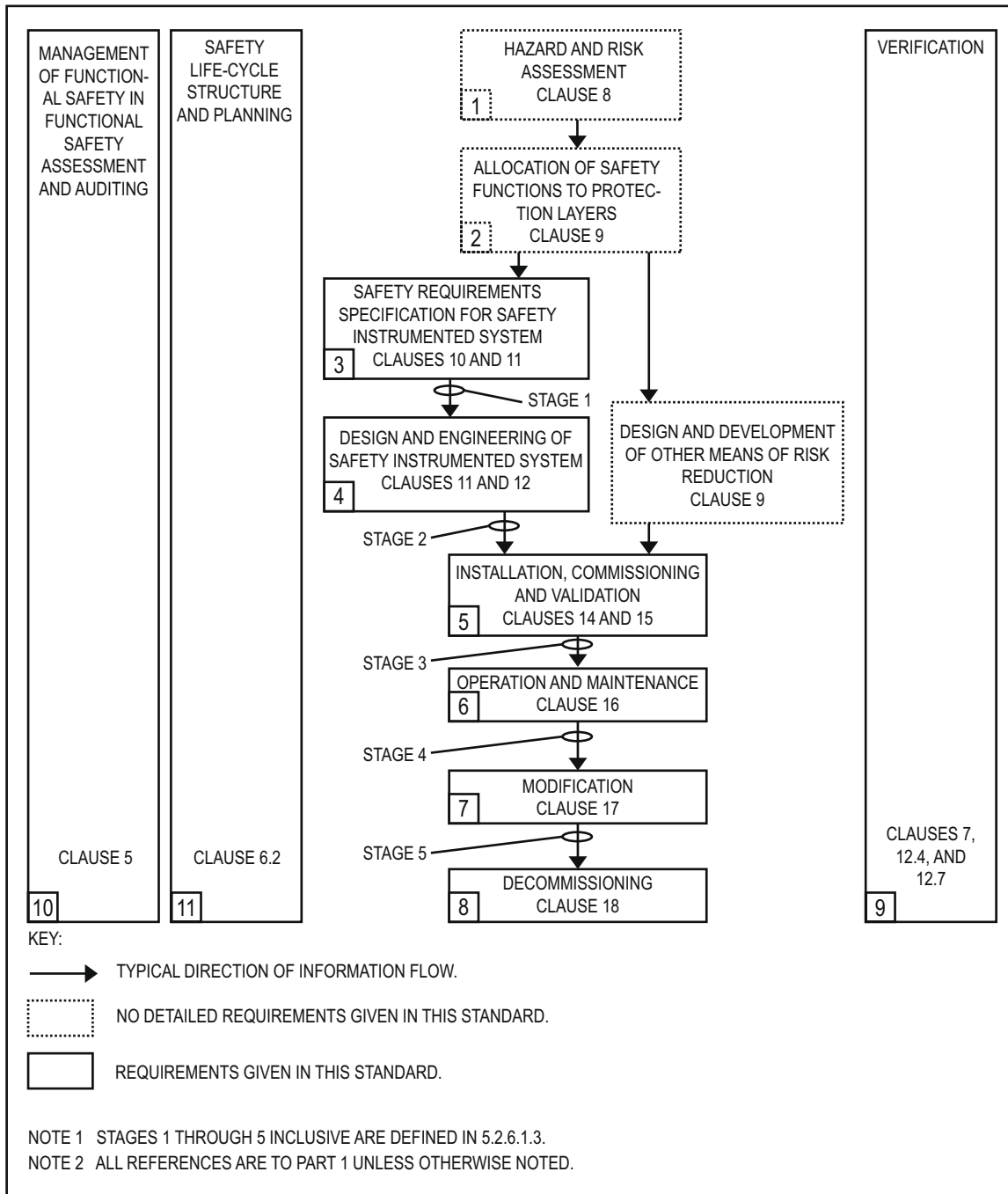
Definitions of Safety Integrity Terms

Term	Definition
Architecture	Arrangement of hardware and/or software elements in a system; for example: (1) Arrangement of safety instrumented system (SIS) subsystems (2) Internal structure of an SIS subsystem (3) Arrangement of software programs
Average Probability of Failure on Demand (PFD_{avg})	Average probability that a safety-instrumented function will fail in such a way that it cannot respond to a potentially dangerous condition. PFD or PFD_{avg} is applied to repairable systems.
Basic Process Control System (BPCS)	System that responds to input signals from the process, its associated equipment, other programmable systems, and/or an operator and generates output signals causing the process and its associated equipment to operate in the desired manner but that does not perform any safety-instrumented functions with a claimed $SIL \geq 1$.
Common Cause Failure	Failure that is the result of one or more events and that causes failure of two or more separate channels in a multiple-channel system, leading to system failure.
MooN	Safety instrumented system, or part thereof, made up of N independent channels that are so connected that M channels are sufficient to perform the safety-instrumented function.
Mean Time to Fail (MTTF)	Mean time to random failure for a component population. MTTF is applied to items that are not repaired, such as bearings and transistors.
Mean Time to Trip Spurious (MTTFS)	Mean time for a safety function to fail in a mode that causes a spurious trip.
Safe Failure Fraction (SFF)	Fraction of the overall random hardware failure rate of a device that results in either a safe failure or a detected dangerous failure.
Safety Instrumented System (SIS)	Instrumented system used to implement one or more safety-instrumented functions. A SIS is composed of any combination of sensor(s), logic solver(s), and final element(s).
Safety Integrity Level (SIL)	Discrete level (one out of four) for specifying the safety integrity requirements of the safety-instrumented functions to be allocated to the safety instrumented systems. SIL 4 has the highest level of safety integrity; SIL 1, the lowest.
Safety Instrumented Function (SIF)	Safety function with a specified safety integrity level that is necessary to achieve functional safety and that can be either a safety instrumented protection function or a safety instrumented control function.
Systematic Failure	Failure related in a deterministic way to a certain cause, which can only be eliminated by a modification of the design or of the manufacturing process, operational procedures, documentation, or other relevant factors.
Tolerable Risk	Risk that is accepted in a given context based on the current values of society.
Validation	Activity of demonstrating that the safety-instrumented function(s) and safety instrumented system(s) under consideration after installation meet in all respects the safety requirements specification.
Verification	Activity of demonstrating for each phase of the relevant safety life-cycle, by analysis and/or tests, that for specific inputs the outputs meet in all respects the objectives and requirements for the specific phase.

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8.8.4.1 Functional Safety Life Cycle

SIS Safety Life-Cycle Phases and Functional Safety Assessment Stages



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8.8.4.2 SIS Safety Life-Cycle Overview

The Safety Instrumented System (SIS) Safety Life-Cycle

Safety Life-Cycle Phase or Activity		Objectives	Requirements Clause or Subclause	Inputs	Outputs
Box # in Previous Image	Title				
1	Hazard and Risk Assessment	To determine the hazards and hazardous events of the process and associated equipment, the sequence of events leading to the hazardous event, the process risks associated with the hazardous event, the requirements for risk reduction, and the safety functions required to achieve the necessary risk reduction	8	Process design, layout, work force arrangements, safety targets	Description of hazards of the required safety function(s) and their associated risk reduction(s)
2	Allocation of Safety Functions to Protection Layers	Allocation of safety functions to protection layers and the associated safety integrity level for each safety-instrumented function	9	Description of required safety-instrumented function(s) and associated safety integrity requirements	Description of allocation of safety requirements (see Clause 9)
3	SIS Safety Requirements Specification	To specify the requirements for each SIS, in terms of the required safety-instrumented functions and their associated safety integrity, in order to achieve the required functional safety	10	Description of allocation of safety requirements (see Clause 9)	SIS safety requirements; software safety requirements
4	SIS Design and Engineering	To design the SIS to meet the requirements for safety-instrumented functions and safety integrity	11, 12.4	SIS safety requirements; software safety requirements	Design of the SIS in conformance with the SIS safety requirements; planning for the SIS integration test

SIS Safety Life-Cycle Overview (cont'd)

Safety Life-Cycle Phase or Activity		Objectives	Requirements Clause or Subclause	Inputs	Outputs
Box # in Previous Image	Title				
5	SIS Installation Commissioning and Validation	To integrate and test the SIS To validate that the SIS meets in all respects the requirements for safety in terms of the required safety-instrumented functions and the required safety integrity	12.3, 14, 15	SIS design SIS integration test plan SIS safety requirements Plan for the safety validation of the SIS	Fully functioning SIS in conformance with the SIS design results of SIS integration tests Results of the installation, commissioning, and validation activities
6	SIS Operation and Maintenance	To ensure that the functional safety of the SIS is maintained during operation and maintenance	16	SIS requirements SIS design Plan for SIS operation and maintenance	Results of the operation and maintenance activities
7	SIS Modification	To make corrections, enhancements, or adaptations to the SIS, ensuring that the required safety integrity level is achieved and maintained	17	Revised SIS safety requirements	Results of SIS modification
8	Decommissioning	To ensure proper review and sector organization, and to ensure SIF remains appropriate	18	As-built safety requirements and process information	SIF placed out of service
9	SIS Verification	To test and evaluate the outputs of a given phase to ensure correctness and consistency with respect to the products and standards provided as inputs to that phase	7, 12.7	Plan for the verification of the SIS for each phase	Results of the verification of the SIS for each phase
10	SIS Functional Safety Assessment	To investigate and arrive at a judgment on the functional safety achieved by the SIS	5	Planning for SIS functional safety assessment SIS safety requirement	Results of SIS functional safety assessment

8.8.4.3 Safety Integrity Levels: Probability of Failure on Demand

Demand Mode of Operation		
Safety Integrity Level (SIL)	Target Average Probability of Failure on Demand	Target Risk Reduction
4	$\geq 10^{-5}$ to $< 10^{-4}$	$> 10,000$ to $\leq 100,000$
3	$\geq 10^{-4}$ to $< 10^{-3}$	> 1000 to $\leq 10,000$
2	$\geq 10^{-3}$ to $< 10^{-2}$	> 100 to ≤ 1000
1	$\geq 10^{-2}$ to $< 10^{-1}$	> 10 to ≤ 100

8.8.4.4 Functional Safety Equations

Average Probability of Failure on Demand (PFD_{avg})

$$PFD_{avg} = \frac{1}{T} \int_0^T PF(t) dt \quad (\text{Rigorous version})$$

$$PFD_{avg} = \frac{\lambda t}{2} \quad (\text{Approximation})$$

Safe Failure Fraction (SFF)

$$SFF = \frac{\lambda_{SD} + \lambda_{SU} + \lambda_{DD}}{\lambda_{SD} + \lambda_{SU} + \lambda_{DD} + \lambda_{DU}}$$

where

λ = failure rate (failures/year)

and subscripts indicate failure mode:

SD = safe detected

SU = safe undetected

DD = dangerous detected

DU = dangerous undetected

8.9 Operation

8.9.1 Operating Procedures

The main types of process operating procedures are

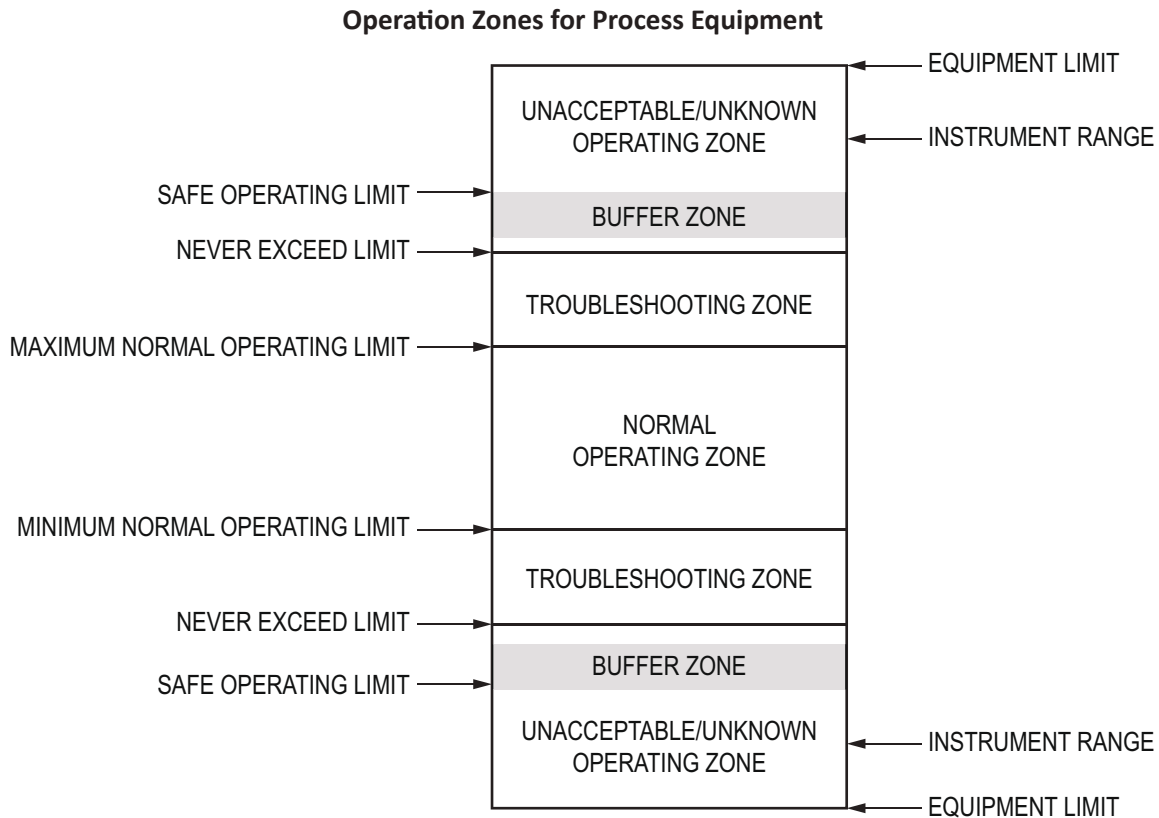
1. **Standard Operating Procedures (SOP)**—Written instructions documenting step-by-step instructions for safely performing a task within operating limits. The SOP covers all modes of operation. The purpose of the standard operating procedure is to ensure operations are always carried out correctly and in the same manner. An SOP should be available at the place where the work is done.
2. **Startup/Shutdown Procedures**—Written procedures for startup and shut-down phased so that interlinked plant operations can resume or stop in a safe and controlled manner.
3. **Emergency or Abnormal Operating Procedures**—Written instructions documenting step-by-step instructions for reaching a safe state following a process in an upset condition. The emergency procedures should cover the PPE, the level of intervention which is safe, and when to evacuate. The procedures will also need to tie in with site emergency plans.

4. **Temporary Operating Procedures**—Written instructions for a finite period of time. At the conclusion of this time, the facility returns to using the Standard Operating Procedures. Temporary operating procedures should include an expiration date.
5. **Maintenance Procedures**—Written instructions that address material control and maintenance practices needed to ensure system operability and integrity. These procedures specify the required maintenance, testing, and inspection frequencies.

Source: Guidelines for Engineering Design for Process Safety, 2nd ed., New York: John Wiley & Sons, Inc., 2012.

The zones for safe operation of process equipment are defined as

1. **Normal Operating Zone:** The minimum or maximum values of an operating parameter that define the boundaries of normal operations. Some examples of operating parameters to be defined include
 - High and low pressure
 - High and low temperature
 - High and low level
 - High and low pH
 - High and low flow
2. **Troubleshooting Zone:** An area that provides time for troubleshooting, so that operations personnel can make adjustments in time to return the operating parameters to the Normal Operating Zone. Human factors and process response time generally indicate zone size. Immediate actions, and in some cases predetermined actions, to avoid Safe Operating Limit (SOL) deviation are taken in this zone.
3. **Buffer Zone:** The upper and lower area of the known safe zone provides a buffer to ensure no operating parameter can reach the Unknown/Unacceptable Operation Zone. Factors that influence Buffer Zone size may include engineering judgment, reliability of instrumentation, operating experience, probability and consequence of human error, and so on. A process will not be operated intentionally in this zone.
4. **Safe Operating Limit (SOL):** A value for an operating parameter that defines the equipment or process unit's safe-operating envelope, beyond which a process will not intentionally be operated due to the risk of imminent, catastrophic equipment failure or loss of containment. Operational or mechanical corrective action ceases and immediate predetermined actions are taken at these operating parameter values in order to bring equipment and process units to a safe state. Each SOL should be documented in the plant's *Process Safety Information*.
5. **Unacceptable or Unknown Operating Zone:** An area beyond the Safe Operating Limit. A process will not be intentionally operated in this zone.



Source: Smith, David J., *Reliability, Maintainability and Risk - Practical Methods for Engineers*, 5th ed., "Appendix A1: Terms Related to Failure," Amsterdam: Elsevier, 1997. www.elsevier.com.

8.9.2 Start-up and Shutdown

Start-up, operation, and shutdown of most different types of plants included the following modes:

1. normal start-up
2. normal operation
3. normal shutdown
4. emergency shutdown

There are further relevant distinctions of shutdowns including:

1. normal shutdown condition
2. hot standby condition
3. emergency shutdown condition
4. major scheduled shutdown, or turnaround, condition
5. prolonged shutdown, or mothballing condition

Source: Adapted from Less, Frank, *Lees' Loss Prevention in the Process Industries*, 4th ed., Chapter 20, Amsterdam: Elsevier, 2012. www.elsevier.com.

8.10 Process Equipment and Reliability

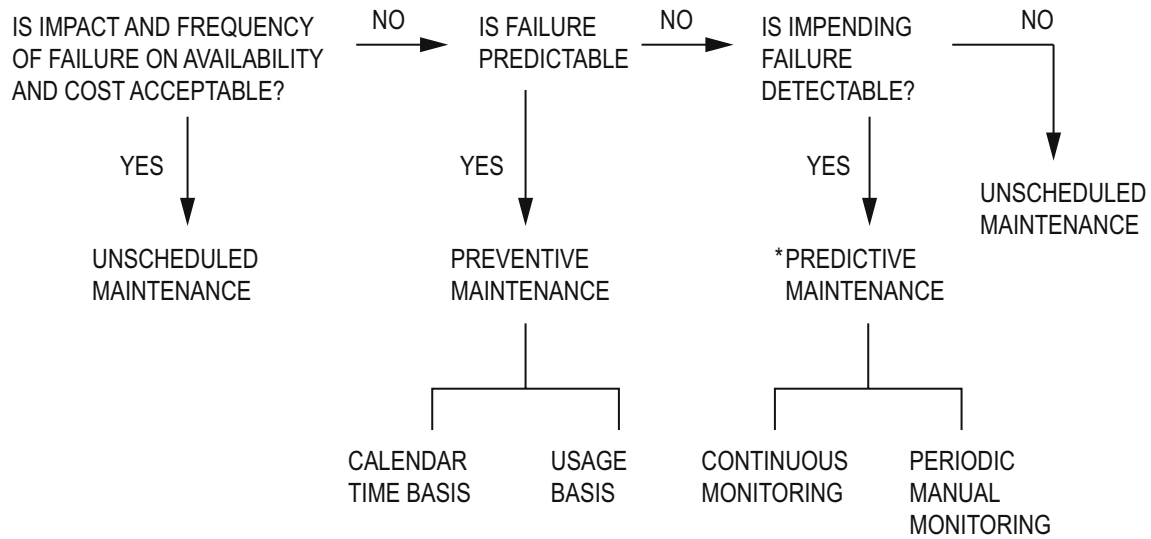
8.10.1 Testing and Inspection

Maintenance and Reliability

Term	Definition	Example or Application
Availability	$\text{Availability (\%)} = \frac{\text{Up Time}}{\text{Total Time}}$ The proportion of time that an item is capable of operating to specification within a large time interval.	The availability of a gas turbine generator was increased to 95% by minimizing the scheduled maintenance duration.
Diversity	The same performance of a function by two or more independent and dissimilar means.	In-line check valves of two different technologies or separate manufacturers are installed to decrease the likelihood of reverse flow from waste-water treatment back to the process.
Failure Modes and Effects Analysis (FMEA)	A qualitative tool for analysis identifying all the ways a particular component can fail and the effects of the failure on the system.	An FMEA identifies internal spring failure from excessive wear on a solenoid valve. The local and system consequences are documented. A recommendation is made for regular inspection to prevent this point of failure.
Mean Time Between Failures (MTBF)	The total cumulative functioning time of a population divided by the number of failures, MTBF is used for items that involve repair and excludes downtime. $\text{MTBF} = \frac{\text{Total Up Time}}{\text{Number of Failures}}$	For 10,000 total hours of recorded uptime, the MTBF for 4 power supplies is 2500 hours.
Predictive Maintenance	The aim of predictive maintenance is, first, to predict when equipment failure may occur and, second, to prevent occurrence of that failure by performing maintenance.	A plant predictive maintenance program could use regular vibration analyses and motor current signature analyses to determine equipment conditions and predict failure.
Preventive Maintenance	Actions carried out for the purpose of keeping equipment or instrumentation in a specified condition.	A preventive maintenance program for a centrifugal pump at a plant could include monthly inspection of the gland packing, bearing lubrication, and pump mountings.
Redundancy	The provision of more than one means of achieving a function. Active/Duty: All items remain operating prior to failure. Standby: Replicated items do not operate until needed.	An active pump runs continuously for long periods of time without having to go through the start-up process. The standby pump remains dormant and is tested regularly to ensure reliability.
Reliability	The probability that the system will not leave the operational state. The availability for a given system is always greater than or equal to the reliability.	Safety-instrumented function; probability of failure on demand.

8.10.2 Maintenance

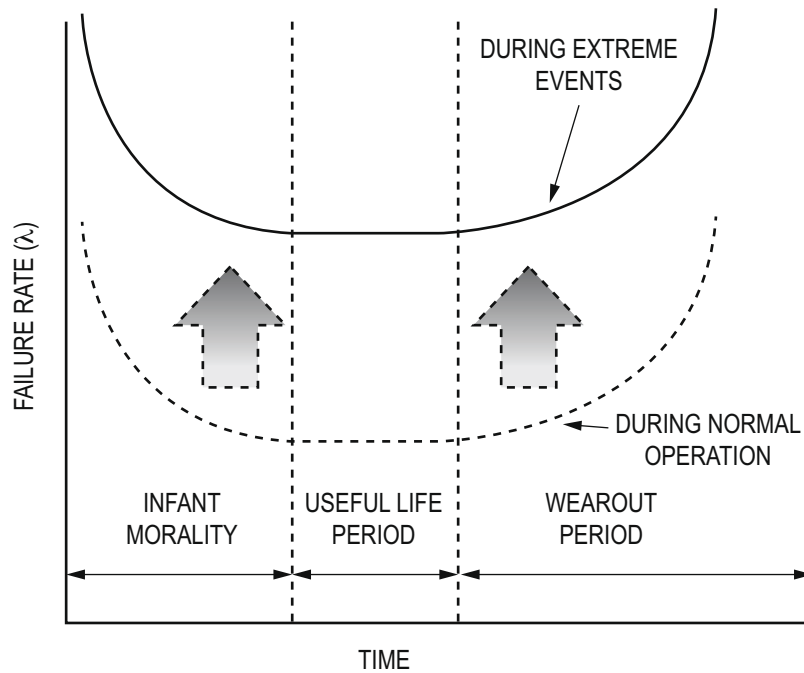
Decision Tree for Optimum Maintenance



* predictive maintenance based on precursor to failure

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Equipment Failure



8.11 Process Improvement and Troubleshooting

The variety and complexity of modern processing industries requires that engineers be able to find ways to improve the processes of their facilities for the benefit of their employers or clients. Process improvement allows for the optimization of utilities, raw materials, and other resources to maximize production and minimize the cost per unit produced.

Engineers who are tasked with process improvement and troubleshooting focus their knowledge and training to make a facility or process work more efficiently and economically.

Work in this area will focus on one of the following types of activities:

- Optimum balance of process variables
- Increased capacity—Debottleneck and/or add equipment
- Improved product quality—Control contamination and deterioration
- Improved mechanical performance—Reduce corrosion and fouling
- Decreased utility and raw material consumption—Steam, power, water, chemicals, and so on
- More efficient maintenance
- Improved safety practices

The use of data is paramount to any of the activities listed above. The modern process-industries plant typically has an abundance of data that is part of the control systems. This data is collected from all aspects and areas of the facility.

One of the most common methods of using data to improve a process is the DMAIC method. The five phases in the DMAIC method are

1. **Define** the problem and system by setting goals and understanding the requirements of the customer and the system.
2. **Measure** the key aspects of the process and gather the data that is available and relevant to the issue, project, or problem to be solved. This data can be used to determine the "as is" state of the process.
3. **Analyze** the data to investigate the process and determine the cause-and-effect relationships in the process. Seek out the root cause(s) of the problem being evaluated.
4. **Improve** or optimize the current process based on data analysis techniques to create a new, future-state process and run pilot trials to establish the process capability.
5. **Control** the new process to ensure that any deviations are corrected quickly before they result in defects or issues.

Data analysis can be a complex activity and techniques in this area include

5 Whys	Analysis of variance
Regression analysis	Correlation
Cause-and-effect diagramming	Control/run charts
Design of experiments	Pareto analysis
Taguchi loss function	Value stream mapping
General linear modeling	Axiomatic design
Cost-benefit analysis	Root-cause analysis
Failure modes, effects, and diagnostic analysis (FMEDA)	

One of the most useful techniques for troubleshooting is root-cause analysis (RCA). RCA is a method of problem-solving used to identify the root cause(s) of faults or problems.

A factor is considered a root cause if removal from the problem-fault sequence prevents the final undesirable event from recurring. A causal factor is one that affects an event's outcome and, if removed, might benefit the process but does not prevent the recurrence of the problem being addressed. RCA is applied to methodically identify and correct the root causes of events, rather than to simply address the symptomatic result. Focusing correction on root causes has the goal of entirely preventing problem recurrence. RCA is typically used as a reactive method for identifying event causes, revealing problems, and solving them. Analysis is most typically done after an event has occurred; however, it can also be used as a predictive tool.

The basic steps in root-cause analysis are:

1. Define the problem or describe the event to prevent in the future.
2. Gather data and evidence, classifying it along a time line.
3. Data-mine for clusters of similar problems that are close to the problem or event.
4. Ask why this happens and identify the causes, giving each sequential step toward the problem or event.
5. Classify all causes into either "causal" or "root."
6. Identify any other items that affect the problem or event.
7. Identify the corrective action(s) that will, with certainty, prevent recurrence of each harmful effect.
8. Identify solutions that prevent recurrence and that are within the control of the institution.
9. Implement the recommended root-cause corrections.
10. Ensure effectiveness by observing the implemented solutions in operation.

Observation is one of the best ways to identify issues that need to be addressed when working and troubleshooting in any type of plant.

8.12 Flammability Data

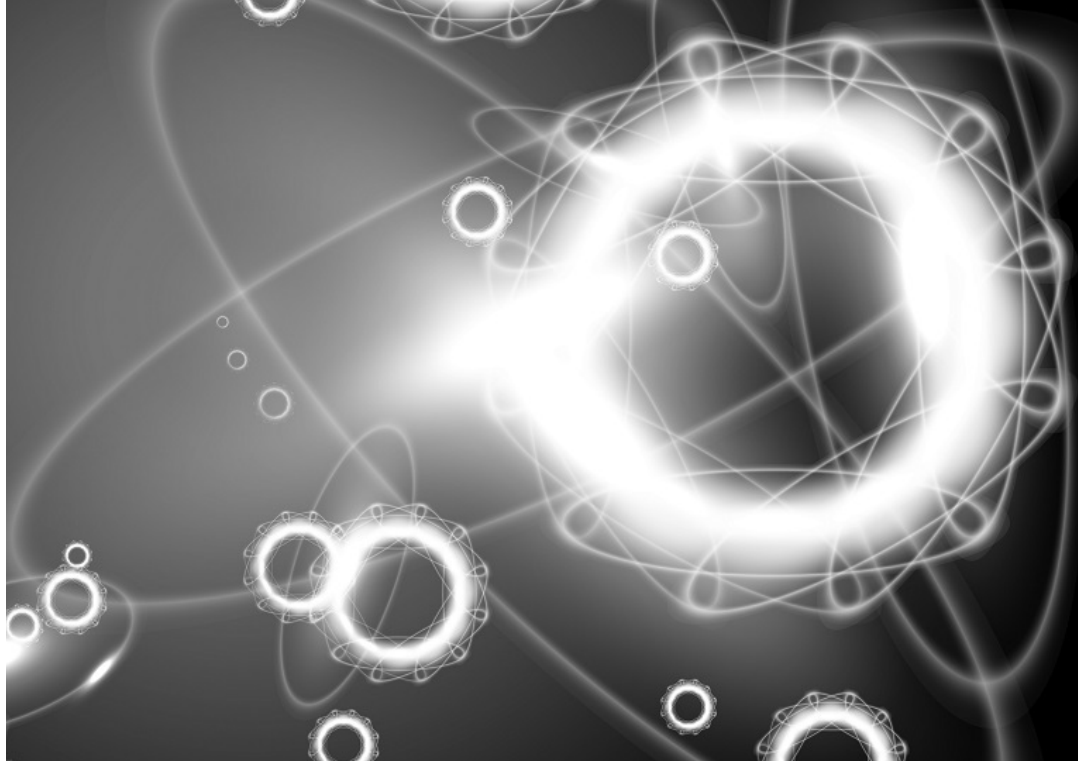
Chemical	CAS No.	Class I Division Group	Type ^a	Flash Point (°C)	AIT (°C)	% LFL	% UFL	Vapor Density (Air=1)	Vapor Pressure ^b (mm Hg)	Class 1 Zone Group ^c	MIE (mJ)	MIC Ratio	MESG (mm)
Acetaldehyde	75-07-0	C ^d	I	−38	175	4	60	1.5	874.9	IIA	0.37	0.98	0.92
Acetic Acid	64-19-7	D ^d	II	39	426		19.9	2.1	15.6	IIA		2.67	1.76
Acetone	67-64-1	D ^d	I	−20	465	2.5	12.8	2	230.7	IIA	1.15	1	1.02
Acetylene	74-86-2	A ^d	GAS		305	2.5	100	0.9	36,600	IIC	0.017	0.28	0.25
Acrylic Acid	79-10-7	D	II	54	488	2.4	8	2.5	4.3	IIB			0.86
Acrylonitrile	107-13-1	D ^d	I	0	481	3	17	1.8	108.5	IIB	0.16	0.78	0.87
Allyl Alcohol	107-18-6	C ^d	I	22	378	2.5	18	2	25.4	IIB			0.84
Allyl Chloride	107-05-1	D	I	−32	485	2.9	11.1	2.6	366	IIA		1.33	1.17
Ammonia	7664-41-7	D ^{d,f}	GAS		651	15	28	0.6	7498	IIA	680	6.85	3.17
Aniline	62-53-3	D	IIIA	70	615	1.2	8.3	3.2	0.7	IIA			
Benzene	71-43-2	D ^d	I	−11	498	1.2	7.8	2.8	94.8	IIA	0.2	1	0.99
n-Butane	3583-47-9	D ^{d,g}	GAS		288	1.9	8.5	2		IIA	0.25	0.94	1.07
1,3-Butadiene	106-99-0	B(D) ^{d,e}	GAS		420	2	11.5	1.9		IIB	0.13	0.76	0.79
1-Butanol	71-36-3	D ^d	I	36	343	1.4	11.2	2.6	7	IIA			0.91
Butylene	25167-67-3	D	I		385	1.6	10	1.9	2214.6	IIA			0.94
Carbon Monoxide	630-08-0	C ^d	GAS		609	12.5	74	0.97		IIB			0.54
Cumene	98-82-8	D	I	36	424	0.9	6.5	4.1	4.6	IIA			1.05
Cyclohexane	110-82-7	D	I	−17	245	1.3	8	2.9	98.8	IIA	0.22	1	0.94
Cyclopropane	75-19-4	D ^d	I		503	2.4	10.4	1.5	5430	IIA	0.17	0.84	0.91
Diethyl Ether (Ethyl Ether)	60-29-7	C ^d	I	−45	160	1.9	36	2.6	538	IIB	0.19	0.88	0.83
Ethane	74-84-0	D ^d	GAS	−29	472	3	12.5	1		IIA	0.24	0.82	0.91
Ethanol (Ethyl Alcohol)	64-17-5	D ^d	I	13	363	3.3	19	1.6	59.5	IIA		0.88	0.89
Ethylene	74-85-1	C ^d	GAS		490	2.7	36	1		IIB	0.07	0.53	0.65
Ethylene Dichloride	107-06-2	D ^d	I	13	413	6.2	16	3.4	79.7				
Ethylene Oxide	75-21-8	B(C) ^{d,e}	I	−20	429	3	100	1.5	1314	IIB	0.065	0.47	0.59
Ethyl Acetate	141-78-6	D ^d	I	−4	427	2	11.5	3	93.2	IIA	0.46		0.99
Ethyl Alcohol	64-17-5	D ^d	I	13	363	3.3	19	1.6	59.5	IIA		0.88	0.89
Ethyl Benzene	100-41-4	D	I	15	432	0.8	6.7	3.7	9.6				
Ethyl Chloride	75-00-3	D	GAS	−50	519	3.8	15.4	2.2					

Chemical	CAS No.	Class I Division Group	Type ^a	Flash Point (°C)	AIT (°C)	% LFL	% UFL	Vapor Density (Air=1)	Vapor Pressure ^b (mm Hg)	Class 1 Zone Group ^c	MIE (mJ)	MIC Ratio	MESG (mm)
Ethyl Mercaptan	75-08-1	C ^d	I	−18	300	2.8	18	2.1	527.4	IIB		0.9	0.9
Formaldehyde (Gas)	50-00-0	B	GAS		430	7	73	1		IIB			0.57
Formic Acid	64-18-6	D	II	50	434	18	57	1.6	42.7	IIA			1.86
Fuel Oil 1 (Jet Fuel)	8008-20-6	D	II or IIIA ^k	38–72 ^k	210	0.7	5						
Fuel Oil 2 (Diesel)			II or IIIA ^k	52–96 ^k	257								
Gasoline	8006-61-9	D ^d	I	−46	280	1.4	7.6	3					
n-Heptane	142-82-5	D ^d	I	−4	204	1	6.7	3.5	45.5	IIA	0.24	0.88	0.91
n-Hexane	110-54-3	D ^{d,g}	I	−23	225	1.1	7.5	3	152	IIA	0.24	0.88	0.93
Hydrogen	1333-74-0	B ^d	GAS		500	4	75	0.1		IIC	0.019	0.25	0.28
Hydrogen Sulfide	7783-06-4	C ^d	GAS		260	4	44	1.2		IIB	0.68		0.9
Isobutane	75-28-5	D ^g	GAS		460	1.8	8.4	2		IIA			0.95
Isoprene	78-79-5	D ^d	I	−54	220	1.5	8.9	2.4	550.6				
Isopropyl Ether	108-20-3	D ^d	I	−28	443	1.4	7.9	3.5	148.7	IIA	1.14		0.94
Kerosene	8008-20-6	D	II	72	210	0.7	5			IIA			
Liquefied Petroleum Gas	68476-85-7	D	I		405								
Methanol (Methyl Alcohol)	67-56-1	D ^d	I	12	385	6	36	1.1	126.3	IIA	0.14	0.82	0.92
Methyl Chloride	74-87-3	D	GAS	−46	632	8.1	17.4	1.7		IIA			1
Methyl Ether	115-10-6	C ^d	GAS	−41	350	3.4	27	1.6		IIB		0.85	0.84
Methyl Ethyl Ketone	78-93-3	D ^d	I	−6	404	1.4	11.4	2.5	92.4	IIB	0.53	0.92	0.84
Naptha (Petroleum)	8030-30-6	D ^{d,h}	I	42	288	1.1	5.9	2.5		IIA			
n-Octane	111-65-9	D ^{d,g}	I	13	206	1	6.5	3.9	14	IIA			0.94
n-Pentane	109-66-0	D ^{d,g}	I	−40	243	1.5	7.8	2.5	513	IIA	0.28	0.97	0.93
Process Gas > 30% H ₂	1333-74-0	B ⁱ	GAS		520	4	75	0.1			0.019	0.45	
Propane	74-98-6	D ^d	GAS		450	2.1	9.5	1.6		IIA	0.25	0.82	0.97
1-Propanol	71-23-8	D ^d	I	15	413	2.2	13.7	2.1	20.7	IIA			0.89
Propylene	115-07-1	D ^d	GAS		460	2.4	10.3	1.5		IIA	0.28		0.91
Styrene	100-42-5	D ^d	I	31	490	0.9	6.8	3.6	6.1	IIA		1.21	
Tetrahydrofuran	109-99-9	C ^d	I	−14	321	2	11.8	2.5	161.6	IIB	0.54		0.87
Toluene	108-88-3	D ^d	I	4	480	1.1	7.1	3.1	28.53	IIA	0.24		

Chemical	CAS No.	Class I Division Group	Type ^a	Flash Point (°C)	AIT (°C)	% LFL	% UFL	Vapor Density (Air=1)	Vapor Pressure ^b (mm Hg)	Class 1 Zone Group ^c	MIE (mJ)	MIC Ratio	MESG (mm)
Triethylamine	121-44-8	C ^d	I	−9	249	1.2	8	3.5	68.5	IIA	0.75		1.05
Vinyl Acetate	108-05-4	D ^d	I	−6	402	2.6	13.4	3	113.4	IIA	0.7		0.94
Vinyl Chloride	75-01-4	D ^d	GAS	−78	472	3.6	33	2.2		IIA			0.96
Xylene	1330-20-7	D ^d	I	25	464	0.9	7	3.7		IIA	0.2		1.09

- a. **Type** designates whether the material is a gas, flammable liquid, or combustible liquid.
- b. **Vapor Pressure** is reflected in units of mm Hg at 25°C (77°F), unless stated otherwise.
- c. **Class I Zone Groups** are based on 1996 IEC TR3 60079-20, Electrical apparatus for explosive gas atmospheres—Part 20: Data for flammable gases and vapors, relating to the use of electrical apparatus, which contains additional data on MESG and group classifications.
- d. Material has been classified by test.
- e. When all conduits run into explosion-proof equipment are provided with explosion-proof seals installed within 450 mm (18 in.) of the enclosure, equipment for the group classification shown in parentheses is permitted.
- f. For classification of areas involving ammonia, see ASHRAE 15, *Safety Code for Mechanical Refrigeration*, and ANSI/CGA G2.1, *Safety Requirements for the Storage and Handling of Anhydrous Ammonia*.
- g. Commercial grades of aliphatic hydrocarbon solvents are mixtures of several isomers of the same chemical formula (or molecular weight). The autoignition temperatures of the individual isomers are significantly different. The electrical equipment should be suitable for the AIT of the solvent mixture.
- h. [deleted]
- i. Petroleum naphtha is a saturated hydrocarbon mixture whose boiling range is 20°C to 135°C (68°F to 275°F). It is also known as benzine, ligroin, petroleum ether, and naphtha.
- j. Fuel and process gas mixtures found by test not to present hazards similar to those of hydrogen may be grouped based on the test results.
- k. [deleted]

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9 PHYSICAL PROPERTIES

9.1 Symbols and Definitions

Symbols

Symbol	Description	Units (U.S.)	Units (SI)
c_p	Heat capacity (at constant pressure)	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}} = \frac{\text{m}^2}{\text{s}^2 \cdot \text{K}}$
c_v	Heat capacity (at constant volume)	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}} = \frac{\text{m}^2}{\text{s}^2 \cdot \text{K}}$
h	Specific enthalpy	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}}$
Δh_{fusion}	Enthalpy of fusion	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}}$
Δh_{vap}	Enthalpy of vaporization	$\frac{\text{Btu}}{\text{lbm}}$	$\frac{\text{J}}{\text{kg}}$
k	Thermal conductivity	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$	$\frac{\text{W}}{\text{m} \cdot \text{K}}$
MW	Molar mass (molecular weight)	$\frac{\text{lbm}}{\text{lb mole}}$	$\frac{\text{g}}{\text{mol}}$
P	Pressure	$\frac{\text{lbf}}{\text{in}^2}$ or psi	$\text{Pa} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
r	Ratio of heat capacities = $\frac{c_p}{c_v}$	dimensionless	
s	Specific entropy	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}}$
T	Temperature	$^\circ\text{F}$ or $^\circ\text{R}$	$^\circ\text{C}$ or K
v	Specific volume	$\frac{\text{ft}^3}{\text{lbm}}$	$\frac{\text{m}^3}{\text{kg}}$
α	Thermal diffusivity	$\frac{\text{ft}^2}{\text{sec}}$	$\frac{\text{m}^2}{\text{s}}$

Symbols (cont'd)

Symbol	Description	Units (U.S.)	Units (SI)
γ	Surface tension	$\frac{\text{dyne}}{\text{cm}}$	$\frac{\text{N}}{\text{m}}$
μ	Dynamic viscosity	$\frac{\text{lbm}}{\text{ft} \cdot \text{sec}}$	$\text{Pa} \cdot \text{s} = \frac{\text{kg}}{\text{m} \cdot \text{s}}$
ν	Kinematic viscosity	$\frac{\text{ft}^2}{\text{sec}}$	$\frac{\text{m}^2}{\text{s}}$
ρ	Density	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{kg}}{\text{m}^3}$
ρ	Electrical resistivity	$\Omega \cdot \text{ft}$	$\Omega \cdot \text{m}$

9.2 Physical Properties of Metals

9.2.1 U.S. Customary Units

Physical Properties of Metals at 68°F (U.S. Units)

Property:	Molar Mass	Density	Heat Capacity	Thermal Conductivity	Thermal Diffusivity	Electrical Resistivity (°C)	Melting Point	Heat of Fusion
U.S. Unit:	$\frac{\text{lbm}}{\text{lb mole}}$	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$	$\frac{\text{ft}^2}{\text{hr}}$	$\Omega \cdot \text{ft} \cdot 10^{-8}$	°F	$\frac{\text{Btu}}{\text{lbm}}$
Aluminum	26.98	168	0.214	136.35	3.55	8.20	1220	138.2
Brass 70%Cu, 30%Zn		532.0	0.091	61.80	1.27		1700	72.2
Bronze 75%Cu, 25%Sn		540.0	0.082	15.00			1200	
Calcium	40.08	95.5	0.152			10.50	1544	83.58
Chromium	52.00	449.1	0.097	55.75		41.67	2939	215.4
Constantan		557.0	0.098	13.10	0.24		2336	
Copper	63.54	557.7	0.093	232.84	3.98	5.09	1983	89
Gold	196.97	1203.7	0.031	184.31	4.52	6.73	1947	28.8
Iron	55.85	491.5	0.109	48.24	0.83	29.20	2804	114.7
Iron, cast		455.0	0.100	29.60	0.65			41.4
Lead	207.20	708.1	0.031	20.80	0.80	62.99	621	10.62
Lithium	6.94	33.3	1.093	49.69		28.05	356	147.4
Magnesium	24.31	108.5	0.250	90.71	3.68	12.93	1202	
Manganese	54.94	466.5	0.120	4.62		452.75	2282	114.6
Mercury (liquid)	200.59	845.7	0.034	4.51		308.72	-38	5.08
Molybdenum	95.94	638.2	0.065	80.31		16.40	4748	186.3
Nickel	58.69	556.1	0.105	54.31	0.87	20.34	2651	132.8
Nichrome V		530.0	0.106	7.06	0.12			
Palladium	106.40	748.8	0.055	41.60		32.81	2829	
Platinum	195.08	1339.1	0.032	41.60	0.09	32.18	3222	43.3
Potassium	39.09	53.8	0.180	60.09		20.01	145	25.0
Rhodium	102.91	775.4	0.058	87.24		14.11	3565	

Physical Properties of Metals at 68°F (U.S. Units) (cont'd)

Property:	Molar Mass	Density	Heat Capacity	Thermal Conductivity	Thermal Diffusivity	Electrical Resistivity (°C)	Melting Point	Heat of Fusion
U.S. Unit:	$\frac{\text{lbm}}{\text{lb mole}}$	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$	$\frac{\text{ft}^2}{\text{hr}}$	$\Omega \cdot \text{ft} \cdot 10^{-8}$	°F	$\frac{\text{Btu}}{\text{lbm}}$
Silver	107.87	655.5	0.056	247.28	6.42	4.82	1762	45.0
Sodium	22.99	60.3	0.295	82.04		13.78	208	37.8
Steel, carbon								
Steel, mild (1%C)		488.0	0.113	24.80	0.45			
Steel, stainless		488.0	0.110	9.40	0.17			
Tin	118.69	454.8	0.055	39.29	1.57	37.73	450	25.2
Titanium	47.88	281.4	0.126	12.71		127.95	3038	122.6
Tungsten	183.85	1202.0	0.034	102.26	2.44	16.08	6129	109.7
Uranium	238.03	1189.3	0.028	15.60	0.53	91.86	2075	
Zinc	65.38	445.4	0.094	67.60	1.55	18.04	786	43.9

9.2.2 SI Units

Physical Properties of Metals at 20°C (SI Units)

Property:	Molar Mass	Density	Heat Capacity	Thermal Conductivity	Thermal Diffusivity	Electrical Resistivity (°C)	Melting Point	Heat of Fusion
SI Unit:	$\frac{\text{g}}{\text{mol}}$	$\frac{\text{kg}}{\text{m}^3}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}}$	$\frac{\text{W}}{\text{m} \cdot \text{K}}$	$\frac{\text{m}^2}{\text{s}}$	$\Omega \cdot \text{m} \cdot 10^{-8}$	°C	$\frac{\text{kJ}}{\text{kg}}$
Aluminum	26.98	2698	896	236.0	91.61	2.50	660	321.5
Brass 70%Cu, 30%Zn		8522	381	107.0	32.77		927	167.9
Bronze 75%Cu, 25%Sn		8650	343	26.0			649	
Calcium	40.08	1530	636			3.20	840	194.4
Chromium	52.00	7194	407	96.5		12.70	1615	501.1
Constantan		8922	410	22.7	6.19		1280	
Copper	63.54	8933	389	403.0	102.71	1.55	1084	207.0
Gold	196.97	19,281	130	319.0	116.64	2.05	1064	67.0
Iron	55.85	7873	456	83.5	21.42	8.90	1540	266.7
Iron, cast		7288	419	51.2	16.77			96.3
Lead	207.20	11,343	130	36.0	20.64	19.20	327	24.7
Lithium	6.94	533	4576	86.0		8.55	180	342.9
Magnesium	24.31	1738	1047	157.0	94.97	3.94	650	
Manganese	54.94	7473	502	8.0		138.00	1250	266.6
Mercury (liquid)	200.59	13,547	142	7.8		94.10	-39	11.8
Molybdenum	95.94	10,222	272	139.0		5.00	2620	433.3

Physical Properties of Metals at 20°C (SI Units) (cont'd)

Property:	Molar Mass	Density	Heat Capacity	Thermal Conductivity	Thermal Diffusivity	Electrical Resistivity (°C)	Melting Point	Heat of Fusion
SI Unit:	$\frac{\text{g}}{\text{mol}}$	$\frac{\text{kg}}{\text{m}^3}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}}$	$\frac{\text{W}}{\text{m} \cdot \text{K}}$	$\frac{\text{m}^2}{\text{s}}$	$\Omega \cdot \text{m} \cdot 10^{-8}$	°C	$\frac{\text{kJ}}{\text{kg}}$
Nickel	58.69	8907	440	94.0	22.45	6.20	1455	308.9
Nichrome V		8490	444	12.2	3.10			
Palladium	106.40	11,995	230	72.0		10.00	1554	
Platinum	195.08	21,450	134	72.0	2.32	9.81	1772	100.8
Potassium	39.09	862	754	104.0		6.10	63	58.1
Rhodium	102.91	12,420	243	151.0		4.30	1963	
Silver	107.87	10,500	235	428.0	165.67	1.47	961	104.7
Sodium	22.99	966	1235	142.0		4.20	98	87.9
Steel, carbon								
Steel, mild (1%C)		7817	473	42.9	11.61			
Steel, stainless		7817	461	16.3	4.39			
Tin	118.69	7285	230	68.0	40.52	11.50	232	58.6
Titanium	47.88	4508	528	22.0		39.00	1670	285.1
Tungsten	183.85	19,254	143	177.0	62.97	4.90	3387	255.1
Uranium	238.03	19,050	117	27.0	13.68	28.00	1135	
Zinc	65.38	7135	394	117.0	40.00	5.50	419	102.1

9.3 Physical Properties of Plastics

9.3.1 U.S. Customary Units

Physical Properties of Plastics (U.S. Units)

Property:	Density	Heat Capacity	Thermal Conductivity
U.S. Unit:	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$
ABS	64–75	0.361–0.370	0.092–0.156
Nylon	64–71	0.330–0.399	0.098–0.196
Polycarbonate	57–78	0.279–0.301	0.110–0.127
Polyethylene	57–60	0.499–0.549	0.243–0.283
Polyester	69–125	0.320–0.499	0.191–0.526
PVC	77–97	0.251	0.081–0.110
Polystyrene foam	1.0–2.0		0.017–0.023
PVDF	109–117	0.28–0.36	0.083–0.110
PFA	134	0.28	0.113
PTFE	130–140		0.142–0.375
FEP	134	0.28	0.113
ETFE	106	0.28	0.138

9.3.2 SI Units

Physical Properties of Plastics (SI Units)

Property:	Density	Heat Capacity	Thermal Conductivity
SI Unit:	$\frac{\text{kg}}{\text{m}^3}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}}$	$\frac{\text{W}}{\text{m} \cdot \text{K}}$
ABS	1020–1200	1510–1550	0.16–0.27
Nylon	1030–1140	1380–1670	0.17–0.34
Polycarbonate	910–1250	1170–1260	0.19–0.22
Polyethylene	913–968	2090–2300	0.42–0.49
Polyester	1100–2010	1340–2090	0.33–0.91
PVC	1240–1550	1050	0.14–0.19
Polystyrene foam	16–32		0.03–0.04
PVDF	1746–1874	1172–1507	0.14–0.19
PFA	2150	1172	0.20
PTFE	2082–2243		0.25–0.65
FEP	2150	1172	0.20
ETFE	1700	1172	0.24

9.3.3 Chemical Resistance of Plastics

Chemical Resistance of Plastics

Plastic:	Polypropylene, Polyethylene	Cellulose Acetate Butyrate (CAB)	Acrylonitrile Butadiene Styrene Polymer (ABS)	Polyvinyl Chloride, Type I (PVC)	Saran	Polyester Glass	Epoxy Glass	Phenolic Asbestos	Fluorocarbons	Chlorinated Polyether (Penton)	Polycarbonate
10% H ₂ SO ₄	Excel.	Good	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.
50% H ₂ SO ₄	Excel.	Poor	Excel.	Excel.	Excel.	Good	Good	Excel.	Excel.	Excel.	Excel.
10% HCl	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.
10% HNO ₃	Excel.	Poor	Good	Excel.	Excel.	Good	Good	Fair	Excel.	Excel.	Excel.
10% Acetic	Excel.	Good	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.
10% NaOH	Excel.	Fair	Excel.	Good	Fair	Fair	Excel.	Poor	Excel.	Excel.	Excel.
50% NaOH	Excel.	Poor	Excel.	Excel.	Fair	Poor	Good	Poor	Excel.	Excel.	Excel.
NH ₄ OH	Excel.	Poor	Excel.	Excel.	Poor	Fair	Excel.	Poor	Excel.	Excel.	Excel.
NaCl	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.
FeCl ₂	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.
CuSO ₄	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.
NH ₄ NO ₃	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Good	Excel.	Excel.	Excel.
Wet H ₂ S	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	
Wet Cl ₂	Poor	Poor	Excel.	Good	Poor	Poor	Poor	Excel.	Excel.	Excel.	
Wet SO ₂	Excel.	Poor	Excel.	Excel.	Good	Excel.	Excel.	Excel.	Excel.	Excel.	
Gasoline	Poor	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.
Benzene	Poor	Poor	Poor	Poor	Fair	Good	Excel.	Excel.	Excel.	Fair	Fair
CCl ₄	Poor	Poor	Poor	Fair	Fair	Excel.	Good	Excel.	Excel.	Fair	Poor
Acetone	Poor	Poor	Poor	Poor	Fair	Poor	Good	Poor	Excel.	Good	Good
Alcohol	Poor	Poor	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.	Excel.

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9.4 Physical Properties of Liquids and Gases—Temperature-Independent Properties

9.4.1 U.S. Customary Units

Temperature-Independent Properties of Liquids and Gases (U.S. Units)

Property:	Formula	Molar Mass	Normal Boiling Point (NBP)	Triple Point	Heat of Vaporization at NBP	Critical Temperature	Critical Pressure	Critical Density
Chemical ↓		$\frac{\text{lbm}}{\text{lb mole}}$	°F	°F	$\frac{\text{Btu}}{\text{lbm}}$	°F	psia	$\frac{\text{lbm}}{\text{ft}^3}$
Acetaldehyde	C ₂ H ₄ O	44.053	69.8	−190.1	253.2	379.1	807.8	17.85
Acetic Acid	C ₂ H ₄ O ₂	60.052	244.2	62.0	167.4	605.8	839.2	21.17
Acetone	C ₃ H ₆ O	58.079	455.1	−138.5	219.3	455.1	681.7	17.34
Acetylene	C ₂ H ₂	26.037	−114.7	564.4	270.5	95.4	900.1	13.82
Air		28.965	−317.6	−352.1		−221.1	549.1	21.39
Ammonia	NH ₃	17.031	−28.0	−107.8	588.8	270.1	1643.7	14.05
Argon	Ar	39.948	−302.5	−301.7	69.3	−188.4	724.1	33.44
Benzene	C ₆ H ₆	78.112	176.1	41.9	169.3	552.0	711.7	19.02
n-Butane	C ₄ H ₁₀	58.122	31.1	−216.9	165.7	305.5	550.6	14.23
iso-Butane	C ₄ H ₁₀	58.122	10.9	−255.0	157.0	274.4	526.3	14.08
1-Butene	C ₄ H ₈	56.106	20.6	−301.6	167.5	295.4	583.1	14.54
1-Butanol	C ₄ H ₁₀ O	74.1216	245.8	−128.7	249.6	553.9	640.2	16.95
1,3-Butadiene	C ₄ H ₆	54.090	23.8	−164.0	178.1	305.5	619.4	15.16
1,2-Butadiene	C ₄ H ₆	54.090	51.7	−213.2	190.4	362.0	702.9	15.45
Carbon dioxide	CO ₂	44.010	−109.2	−69.8	246.5	87.8	1070.0	29.19
Carbon monoxide	CO	28.010	−312.7	−337.0	92.3	−220.5	506.8	18.97
Carbonyl sulfide	COS	60.075	−58.3	−217.9	132.8	222.1	923.7	27.78
Carbon tetrachloride	CCl ₄	153.823	170.0	−9.1	83.0	541.8	661.4	34.79
Chlorine	Cl ₂	70.906	−29.2	−149.7	121.5	290.7	1157.1	34.01
Cumene (isopropyl-benzene)	C ₉ H ₁₂	120.192	306.2	−140.8	131.9	676.4	462.2	17.78
Cyclohexane	C ₆ H ₁₂	84.160	177.3	44.1	153.2	536.8	591.8	16.94
n-Decane	C ₁₀ H ₂₂	142.282	345.4	−21.4	118.8	652.2	305.0	14.57
Diethyl ether	C ₄ H ₁₀ O	74.122	94.0	−177.3	154.2	380.4	527.9	16.52
Ethane	C ₂ H ₆	30.069	−127.4	−297.0	210.4	89.9	706.7	12.87
Ethanol	C ₂ H ₆ O	46.068	172.9	172.9	365.7	465.5	890.1	17.11
Ethyl acetate	C ₄ H ₈ O ₂	88.105	170.7	−118.4	156.9	482.3	562.7	19.23
Ethylbenzene	C ₈ H ₁₀	106.165	277.1	−138.9	144.2	651.2	525.4	18.17
Ethylene	C ₂ H ₄	28.053	−154.8	−272.5	207.4	48.6	731.3	13.37

Temperature-Independent Properties of Liquids and Gases (U.S. Units) (cont'd)

Property:	Formula	Molar Mass	Normal Boiling Point (NBP)	Triple Point	Heat of Vaporization at NBP	Critical Temperature	Critical Pressure	Critical Density
Chemical ↓		$\frac{\text{lbm}}{\text{lb mole}}$	°F	°F	$\frac{\text{Btu}}{\text{lbm}}$	°F	psia	$\frac{\text{lbm}}{\text{ft}^3}$
Ethylene glycol	C ₂ H ₆ O ₂	62.000	387.0	7.9				
Fluorine	F ₂	37.997	-306.6	-363.4	75.0	-199.7	770.2	37.01
Formaldehyde	CH ₂ O	30.026	-2.7	-180.4	329.7	296.3	955.8	22.02
Helium	He	4.003	-781.7	-788.4	8.8	-450.3	33.1	4.34
<i>n</i> -Heptane	C ₇ H ₁₆	100.202	209.1	-131.1	136.2	512.6	396.8	14.48
<i>n</i> -Hexane	C ₆ H ₁₄	86.175	155.7	-139.6	144.0	453.8	436.9	14.56
Hydrogen	H ₂	2.016	-423.0	-434.6	192.9	-400.0	188.0	1.95
Hydrogen chloride	HCl	36.461	-121.0	-173.5	190.7	124.5	1202.1	26.86
Hydrogen sulfide	H ₂ S	34.081	-76.5	-121.8	234.9	211.9	1305.3	21.68
Methane	CH ₄	16.043	-258.7	-296.4	219.6	-116.7	667.1	10.15
Methanol	CH ₄ O	32.042	148.5	-143.8	473.1	462.8	1172.5	17.09
Methyl acetate	C ₃ H ₆ O ₂	74.07854	134.5	-144.4	176.7	452.1	688.9	20.28
Methyl amine	CH ₅ N	31.057	20.6	-136.2	361.0	314.4	1082.0	12.59
Methyl ethyl ketone	C ₄ H ₈ O	72.106	175.4	-124.0	188.5	504.2	601.9	16.85
Neon	Ne	20.180	-410.9	-415.5	36.9	-379.6	398.9	30.09
Nitrobenzene	C ₆ H ₅ NO ₂	123.109	411.5	42.4	190.0			
Nitrogen	N ₂	28.013	-320.4	-346.0	85.6	-232.5	492.5	19.56
<i>n</i> -Nonane	C ₉ H ₂₀	128.255	303.4	-64.2	124.1	610.5	330.8	14.49
<i>n</i> -Octane	C ₈ H ₁₈	114.229	258.1	-70.2	129.9	564.2	360.7	14.66
iso-Octane (2,2,4-trimethylpentane)	C ₈ H ₁₈	114.229	210.6	-161.3	115.3	519.5	373.0	15.12
Oxygen	O ₂	31.999	-297.3	-361.8	91.6	-181.4	731.4	27.23
iso-Pentane	C ₅ H ₁₂	72.149	82.1	-256.9	147.6	369.0	489.9	14.73
<i>n</i> -Pentane	C ₅ H ₁₂	72.149	96.9	-201.4	153.7	385.8	488.8	14.48
Propane	C ₃ H ₈	44.096	-43.8	-305.7	183.0	206.1	616.6	13.76
1-Propanol	C ₃ H ₈ O	60.095	207.0	-195.2	297.5	506.6	749.7	17.13
Propylene	C ₃ H ₆	42.080	-53.7	-301.4	188.7	195.9	660.6	14.33
Sulfur dioxide	SO ₂	64.064	14.0	-103.8	167.3	315.5	1143.5	32.77
Styrene	C ₈ H ₈	104.149	293.5	-23.2	151.2	683.7	563.2	18.24
Toluene	C ₇ H ₈	92.138	231.1	-139.3	155.1	605.5	598.5	18.23
Water	H ₂ O	18.015	212.0	32.0	970.1	705.1	3200.1	20.10
<i>p</i> -Xylene	C ₈ H ₁₀	106.165	281.0	55.9	144.6	649.4	512.2	17.85
<i>m</i> -Xylene	C ₈ H ₁₀	106.165	282.3	-54.1	146.2	650.7	512.7	17.66
<i>o</i> -Xylene	C ₈ H ₁₀	106.165	291.9	-13.3	147.3	674.8	542.1	17.79

9.4.2 SI Units

Temperature-Independent Properties of Liquids and Gases (SI Units)

Property:	Formula	Molar Mass	Normal Boiling Point (NBP)	Triple Point	Heat of Vaporization at NBP	Critical Temperature	Critical Pressure	Critical Density
Chemical ↓		$\frac{\text{g}}{\text{mol}}$	$^{\circ}\text{C}$	$^{\circ}\text{C}$	$\frac{\text{kJ}}{\text{kg}}$	$^{\circ}\text{C}$	MPa	$\frac{\text{kg}}{\text{m}^3}$
Acetaldehyde	$\text{C}_2\text{H}_4\text{O}$	44.053	21.0	-123.4	588.8	192.9	5.57	286
Acetic Acid	$\text{C}_2\text{H}_4\text{O}_2$	60.052	117.9	16.7	389.3	318.8	5.79	339
Acetone	$\text{C}_3\text{H}_6\text{O}$	58.079	235.1	-94.7	510.0	235.1	4.70	278
Acetylene	C_2H_2	26.037	-81.5	295.8	629.3	35.2	6.21	221
Air		28.965	-194.2	-213.4		-140.6	3.79	343
Ammonia	NH_3	17.031	-33.3	-77.7	1,369.5	132.3	11.33	225
Argon	Ar	39.948	-185.8	-185.4	161.2	-122.5	4.99	536
Benzene	C_6H_6	78.112	80.1	5.5	393.8	288.9	4.91	305
n-Butane	C_4H_{10}	58.122	-0.5	-138.3	385.5	152.0	3.80	228
Isobutane	C_4H_{10}	58.122	-11.7	-159.4	365.2	134.7	3.63	226
1-Butene	C_4H_8	56.106	-6.35	-185.35	389.7	146.35	4.02	233
1-Butanol	$\text{C}_4\text{H}_{10}\text{O}$	74.1216	118.75	-89.3	580.573	289.95	4.414	272
1,3-Butadiene	C_4H_6	54.090	-4.6	-108.9	414.3	151.9	4.27	243
1,2-Butadiene	C_4H_6	54.090	11.0	-136.2	442.9	183.4	4.85	247
Carbon dioxide	CO_2	44.010	-78.5	-56.6	573.4	31.0	7.38	468
Carbon monoxide	CO	28.010	-191.5	-205.0	214.7	-140.3	3.49	304
Carbonyl sulfide	COS	60.075	-50.2	-138.8	308.9	105.6	6.37	445
Carbon tetrachloride	CCl_4	153.823	76.6	-22.82	193.1	283.2	4.56	557
Chlorine	Cl_2	70.906	-34.0	-100.9	282.6	143.7	7.98	545
Cumene (isopropylbenzene)	C_9H_{12}	120.192	152.3	-96.0	306.8	358.0	3.19	285
Cyclohexane	C_6H_{12}	84.160	80.7	6.7	356.3	280.5	4.08	271
n-Decane	$\text{C}_{10}\text{H}_{22}$	142.282	174.1	-29.7	276.3	344.6	2.10	233
Diethyl ether	$\text{C}_4\text{H}_{10}\text{O}$	74.122	34.4	-116.3	358.6	193.6	3.64	265
Ethane	C_2H_6	30.069	-88.6	-182.8	489.4	32.2	4.87	206
Ethanol	$\text{C}_2\text{H}_6\text{O}$	46.068	78.3	78.3	850.6	240.9	6.14	274
Ethyl acetate	$\text{C}_4\text{H}_8\text{O}_2$	88.105	77.1	-83.6	365.0	250.2	3.88	308
Ethylbenzene	C_8H_{10}	106.165	136.2	-95.0	335.4	344.0	3.62	291
Ethylene	C_2H_4	28.053	-103.8	-169.2	482.4	9.2	5.04	214
Ethylene glycol	$\text{C}_2\text{H}_6\text{O}_2$	62.000	197.2	-13.4				
Fluorine	F_2	37.997	-188.1	-219.7	174.4	-128.7	5.31	593
Formaldehyde	CH_2O	30.026	-19.3	-118.0	767.0	146.9	6.59	353
Helium	He	4.003	-452.1	-455.8	20.6	-268.0	0.23	70
n-Heptane	C_7H_{16}	100.202	98.4	-90.6	316.8	267.0	2.74	232
n-Hexane	C_6H_{14}	86.175	68.7	-95.3	334.9	234.3	3.01	233

Temperature-Independent Properties of Liquids and Gases (SI Units) (cont'd)

Property:	Formula	Molar Mass	Normal Boiling Point (NBP)	Triple Point	Heat of Vaporization at NBP	Critical Temperature	Critical Pressure	Critical Density
Chemical ↓		$\frac{\text{g}}{\text{mol}}$	$^{\circ}\text{C}$	$^{\circ}\text{C}$	$\frac{\text{kJ}}{\text{kg}}$	$^{\circ}\text{C}$	MPa	$\frac{\text{kg}}{\text{m}^3}$
Hydrogen	H ₂	2.016	-252.8	-259.2	448.7	-240.0	1.30	31
Hydrogen chloride	HCl	36.461	-85.0	-114.1	443.6	51.4	8.29	430
Hydrogen sulfide	H ₂ S	34.081	-60.3	-85.5	546.4	100.0	9.00	347
Methane	CH ₄	16.043	-161.5	-182.5	510.8	-82.6	4.60	163
Methanol	CH ₄ O	32.042	64.7	-97.7	1100.5	239.4	8.08	274
Methyl acetate	C ₃ H ₆ O ₂	74.07854	56.9	-98.0	411.1	233.4	4.75	325
Methyl amine	CH ₅ N	31.057	-6.3	-93.5	839.8	156.9	7.46	202
Methyl ethyl ketone	C ₄ H ₈ O	72.106	79.6	-86.7	438.3	262.4	4.15	270
Neon	Ne	20.180	-246.0	-248.6	85.8	-228.7	2.75	482
Nitrobenzene	C ₆ H ₅ NO ₂	123.109	210.9	5.8	442.7			
Nitrogen	N ₂	28.013	-195.8	-210.0	199.2	-147.0	3.40	313
<i>n</i> -Nonane	C ₉ H ₂₀	128.255	150.8	-53.5	288.7	321.4	2.28	232
<i>n</i> -Octane	C ₈ H ₁₈	114.229	125.6	-56.8	302.1	295.7	2.49	235
iso-Octane (2,2,4-trimethylpentane)	C ₈ H ₁₈	114.229	99.2	-107.4	268.2	270.9	2.57	242
Oxygen	O ₂	31.999	-183.0	-218.8	213.1	-118.6	5.04	436
iso-Pentane	C ₅ H ₁₂	72.149	27.8	-160.5	343.3	187.2	3.38	236
<i>n</i> -Pentane	C ₅ H ₁₂	72.149	36.1	-129.7	357.5	196.6	3.37	232
Propane	C ₃ H ₈	44.096	-42.1	-187.6	425.7	96.7	4.25	220
1-Propanol	C ₃ H ₈ O	60.095	97.2	-126.2	692.0	263.7	5.17	274
Propylene	C ₃ H ₆	42.080	-47.6	-185.2	438.9	91.1	4.55	230
Sulfur dioxide	SO ₂	64.064	-10.0	-75.5	389.1	157.5	7.88	525
Styrene	C ₈ H ₈	104.149	145.3	-30.7	351.7	362.1	3.88	292
Toluene	C ₇ H ₈	92.138	110.6	-95.2	360.8	318.6	4.13	292
Water	H ₂ O	18.015	100.0	0.0	2256.5	705.1	22.06	322
<i>p</i> -Xylene	C ₈ H ₁₀	106.165	138.3	13.3	336.3	343.0	3.53	286
<i>m</i> -Xylene	C ₈ H ₁₀	106.165	139.1	-47.9	340.1	343.7	3.53	283
<i>o</i> -Xylene	C ₈ H ₁₀	106.165	144.4	-25.2	342.6	357.1	3.74	285

Source for tables in Section 8.4: "Table of Physical Properties for Hydrocarbons and Other Compounds of Interest to the Natural Gas and Natural Gas Liquids Industries," GPS Standard 2145-16, Tulsa, OK: GPA Midstream Association, 2016, pp. 4–9, and NIST Chemistry Web Book, NIST Standard Reference Database Number 69, P.J. Linstrom and W.G. Mallard, eds.

9.5 Physical Properties of Liquids and Gases—Temperature-Dependent Properties

9.5.1 U.S. Customary Units

Temperature-Dependent Physical Properties of Gases at 14.7 psia (U.S. Units)

Gas	Temperature	Density	Heat Capacity (Cp)	Thermal Conductivity	Thermal Diffusivity	Viscosity	Prandtl Number
	°F	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{Btu}}{\text{ft} \cdot \text{hr} \cdot ^\circ\text{F}}$	$\frac{\text{ft}^2}{\text{hr}}$	$\frac{\text{lbm}}{\text{ft} \cdot \text{sec}}$	
Nitrogen	0	0.0835	0.248	0.0129	0.624	10.6E–6	0.730
	200	0.0582	0.249	0.0175	1.21	13.9E–6	0.713
	400	0.0446	0.251	0.0217	1.93	16.8E–6	0.704
	600	0.0362	0.256	0.0256	2.76	19.5E–6	0.701
	800	0.0304	0.262	0.0294	3.69	22.0E–6	0.704
	1000	0.0263	0.269	0.0331	4.68	24.3E–6	0.709
Oxygen	0	0.0955	0.218	0.0133	0.637	12.2E–6	0.722
	200	0.0664	0.222	0.0184	1.24	16.3E–6	0.710
	400	0.0510	0.230	0.0231	1.97	19.9E–6	0.714
	600	0.0414	0.239	0.0276	2.79	23.1E–6	0.721
	800	0.0348	0.246	0.0318	3.71	26.1E–6	0.727
	1000	0.0300	0.252	0.0359	4.74	28.8E–6	0.730
Carbon Monoxide	0	0.0835	0.248	0.0126	0.608	10.5E–6	0.745
	200	0.0581	0.249	0.0172	1.18	13.9E–6	0.727
	400	0.0446	0.253	0.0213	1.88	16.8E–6	0.719
	600	0.0362	0.259	0.0251	2.68	19.4E–6	0.721
	800	0.0304	0.266	0.0286	3.54	21.7E–6	0.726
	1000	0.0263	0.273	0.0320	4.47	23.9E–6	0.733
Carbon Dioxide	0	0.131	0.190	0.00763	0.307	8.65E–6	0.774
	200	0.0914	0.219	0.0127	0.632	12.2E–6	0.760
	400	0.0701	0.239	0.0179	1.07	15.3E–6	0.739
	600	0.0569	0.255	0.0230	1.59	18.2E–6	0.723
	800	0.0479	0.268	0.0279	2.18	20.8E–6	0.716
	1000	0.0413	0.279	0.0325	2.82	23.2E–6	0.715
Sulfur Dioxide	0	0.195*	0.143	0.00443	0.159	7.38E–6	0.856
	200	0.134	0.158	0.00738	0.349	10.7E–6	0.823
	400	0.102	0.171	0.0107	0.612	13.8E–6	0.791
	600	0.0829	0.182	0.0142	0.945	16.6E–6	0.764
	800	0.0697	0.190	0.0177	1.34	19.3E–6	0.746
	1000	0.0601	0.196	0.0208	1.76	21.8E–6	0.741
Air	0	0.0864	0.239	0.0132	0.639	11.06E–6	0.716
	200	0.0601	0.240	0.0178	1.23	14.5E–6	0.705
	400	0.0461	0.244	0.0220	1.95	17.5E–6	0.698
	600	0.0374	0.249	0.0259	2.78	20.1E–6	0.697
	800	0.0315	0.256	0.0297	3.69	22.5E–6	0.698
	1000	0.0272	0.262	0.0333	4.68	24.7E–6	0.698

Temperature-Dependent Physical Properties of Gases at 14.7 psia (U.S. Units) (cont'd)

Gas	Temperature	Density	Heat Capacity (Cp)	Thermal Conductivity	Thermal Diffusivity	Viscosity	Prandtl Number
	°F	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{Btu}}{\text{ft} \cdot \text{hr} \cdot ^\circ\text{F}}$	$\frac{\text{ft}^2}{\text{hr}}$	$\frac{\text{lbm}}{\text{ft} \cdot \text{sec}}$	
Hydrogen	0	0.00600	3.37	0.0911	4.50	5.38E-6	0.718
	200	0.00420	3.45	0.121	8.34	6.89E-6	0.708
	400	0.00320	3.47	0.148	13.3	8.27E-6	0.697
	600	0.00260	3.47	0.174	19.3	9.54E-6	0.685
	800	0.00220	3.48	0.199	25.9	10.7E-6	0.678
	1000	0.00190	3.51	0.222	33.4	11.9E-6	0.675
Ammonia	0	0.0514	0.484	0.0117	0.471	5.76E-6	0.857
	200	0.0355	0.529	0.0193	1.03	8.48E-6	0.837
	400	0.0272	0.581	0.0278	1.76	11.2E-6	0.843
	600	0.0220	0.630	0.0371	2.67	13.9E-6	0.851
	800	0.0185	0.677	0.0471	3.76	16.6E-6	0.860
	1000	0.0160	0.722	0.0577	4.99	19.3E-6	0.869
Helium	0	0.0119	1.24	0.0802	5.43	12.0E-6	0.669
	200	0.00830	1.24	0.102	9.94	15.4E-6	0.671
	400	0.00640	1.24	0.123	15.5	18.5E-6	0.672
	600	0.00520	1.24	0.142	22.0	21.4E-6	0.672
	800	0.00440	1.24	0.160	29.4	24.2E-6	0.672
	1000	0.00380	1.24	0.178	37.8	26.8E-6	0.672
Neon	0	0.0601	0.246	0.0253	1.71	19.0E-6	0.664
	200	0.0419	0.246	0.0325	3.15	24.3E-6	0.662
	400	0.0321	0.246	0.0390	4.93	29.1E-6	0.660
	600	0.0261	0.246	0.0449	7.00	33.5E-6	0.660
	800	0.0219	0.246	0.0505	9.38	37.6E-6	0.659
	1000	0.0189	0.246	0.0558	12.0	41.5E-6	0.659
Argon	0	0.119	0.124	0.00903	0.609	13.4E-6	0.663
	200	0.0830	0.124	0.0121	1.17	18.0E-6	0.666
	400	0.0636	0.124	0.0148	1.87	22.1E-6	0.667
	600	0.0516	0.124	0.0173	2.69	25.8E-6	0.667
	800	0.0434	0.124	0.0196	3.63	29.2E-6	0.667
	1000	0.0375	0.124	0.0217	4.66	32.4E-6	0.667
Fluorine	0	0.113	0.192	0.0127	0.585	13.6E-6	0.740
	200	0.0789	0.204	0.0179	1.11	18.4E-6	0.758
	400	0.0605	0.214	0.0229	1.77	22.7E-6	0.763
	600	0.0491	0.220	0.0279	2.58	26.7E-6	0.759
	800	0.0413	0.225	0.0328	3.53	30.4E-6	0.751
	1000	0.0356	0.229	0.0376**	4.62	33.9E-6	0.741

Temperature-Dependent Physical Properties of Gases at 14.7 psia (U.S. Units) (cont'd)

Gas	Temperature	Density	Heat Capacity (Cp)	Thermal Conductivity	Thermal Diffusivity	Viscosity	Prandtl Number
	°F	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{Btu}}{\text{ft} \cdot \text{hr} \cdot ^\circ\text{F}}$	$\frac{\text{ft}^2}{\text{hr}}$	$\frac{\text{lbm}}{\text{ft} \cdot \text{sec}}$	
Chlorine	0	0.215	0.112	0.00429	0.178	7.72E-6	0.724
	200	0.148	0.118	0.00649	0.372	11.0E-6	0.720
	400	0.113	0.121	0.00861	0.629	14.1E-6	0.713
	600	0.0918	0.123	0.0106	0.943	17.0E-6	0.708
	800	0.0772	0.124	0.0126	1.31	19.8E-6	0.705
	1000	0.0666	0.125	0.0144	1.73	22.5E-6	0.704
Methane	0	0.0480	0.512	0.0164	0.667	6.57E-6	0.739
	200	0.0333	0.578	0.0256	1.33	8.93E-6	0.727
	400	0.0256	0.673	0.0360	2.09	11.0E-6	0.741
	600	0.0207	0.772	0.0475	2.97	12.9E-6	0.755
	800	0.0174	0.866	0.0599**	3.97	14.6E-6	0.762
	1000	0.0150	0.953	0.0731**	5.11	16.3E-6	0.763
Ethane	0	0.0907	0.377	0.00939	0.275	5.43E-6	0.786
	200	0.0627	0.485	0.0179	0.590	7.59E-6	0.739
	400	0.0480	0.599	0.0282	0.981	9.56E-6	0.731
	600	0.0389	0.700	0.0399	1.46	11.4E-6	0.720
	800	0.0327	0.787	0.0526	2.04	13.1E-6	0.706
	1000	0.0282	0.863	0.0663	2.72	14.7E-6	0.691
Propane	0	0.135	0.353	0.00777	0.163	4.87E-6	0.797
	200	0.0923	0.470	0.0151	0.348	6.74E-6	0.757
	400	0.0705	0.589	0.0242	0.583	8.56E-6	0.750
	600	0.0571	0.690	0.0348	0.884	10.3E-6	0.738
	800	0.0480	0.773	0.0467	1.26	12.1E-6	0.720
	1000	0.0414	0.844	0.0597	1.71	13.8E-6	0.702
Acetylene	0	0.0784	1.60	0.0193	0.0377	9.36E-6	0.774
	200	0.0542	0.442	0.0171	0.715	8.38E-6	0.778
	400	0.0415	0.494	0.0241	1.18	10.6E-6	0.786
	600	0.0337	0.531	0.0309	1.73	12.7E-6	0.787
	800	0.0283	0.558	0.0375	2.37	14.6E-6**	0.783
	1000	0.0244	0.581	0.0441	3.11	16.4E-6**	0.777

Temperature-Dependent Physical Properties of Gases at 14.7 psia (U.S. Units) (cont'd)

Gas	Temperature	Density	Heat Capacity (Cp)	Thermal Conductivity	Thermal Diffusivity	Viscosity	Prandtl Number
	°F	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{Btu}}{\text{ft} \cdot \text{hr} \cdot ^\circ\text{F}}$	$\frac{\text{ft}^2}{\text{hr}}$	$\frac{\text{lbm}}{\text{ft} \cdot \text{sec}}$	
Ethylene	0	0.0844	0.332	0.00930	0.332	5.90E-6	0.758
	200	0.0584	0.424	0.0170	0.685	8.31E-6	0.749
	400	0.0447	0.515	0.0266	1.16	10.5E-6	0.730
	600	0.0363	0.595	0.0380	1.76	12.4E-6	0.702
	800	0.0305	0.665	0.0508**	2.51	14.2E-6	0.669
	1000	0.0263	0.725	0.0651**	3.41	15.8E-6	0.635
Propylene	0	0.128	0.329	0.00765	0.182	4.94E-6	0.763
	200	0.0880	0.428	0.0146	0.386	7.10E-6	0.751
	400	0.0673	0.518	0.0229	0.656	9.08E-6	0.741
	600	0.0545	0.600	0.0323	0.986	10.9E-6	0.730
	800	0.0458	0.674	0.0425	1.38	12.6E-6	0.718
	1000	0.0395	0.738	0.0535	1.84	14.2E-6	0.703
Hydrogen Sulfide	0	0.1027	0.237	0.0065	0.267	7.27E-6	0.956
	200	0.0711	0.246	0.0108	0.616	10.5E-6	0.862
	400	0.0544	0.258	0.0144	1.03	13.7E-6	0.884
	600	0.0441	0.272	0.0184	1.53	16.9E-6**	0.901
	800	0.0371	0.286	0.0229**	2.16	20.2E-6**	0.908
	1000	0.0320	0.300	0.0279**	2.90	23.4E-6**	0.908

* The vapor pressure of sulfur dioxide at 0°F is 10.2 psia. Hypothetical vapor density at 0°F and 14.7 psia is reported in the table.

** Extrapolated values

Sources: These data are provide courtesy of the American Institute of Chemical Engineering (AIChE) and its thermophysical property research consortium, the Design Institute for Physical Properties (DIPPR®) using DIPPR® 2016 version. Vapor densities were obtained using SRK equation of state with DIPPR 801 values for critical temperature, critical pressure, and acentric factor.

These data are provided for the sole purpose of the preparation for and taking of NCEES engineering exams with no warrantee expressed or implied.

9.5.2 SI Units

Temperature-Dependent Physical Properties of Gases at 0.1 MPa (SI Units)

Gas	Temperature	Density	Heat Capacity (Cp)	Thermal Conductivity	Thermal Diffusivity	Viscosity	Prandtl Number
	°C	$\frac{\text{kg}}{\text{m}^3}$	$\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$	$\frac{\text{W}}{\text{m} \cdot \text{K}}$	$\frac{\text{m}^2}{\text{hr}}$	$\mu\text{Pa} \cdot \text{s}$	
Nitrogen	0	1.23	1.04	0.0237	0.0665	16.6	0.727
	100	0.903	1.04	0.0307	0.117	21.0	0.712
	200	0.712	1.05	0.0372	0.179	24.9	0.704
	300	0.588	1.07	0.0434	0.249	28.5	0.701
	400	0.500	1.09	0.0494	0.326	31.8	0.703
	500	0.436	1.12	0.0551	0.408	35.0	0.707
Oxygen	0	1.41	0.914	0.0244	0.0682	19.2	0.718
	100	1.03	0.933	0.0323	0.121	24.6	0.710
	200	0.813	0.963	0.0397	0.182	29.4	0.713
	300	0.671	0.995	0.0466	0.251	33.8	0.720
	400	0.572	1.02	0.0533	0.328	37.8	0.725
	500	0.498	1.05	0.0597	0.412	41.5	0.729
Carbon Monoxide	0	1.23	1.04	0.0231	0.0650	16.5	0.741
	100	0.903	1.04	0.0301	0.115	20.9	0.726
	200	0.712	1.06	0.0365	0.175	24.8	0.719
	300	0.588	1.08	0.0425	0.241	28.3	0.720
	400	0.500	1.11	0.0481	0.313	31.5	0.725
	500	0.436	1.13	0.0535	0.390	34.5	0.731
Carbon Dioxide	0	1.94	0.816	0.0145	0.0331	13.8	0.772
	100	1.42	0.924	0.0224	0.0617	18.4	0.758
	200	1.12	1.00	0.0306	0.0985	22.6	0.740
	300	0.924	1.06	0.0386	0.142	26.5	0.725
	400	0.786	1.11	0.0464	0.192	30.0	0.717
	500	0.685	1.15	0.0536	0.245	33.3	0.715
Sulfur Dioxide	0	2.87	0.609	0.00843	0.0174	11.8	0.851
	100	2.08	0.664	0.0131	0.0342	16.2	0.821
	200	1.63	0.714	0.0183	0.0565	20.3	0.792
	300	1.35	0.756	0.0238	0.0842	24.2	0.767
	400	1.15	0.788	0.0292	0.117	27.8	0.749
	500	0.997	0.813	0.0343	0.152	31.2	0.741
Air	0	1.28	1.00	0.0242	0.0681	17.2	0.714
	100	0.933	1.01	0.0312	0.120	21.8	0.704
	200	0.736	1.02	0.0377	0.181	25.8	0.698
	300	0.608	1.04	0.0439	0.250	29.4	0.697
	400	0.517	1.06	0.0498	0.326	32.7	0.697
	500	0.450	1.09	0.0556	0.408	35.7	0.698

Temperature-Dependent Physical Properties of Gases at 0.1 MPa (SI Units) (cont'd)

Gas	Temperature	Density	Heat Capacity (Cp)	Thermal Conductivity	Thermal Diffusivity	Viscosity	Prandtl Number
	°C	$\frac{\text{kg}}{\text{m}^3}$	$\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$	$\frac{\text{W}}{\text{m} \cdot \text{K}}$	$\frac{\text{m}^2}{\text{hr}}$	$\mu\text{Pa} \cdot \text{s}$	
Hydrogen	0	0.0887	14.2	0.166	0.475	8.39	0.716
	100	0.0649	14.5	0.212	0.814	10.4	0.708
	200	0.0512	14.5	0.255	1.23	12.2	0.697
	300	0.0423	14.5	0.295	1.73	13.9	0.687
	400	0.0360	14.6	0.334	2.29	15.6	0.679
	500	0.0314	14.6	0.371	2.91	17.1	0.675
Ammonia	0	0.758	2.05	0.0222	0.0516	9.21	0.849
	100	0.551	2.23	0.0342	0.100	12.9	0.837
	200	0.434	2.42	0.0475	0.163	16.5	0.843
	300	0.358	2.61	0.0618	0.238	20.1	0.850
	400	0.304	2.79	0.0772	0.327	23.7	0.858
	500	0.265	2.96	0.0934	0.429	27.4	0.866
Helium	0	0.176	5.19	0.145	0.571	18.7	0.669
	100	0.129	5.19	0.179	0.964	23.2	0.671
	200	0.102	5.19	0.211	1.44	27.3	0.672
	300	0.0840	5.19	0.241	1.99	31.3	0.672
	400	0.0715	5.19	0.270	2.62	35.0	0.672
	500	0.0623	5.19	0.298	3.32	38.6	0.672
Neon	0	0.888	1.030	0.0459	0.181	29.6	0.663
	100	0.650	1.030	0.0570	0.306	36.6	0.661
	200	0.513	1.030	0.0670	0.457	43.0	0.660
	300	0.423	1.030	0.0764	0.630	48.9	0.660
	400	0.361	1.030	0.0852	0.826	54.5	0.659
	500	0.314	1.030	0.0935	1.04	59.8	0.659
Argon	0	1.76	0.520	0.0165	0.0649	21.1	0.663
	100	1.29	0.520	0.0212	0.114	27.1	0.666
	200	1.02	0.520	0.0254	0.173	32.6	0.667
	300	0.838	0.520	0.0293	0.242	37.6	0.667
	400	0.714	0.520	0.0329	0.319	42.2	0.667
	500	0.621	0.520	0.0364	0.405	46.6	0.667
Fluorine	0	1.67	0.812	0.0235	0.0622	21.5	0.744
	100	1.22	0.858	0.0314	0.108	27.8	0.759
	200	0.966	0.893	0.0393	0.164	33.6	0.763
	300	0.797	0.919	0.0470	0.231	38.9	0.760
	400	0.679	0.938	0.0547	0.309	43.9	0.753
	500	0.591	0.953	0.0623*	0.398	48.7	0.745

Temperature-Dependent Physical Properties of Gases at 0.1 MPa (SI Units) (cont'd)

Gas	Temperature	Density	Heat Capacity (Cp)	Thermal Conductivity	Thermal Diffusivity	Viscosity	Prandtl Number
	°C	$\frac{\text{kg}}{\text{m}^3}$	$\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$	$\frac{\text{W}}{\text{m} \cdot \text{K}}$	$\frac{\text{m}^2}{\text{hr}}$	$\mu\text{Pa} \cdot \text{s}$	
Chlorine	0	3.17	0.473	0.00803	0.0193	12.3	0.724
	100	2.30	0.494	0.0115	0.0363	16.7	0.720
	200	1.81	0.506	0.0148	0.0581	20.8	0.714
	300	1.49	0.514	0.0179	0.0844	24.8	0.709
	400	1.27	0.519	0.0210	0.115	28.5	0.706
	500	1.10	0.523	0.0239	0.149	32.1	0.704
Methane	0	0.708	2.17	0.0307	0.0720	10.4	0.733
	100	0.517	2.44	0.0453	0.129	13.5	0.727
	200	0.408	2.80	0.0615	0.194	16.3	0.741
	300	0.337	3.17	0.0793	0.267	18.8	0.754
	400	0.287	3.53	0.0983	0.350	21.2	0.761
	500	0.250	3.87	0.119*	0.442	23.4	0.764
Ethane	0	1.34	1.64	0.0184	0.0301	8.62	0.772
	100	0.973	2.06	0.0320	0.0575	11.5	0.738
	200	0.765	2.49	0.0481	0.0908	14.1	0.731
	300	0.631	2.88	0.0661	0.131	16.6	0.722
	400	0.537	3.21	0.0857	0.179	18.9	0.709
	500	0.468	3.51	0.107	0.234	21.1	0.696
Propane	0	1.98	1.55	0.0152	0.0178	7.70	0.785
	100	1.43	2.00	0.0270	0.0339	10.2	0.756
	200	1.125	2.45	0.0412	0.0539	12.6	0.750
	300	0.927	2.83	0.0575	0.0788	15.0	0.740
	400	0.788	3.16	0.0757	0.109	17.4	0.724
	500	0.686	3.44	0.0955	0.146	19.7	0.708
Acetylene	0	1.16	1.60	0.0193	0.0377	9.36	0.774
	100	0.842	1.87	0.0304	0.0696	12.7	0.778
	200	0.663	2.06	0.0412	0.109	15.7	0.786
	300	0.547	2.20	0.0518	0.155	18.5	0.788
	400	0.465	2.31	0.0622	0.208	21.1	0.785
	500	0.405	2.40	0.0724	0.268	23.5*	0.779
Ethylene	0	1.24	1.45	0.0180	0.0359	9.38	0.755
	100	0.907	1.80	0.0303	0.0668	12.6	0.748
	200	0.714	2.14	0.0453	0.107	15.5	0.731
	300	0.589	2.45	0.0628	0.157	18.1	0.706
	400	0.501	2.72	0.0824*	0.218	20.5	0.677
	500	0.436	2.95	0.104*	0.290	22.8	0.646

Temperature-Dependent Physical Properties of Gases at 0.1 MPa (SI Units) (cont'd)

Gas	Temperature	Density	Heat Capacity (Cp)	Thermal Conductivity	Thermal Diffusivity	Viscosity	Prandtl Number
	°C	$\frac{\text{kg}}{\text{m}^3}$	$\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$	$\frac{\text{W}}{\text{m} \cdot \text{K}}$	$\frac{\text{m}^2}{\text{hr}}$	$\mu\text{Pa} \cdot \text{s}$	
Propylene	0	1.89	1.44	0.0150	0.0198	7.88	0.760
	100	1.37	1.82	0.0260	0.0376	10.7	0.751
	200	1.07	2.15	0.0390	0.0608	13.4	0.741
	300	0.884	2.47	0.0535	0.0882	15.9	0.732
	400	0.752	2.75	0.0692	0.120	18.1	0.722
	500	0.655	3.00	0.0860	0.158	20.3	0.709
Hydrogen Sulfide	0	1.51	1.00	0.0126	0.0302	11.6	0.911
	100	1.10	1.03	0.0190	0.0601	15.9	0.862
	200	0.868	1.08	0.0247	0.0949	20.2	0.883
	300	0.716	1.13	0.0308	0.137	24.5*	0.899
	400	0.609	1.18	0.0376*	0.188	28.9*	0.907
	500	0.530	1.24	0.0452*	0.248	33.2*	0.909

* Extrapolated values

Sources: These data are provide courtesy of the American Institute of Chemical Engineering (AIChE) and its thermophysical property research consortium, the Design Institute for Physical Properties (DIPPR®) using DIPPR® 2016 version. Vapor densities were obtained using SRK equation of state with DIPPR 801 values for critical temperature, critical pressure, and acentric factor.

These data are provided for the sole purpose of the preparation for and taking of NCEES engineering exams with no warrantee expressed or implied.

9.6 Physical Properties of Air

9.6.1 Dry Atmospheric Air Composition

Composition of Dry Atmospheric Air

Component	Mole Fraction	Molar Mass $\left(\frac{\text{lb}}{\text{lb mole}} \text{ or } \frac{\text{g}}{\text{mol}} \right)$
Nitrogen	0.780848	28.0134
Oxygen	0.209390	31.9988
Argon	0.009332	39.948
Carbon dioxide	0.000400	44.0095
Neon	18.2×10^{-6}	20.1797
Helium	5.2×10^{-6}	4.0026
Methane	1.5×10^{-6}	16.0325
Krypton	1.1×10^{-6}	83.798
Hydrogen	0.5×10^{-6}	2.01588
Nitrous oxide	0.3×10^{-6}	44.0128
Carbon monoxide	0.2×10^{-6}	28.0101
Xenon	0.1×10^{-6}	131.294
Total	1.0	28.96546

9.6.2 Dry Atmospheric Air Properties

Properties of Dry Atmospheric Air

Property	U.S. Units*	SI Units**
Molar mass	28.965 $\frac{\text{lb}}{\text{lb mole}}$	28.965 $\frac{\text{g}}{\text{mol}}$
NBP temperature	-317.64 °F	78.903 K
Triple point temperature	-352.12 °F	59.75 K
Critical temperature	-221.12 °F	132.53 K
Critical pressure	549.11 psia	3.7860 MPa
Critical density	21.393 $\frac{\text{lbm}}{\text{ft}^3}$	342.68 $\frac{\text{kg}}{\text{m}^3}$
Density of liquid at NBP	54.637 $\frac{\text{lbm}}{\text{ft}^3}$ 7.3039 $\frac{\text{lbm}}{\text{gal}}$	875.21 $\frac{\text{kg}}{\text{m}^3}$
Volume of liquid at NBP	0.13691 $\frac{\text{gal}}{\text{lbm}}$	0.0011426 $\frac{\text{m}^3}{\text{kg}}$
Density of ideal gas	0.07633 $\frac{\text{lbm}}{\text{ft}^3}$	1.2250 $\frac{\text{kg}}{\text{m}^3}$
Volume of ideal gas	13.101 $\frac{\text{ft}^3}{\text{lbm}}$	0.81631 $\frac{\text{m}^3}{\text{kg}}$
Speed of sound in air $p = 14.696 \text{ psia}, T = 32^\circ\text{F}$ $p = 0.1 \text{ MPa}, T = 0^\circ\text{C}$	1090 $\frac{\text{ft}}{\text{sec}}$	330 $\frac{\text{m}}{\text{s}}$
Speed of sound in air $p = 14.696 \text{ psia}, T = 68^\circ\text{F}$ $p = 0.1 \text{ MPa}, T = 20^\circ\text{C}$	1130 $\frac{\text{ft}}{\text{sec}}$	343 $\frac{\text{m}}{\text{s}}$

* U.S. unit values are given at 60°F and 14.696 psia, except where noted otherwise.

** SI unit values are given at 15°C and 0.101325 MPa, except where noted otherwise.

9.6.3 Temperature-Dependent Properties of Air (U.S. Customary Units)

Temperature-Dependent Properties of Air at 14.7 psia (U.S. Units)

Temperature	Density	Heat Capacity (c_p)	Heat Capacity Ratio	Viscosity	Thermal Conductivity	Prandtl Number	Thermal Diffusivity
°F	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$		$\frac{\text{lbm}}{\text{ft} \cdot \text{sec}}$	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$		$\frac{\text{ft}^2}{\text{hr}}$
-50	0.094	0.2400	1.40	9.98E-06	0.0120	0.719	0.530
0	0.086	0.2400	1.40	1.09E-05	0.0132	0.714	0.642
32	0.081	0.2400	1.40	1.15E-05	0.0140	0.710	0.726
100	0.071	0.2400	1.40	1.28E-05	0.0156	0.708	0.917
200	0.060	0.2401	1.40	1.44E-05	0.0179	0.701	1.233
300	0.052	0.2424	1.39	1.60E-05	0.0202	0.692	1.599
400	0.046	0.2452	1.39	1.74E-05	0.0225	0.685	1.989
500	0.041	0.2476	1.38	1.89E-05	0.0247	0.681	2.415
600	0.0374	0.2507	1.38	2.01E-05	0.0268	0.678	2.857
700	0.034	0.2533	1.37	2.14E-05	0.0288	0.677	3.323
800	0.0286	0.2567	1.37	2.25E-05	0.0307	0.679	4.173

9.6.4 Temperature-Dependent Properties of Air (SI Units)

Temperature-Dependent Properties of Air at 0.1 MPa (SI Units)

Temperature	Density	Heat Capacity (c_p)	Heat Capacity Ratio	Viscosity	Thermal Conductivity	Prandtl Number	Thermal Diffusivity
°C	$\frac{\text{kg}}{\text{m}^3}$	$\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$		$\mu\text{Pa} \cdot \text{s}$	$\frac{\text{W}}{\text{m} \cdot \text{K}}$		$\frac{\text{m}^2}{\text{s}}$
-50	1.534	1.005	1.40	14.6	0.0204	0.722	1.32E-05
0	1.293	1.005	1.40	17.2	0.0243	0.711	1.87E-05
20	1.205	1.005	1.40	18.2	0.0257	0.712	2.12E-05
40	1.127	1.005	1.40	19.1	0.0271	0.709	2.39E-05
60	1.067	1.009	1.40	20.2	0.0285	0.714	2.65E-05
80	1.000	1.009	1.40	20.9	0.0299	0.707	2.96E-05
100	0.946	1.009	1.40	21.8	0.0314	0.701	3.29E-05
120	0.898	1.013	1.40	22.7	0.0328	0.700	3.61E-05
140	0.854	1.013	1.40	23.5	0.0343	0.695	3.96E-05
160	0.815	1.017	1.39	24.3	0.0358	0.691	4.32E-05
180	0.779	1.022	1.39	25.2	0.0372	0.691	4.67E-05
200	0.746	1.026	1.39	25.8	0.0386	0.687	5.04E-05
250	0.675	1.034	1.38	27.8	0.0421	0.683	6.03E-05
300	0.616	1.047	1.38	29.5	0.0454	0.680	7.04E-05
350	0.566	1.055	1.37	31.2	0.0485	0.678	8.12E-05
400	0.524	1.068	1.37	32.8	0.0515	0.679	9.20E-05

9.6.5 Psychrometric Chart (U.S. Customary Units)

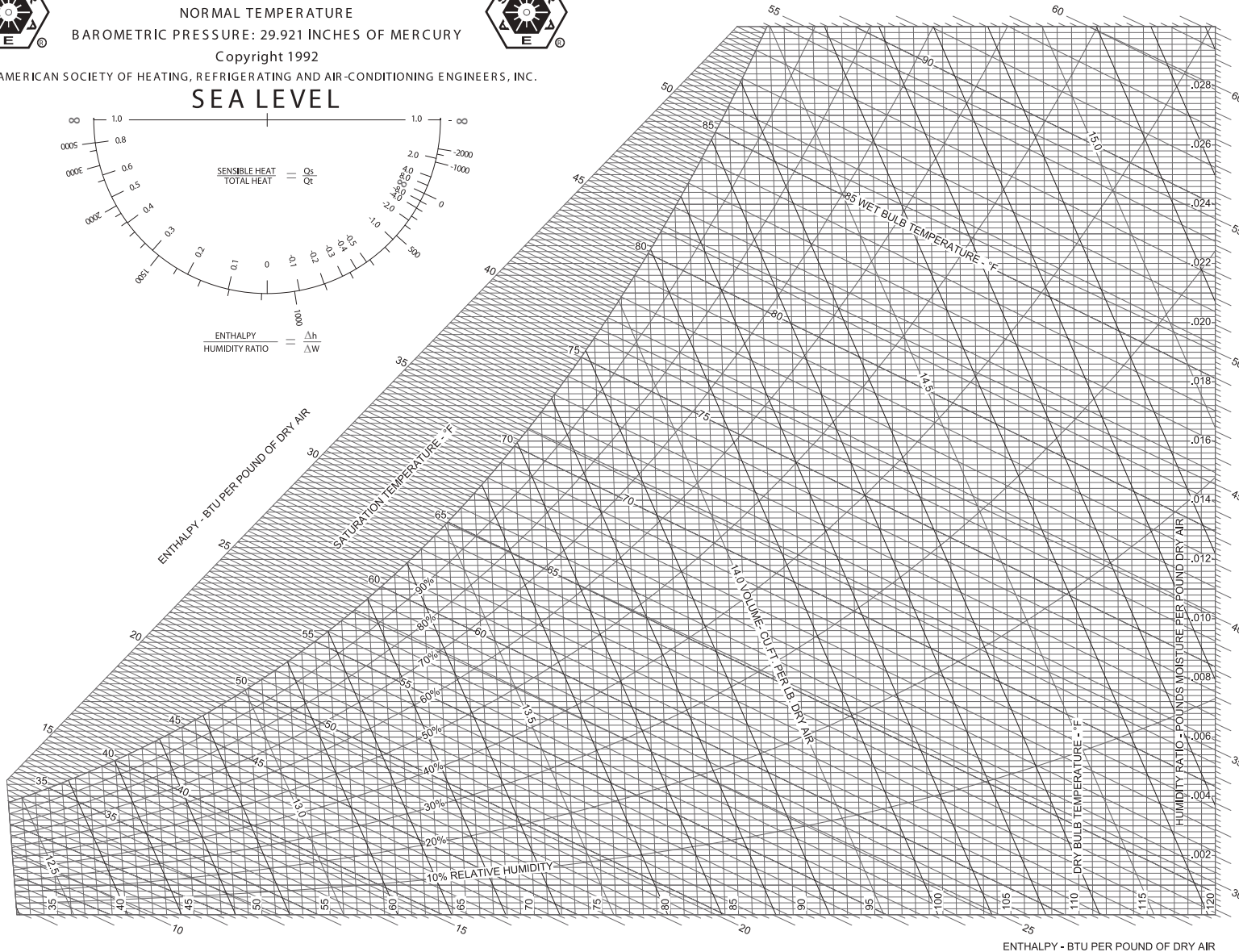
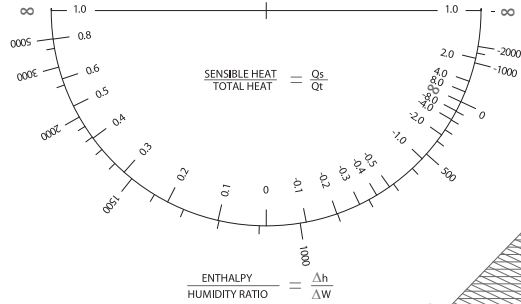


ASHRAE PSYCHROMETRIC CHART NO.1
NORMAL TEMPERATURE
BAROMETRIC PRESSURE: 29.921 INCHES OF MERCURY
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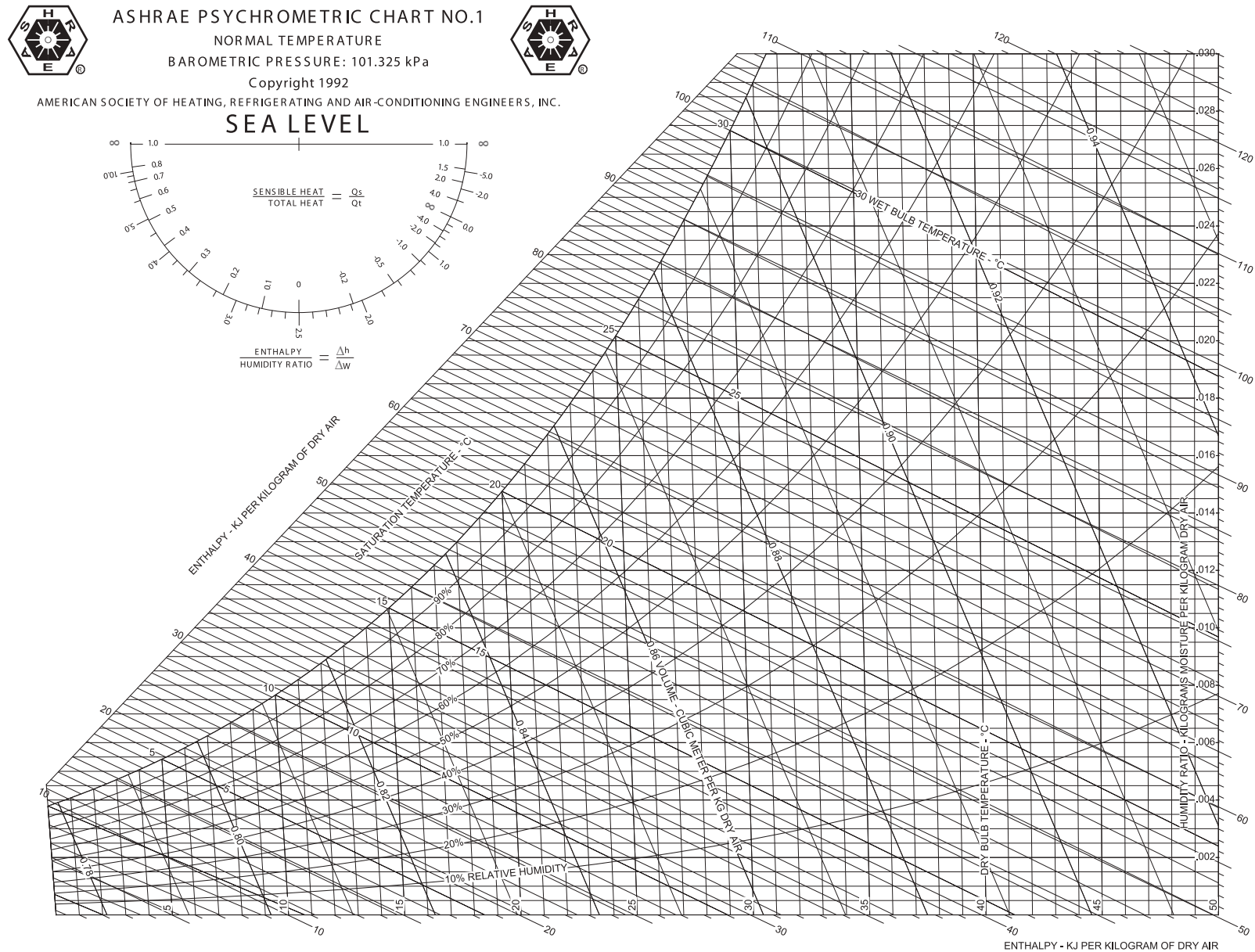
AMERICAN SOCIETY OF HEATING, REFRIGERATING AND AIR-CONDITIONING ENGINEERS, INC.

SEA LEVEL



Source: American Society of Heating, Refrigerating, and Air-Conditioning Engineers, 1992.

9.6.6 Psychrometric Chart (SI Units)



Source: American Society of Heating, Refrigerating, and Air-Conditioning Engineers, 1992.

9.7 Physical Properties of Water

9.7.1 U.S. Customary Units

Physical Properties of Liquid Water (U.S. Units)

Temperature	Vapor Pressure	Density	Heat Capacity	Viscosity	Thermal Conductivity	Prandtl Number	Surface Tension
°F	psia	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{lbm}}{\text{ft} \cdot \text{sec}}$	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$		$\frac{\text{dyne}}{\text{cm}}$
32.02	0.08872	62.415	1.0086	1.204E-03	0.3244	13.47	75.65
40	0.12173	62.423	1.0055	1.038E-03	0.3293	11.42	75.02
50	0.17814	62.406	1.0028	8.776E-04	0.3353	9.45	74.22
60	0.2564	62.364	1.0010	7.533E-04	0.3413	7.95	73.40
70	0.36336	62.299	0.9999	6.552E-04	0.3471	6.79	72.57
80	0.50747	62.213	0.9993	5.761E-04	0.3527	5.88	71.71
90	0.69904	62.110	0.9990	5.114E-04	0.3579	5.14	70.84
100	0.95051	61.991	0.9989	4.577E-04	0.3628	4.54	69.96
110	1.2767	61.857	0.9991	4.127E-04	0.3672	4.04	69.05
120	1.695	61.710	0.9993	3.744E-04	0.3713	3.63	68.13
130	2.2259	61.549	0.9998	3.417E-04	0.3750	3.28	67.19
140	2.893	61.377	1.0003	3.134E-04	0.3783	2.98	66.24
150	3.7232	61.193	1.0009	2.888E-04	0.3813	2.73	65.27
160	4.7472	60.998	1.0016	2.673E-04	0.3839	2.51	64.28
170	5.9998	60.793	1.0025	2.484E-04	0.3862	2.32	63.28
180	7.5195	60.578	1.0035	2.316E-04	0.3881	2.16	62.26
190	9.3496	60.354	1.0046	2.168E-04	0.3898	2.01	61.23
200	11.538	60.120	1.0059	2.035E-04	0.3912	1.88	60.19
210	14.136	59.877	1.0073	1.916E-04	0.3924	1.77	59.13
220	17.201	59.626	1.0088	1.808E-04	0.3934	1.67	58.05
230	20.795	59.366	1.0106	1.712E-04	0.3941	1.58	56.96
240	24.986	59.097	1.0125	1.624E-04	0.3947	1.50	55.86
250	29.844	58.820	1.0147	1.544E-04	0.3951	1.43	54.74
260	35.447	58.535	1.0170	1.471E-04	0.3953	1.36	53.62
270	41.878	58.241	1.0196	1.405E-04	0.3953	1.30	52.47
280	49.222	57.940	1.0224	1.344E-04	0.3952	1.25	51.32
290	57.574	57.630	1.0254	1.288E-04	0.3949	1.20	50.16
300	67.029	57.312	1.0287	1.236E-04	0.3944	1.16	48.98
320	89.667	56.650	1.0362	1.144E-04	0.3931	1.09	46.59
340	118.02	55.955	1.0449	1.065E-04	0.3912	1.02	44.16

Chapter 9: Physical Properties

Physical Properties of Liquid Water (U.S. Units) (cont'd)

Temperature	Vapor Pressure	Density	Heat Capacity	Viscosity	Thermal Conductivity	Prandtl Number	Surface Tension
°F	psia	$\frac{\text{lbm}}{\text{ft}^3}$	$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	$\frac{\text{lbm}}{\text{ft} \cdot \text{sec}}$	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{F}}$		$\frac{\text{dyne}}{\text{cm}}$
360	153.03	55.225	1.0550	9.958E-05	0.3888	0.97	41.69
380	195.74	54.458	1.0666	9.354E-05	0.3857	0.93	39.19
400	247.26	53.652	1.0802	8.820E-05	0.3819	0.90	36.66
420	308.76	52.804	1.0959	8.343E-05	0.3776	0.87	34.10
440	381.48	51.912	1.1143	7.913E-05	0.3725	0.85	31.52
460	466.75	50.971	1.1358	7.523E-05	0.3666	0.84	28.92
480	565.95	49.976	1.1612	7.165E-05	0.3599	0.83	26.30
500	680.55	48.920	1.1916	6.833E-05	0.3522	0.83	23.69
520	812.10	47.795	1.2285	6.521E-05	0.3436	0.84	21.08
540	962.24	46.590	1.2740	6.225E-05	0.3340	0.85	18.47
560	1132.7	45.290	1.3317	5.940E-05	0.3233	0.88	15.89
580	1325.5	43.876	1.4072	5.661E-05	0.3118	0.92	13.35
600	1542.5	42.318	1.5100	5.382E-05	0.2995	0.98	10.846
620	1786.2	40.572	1.6588	5.097E-05	0.2867	1.06	8.415
640	2059.2	38.566	1.8958	4.796E-05	0.2735	1.20	6.078
660	2364.9	36.152	2.3480	4.463E-05	0.2600	1.45	3.876
680	2707.3	32.936	3.5861	4.054E-05	0.2461	2.13	1.877
700	3093.0	27.283	15.5790	3.399E-05	0.2547	7.48	0.2565
702	3134.5	26.085	28.2960	3.268E-05	0.2765	12.04	0.1375
704	3176.6	24.196	97.4060	3.180E-05	0.3584	31.11	0.0375
705.1	3200.1	20.102		3.333E-05			

9.7.2 SI Units

Physical Properties of Liquid Water (SI Units)

Temperature	Vapor Pressure	Density	Heat Capacity	Viscosity	Thermal Conductivity	Prandtl Number	Surface Tension
°C	MPa	$\frac{\text{kg}}{\text{m}^3}$	$\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$	$\text{Pa} \cdot \text{s}$	$\frac{\text{W}}{\text{m} \cdot \text{K}}$		$\frac{\text{N}}{\text{m}} \cdot 10^{-3}$
0.01	0.000612	999.79	4.2199	1.791E-03	0.5610	13.47	75.65
5	0.000873	999.92	4.2055	1.518E-03	0.5705	11.19	74.94
10	0.001228	999.65	4.1955	1.306E-03	0.5800	9.45	74.22
15	0.001706	999.06	4.1888	1.138E-03	0.5893	8.09	73.49
20	0.002339	998.16	4.1844	1.002E-03	0.5984	8.09	72.74
25	0.00317	997.00	4.1816	8.901E-04	0.6072	6.13	71.97
30	0.004247	995.61	4.1801	7.974E-04	0.6155	5.42	71.19
35	0.005629	993.99	4.1795	7.193E-04	0.6233	4.82	70.40
40	0.007385	992.18	4.1796	6.530E-04	0.6306	4.33	69.60
45	0.009595	990.17	4.1804	5.961E-04	0.6373	3.91	68.78
50	0.012352	988.00	4.1815	5.468E-04	0.6436	3.55	67.94
55	0.015762	985.66	4.1831	5.040E-04	0.6492	3.25	67.10
60	0.019946	983.16	4.1851	4.664E-04	0.6544	2.98	66.24
65	0.025042	980.52	4.1875	4.332E-04	0.6590	2.75	65.37
70	0.031201	977.73	4.1902	4.039E-04	0.6631	2.55	64.48
75	0.038595	974.81	4.1933	3.777E-04	0.6668	2.38	63.58
80	0.047414	971.77	4.1969	3.543E-04	0.6700	2.22	62.67
85	0.057867	968.59	4.2008	3.333E-04	0.6728	2.08	61.75
90	0.070182	965.30	4.2053	3.144E-04	0.6753	1.96	60.82
95	0.084608	961.88	4.2102	2.973E-04	0.6773	1.85	59.87
100	0.10142	958.35	4.2157	2.817E-04	0.6791	1.75	58.91
110	0.14338	950.95	4.2283	2.547E-04	0.6817	1.58	56.96
120	0.19867	943.11	4.2435	2.321E-04	0.6832	1.44	54.97
130	0.27028	934.83	4.2615	2.129E-04	0.6837	1.33	52.93
140	0.36154	926.13	4.2826	1.965E-04	0.6833	1.23	50.86
150	0.47616	917.01	4.3071	1.825E-04	0.6820	1.15	48.74
160	0.61823	907.45	4.3354	1.702E-04	0.6800	1.09	46.59
170	0.79219	897.45	4.3678	1.596E-04	0.6771	1.03	44.41
180	1.0028	887.00	4.4050	1.501E-04	0.6733	0.98	42.19
190	1.2552	876.08	4.4474	1.418E-04	0.6688	0.94	39.95

Physical Properties of Liquid Water (SI Units) (cont'd)

Temperature	Vapor Pressure	Density	Heat Capacity	Viscosity	Thermal Conductivity	Prandtl Number	Surface Tension
°C	MPa	$\frac{\text{kg}}{\text{m}^3}$	$\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$	$\text{Pa} \cdot \text{s}$	$\frac{\text{W}}{\text{m} \cdot \text{K}}$		$\frac{\text{N}}{\text{m}} \cdot 10^{-3}$
200	1.5549	864.66	4.4958	1.343E-04	0.6633	0.91	37.68
210	1.9077	852.72	4.5512	1.276E-04	0.6570	0.88	35.38
220	2.3196	840.22	4.6146	1.215E-04	0.6497	0.86	33.07
230	2.7971	827.12	4.6876	1.160E-04	0.6413	0.85	30.74
240	3.3469	813.37	4.7719	1.109E-04	0.6319	0.84	28.39
250	3.9762	798.89	4.8701	1.061E-04	0.6212	0.83	26.04
260	4.6923	783.63	4.9856	1.017E-04	0.6092	0.83	23.69
270	5.503	767.46	5.1230	9.750E-05	0.5959	0.84	21.34
280	6.4166	750.28	5.2889	9.351E-05	0.5812	0.85	18.99
290	7.4418	731.91	5.4931	8.966E-05	0.5650	0.87	16.66
300	8.5879	712.14	5.7504	8.590E-05	0.5474	0.90	14.36
310	9.8651	690.67	6.0848	8.217E-05	0.5288	0.95	12.09
320	11.284	667.09	6.5373	7.841E-05	0.5092	1.01	9.864
330	12.858	640.77	7.1863	7.454E-05	0.4891	1.10	7.703
340	14.601	610.67	8.2080	7.043E-05	0.4685	1.23	5.626
350	16.529	574.71	10.1160	6.588E-05	0.4474	1.49	3.665
360	18.666	527.59	15.0040	6.033E-05	0.4257	2.13	1.877
370	21.044	451.43	45.1550	5.207E-05	0.4250	5.53	0.388
371	21.297	438.64	62.3510	5.075E-05	0.4384	7.22	0.269
372	21.554	422.26	102.1500	4.908E-05	0.4674	10.72	0.160
373	21.814	398.68	243.7800	4.781E-05	0.5479	21.27	0.065
373.94	22.064	322.00		4.854E-05			

9.7.3 Properties of Water

Properties of Water

Property	U.S. Units	SI Units
Molar mass	18.01528 $\frac{\text{lb}}{\text{lb mole}}$	18.01528 $\frac{\text{g}}{\text{mol}}$
Boiling temperature	212°F	373.15 K
Triple point temperature	32°F	273.15 K
Triple point pressure	0.0887 psia	611.657 Pa
Critical temperature	705.1°F	647.09 K
Critical pressure	3200.1 psia	22.06 MPa
Critical density	20.102 $\frac{\text{lbm}}{\text{ft}^3}$	322.00 $\frac{\text{kg}}{\text{m}^3}$
Maximum density of liquid (4°C = 39°F)	62.426 $\frac{\text{lbm}}{\text{ft}^3}$ 8.3455 $\frac{\text{lbm}}{\text{gal}}$	1000 $\frac{\text{kg}}{\text{m}^3}$
Minimum volume of liquid (4°C = 39°F)	0.11983 $\frac{\text{gal}}{\text{lbm}}$	0.001 $\frac{\text{m}^3}{\text{kg}}$
Heat of vaporization (100°C = 212°F)	970.17 $\frac{\text{Btu}}{\text{lbm}}$	2257 $\frac{\text{kJ}}{\text{kg}}$
Density of ice (0°C = 32°F)	57.227 $\frac{\text{lbm}}{\text{ft}^3}$	916.7 $\frac{\text{kg}}{\text{m}^3}$
Latent heat of fusion (0°C = 32°F)	143.38 $\frac{\text{Btu}}{\text{lbm}}$	333.55 $\frac{\text{kJ}}{\text{kg}}$
Dielectric constant of liquid (0°C = 32°F)	87.90	87.90
Dielectric constant of liquid (100°C = 212°F)	55.53	55.53
Refractive index of liquid (20°C = 68°F)*	1.3333	1.3333

*Refractive index at Sodium D line

Source: Harvey, Allan H., and Eric W. Lemmon, *NIST/ASME Steam Properties, Version 3.0*, Gaithersburg: National Institute of Standards and Technology, 2013.

9.8 Steam Tables

Source for all tables in this section: GPSA Engineering Data Book, 13th ed., Vol. 2, Tulsa, OK: GPSA, 2012, Figures 24-30 and 24-31 on pp. 24-35 through 24-38.

9.8.1 Properties of Saturated Steam (U.S. Customary Units)

Saturated Steam (U.S. Units)—Temperature Table

Temperature	Pressure	Specific Volume, v		Specific Enthalpy, h		Specific Entropy, s	
		$\frac{\text{ft}^3}{\text{lbm}}$		$\frac{\text{Btu}}{\text{lbm}}$		$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	
$^\circ\text{F}$	psia	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor
32.018	0.08865	0.016022	3302.4	0.000	1075.5	0.0000	2.1872
35	0.09991	0.016020	2948.1	3.002	1076.8	0.0061	2.1767
40	0.12163	0.016019	2445.8	8.027	1079.0	0.0162	2.1594
45	0.14744	0.016020	2037.8	13.044	1081.2	0.0262	2.1426
50	0.17796	0.016023	1704.8	18.054	1083.4	0.0361	2.1262
55	0.21392	0.016027	1432.0	23.059	1085.6	0.0458	2.1102
60	0.25611	0.016033	1207.6	28.060	1087.7	0.0555	2.0946
65	0.30545	0.016041	1022.1	33.057	1089.9	0.0651	2.0794
70	0.36292	0.016050	868.4	38.052	1092.1	0.0745	2.0645
75	0.42964	0.016060	740.3	43.045	1094.3	0.0839	2.0500
80	0.50683	0.016072	633.3	48.037	1096.4	0.0932	2.0359
85	0.59583	0.016085	543.6	53.027	1098.6	0.1024	2.0221
90	0.69813	0.016099	468.1	58.018	1100.8	0.1115	2.0086
95	0.81534	0.016114	404.4	63.008	1102.9	0.1206	1.9954
100	0.94294	0.016130	350.4	67.999	1105.1	0.1295	1.9825
110	1.2750	0.016165	265.39	77.98	1109.3	0.1472	1.9577
120	1.6927	0.016204	203.26	87.97	1113.6	0.1646	1.9339
130	2.2230	0.016247	157.33	97.96	1117.8	0.1817	1.9112
140	2.8892	0.016293	122.98	123.00	1122.0	0.1985	1.8895
150	3.7184	0.016343	97.07	117.95	1126.1	0.2150	1.8686
160	4.7414	0.016395	77.27	127.96	1130.2	0.2313	1.8487
170	5.9926	0.016451	62.08	137.97	1134.2	0.2473	1.8295
180	7.5110	0.016510	50.225	148.00	1138.2	0.2631	1.8111
190	9.3400	0.016572	40.957	158.04	1142.1	0.2787	1.7934
200	11.5260	0.016637	33.639	168.09	1146.0	0.2940	1.7764
210	14.1230	0.016705	27.816	178.15	1149.7	0.3091	1.7600
212	14.6960	0.016719	26.799	180.17	1150.5	0.3121	1.7568
220	17.1860	0.016775	23.148	188.23	1153.4	0.3241	1.7442
230	20.7790	0.016849	19.381	198.33	1157.1	0.3388	1.7290
240	24.9680	0.016926	16.321	208.45	1160.6	0.3533	1.7142

Saturated Steam (U.S. Units)—Temperature Table (cont'd)

Temperature	Pressure	Specific Volume, v		Specific Enthalpy, η		Specific Entropy, σ	
		$\frac{\text{ft}^3}{\text{lbm}}$		$\frac{\text{Btu}}{\text{lbm}}$		$\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	
$^\circ\text{F}$	psia	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor
250	29.8250	0.017066	13.819	218.59	1164.0	0.3677	1.7000
260	35.4270	0.017089	11.762	228.76	1167.4	0.3819	1.6862
270	41.8560	0.017175	10.060	238.95	1170.6	0.3960	1.6729
280	49.2000	0.017264	8.644	249.17	1173.8	0.4098	1.6599
290	57.5500	0.01736	7.4603	259.4	1167.8	0.4236	1.6473
300	67.0050	0.01745	6.4658	269.7	1179.7	0.4372	1.6351
320	89.6430	0.01766	4.9138	290.4	1185.2	0.4640	1.6116
340	117.9920	0.01787	3.7878	311.3	1190.1	0.4902	1.5892
360	153.01	0.01811	2.9573	332.3	1194.4	0.5161	1.5678
380	195.73	0.01836	2.3353	353.6	1198.0	0.5416	1.5473
400	247.26	0.01864	1.8630	375.1	1201.0	0.5667	1.5274
420	308.78	0.01894	1.4997	396.9	1203.1	0.5915	1.5080
440	381.54	0.01926	1.2169	419.0	1204.4	0.6161	1.4890
460	466.87	0.01961	0.99424	441.5	1204.8	0.6405	1.4704
480	566.15	0.02000	0.81717	464.5	1204.1	0.6648	1.4518
500	680.86	0.02043	0.67492	487.9	1202.2	0.6890	1.4333
520	812.53	0.02091	0.55956	512.0	1199.0	0.7133	1.4146
540	962.79	0.02146	0.46513	536.8	1194.3	0.7378	1.3954
560	1133.38	0.02207	0.38714	562.4	1187.7	0.7625	1.3757
580	1326.17	0.02279	0.32216	589.1	1179.0	0.7876	1.3550
600	1543.2	0.02364	0.26747	617.1	1167.7	0.8134	1.3330
620	1786.9	0.02466	0.22081	646.9	1153.2	0.8403	1.3092
640	2059.9	0.02595	0.18021	679.1	1133.7	0.8686	1.2821
660	2065.7	0.02768	0.14431	714.9	1107.0	0.8995	1.2498
680	2708.6	0.03037	0.11117	758.5	1068.5	0.9365	1.2086
700	3094.3	0.03662	0.07519	825.2	991.7	0.9924	1.1359
702	3135.5	0.03824	0.06997	835.0	979.7	1.0006	1.1210
704	3177.2	0.04108	0.06300	854.2	956.2	1.0169	1.1046
705.47	3208.2	0.05078	0.05078	906.0	906.0	1.0612	1.0612

Saturated Steam (U.S. Units)—Pressure Table

Pressure	Temperature	Specific Volume, v $\frac{\text{ft}^3}{\text{lbm}}$		Specific Enthalpy, h $\frac{\text{Btu}}{\text{lbm}}$		Specific Entropy, s $\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	
		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor
0.1	35.02	0.016020	2,945.5	3.03	1076.8	0.0061	2.1766
0.2	53.16	0.016025	1,526.3	21.22	1084.7	0.0422	2.1060
0.3	64.48	0.016040	1,039.7	32.54	1089.7	0.0641	2.0809
0.4	72.87	0.016056	792.1	40.92	1093.3	0.0799	2.0562
0.6	85.22	0.016085	540.1	53.25	1098.7	0.1028	2.0215
0.8	94.38	0.016112	411.69	62.39	1102.6	0.1195	1.9970
1	101.74	0.016136	333.60	69.73	1105.8	0.1326	1.9781
2	126.07	0.016230	173.76	94.03	1116.2	0.1750	1.9200
3	141.47	0.016300	118.73	109.42	1122.6	0.2009	1.8864
4	152.96	0.016358	90.64	120.92	1127.3	0.2199	1.8626
6	170.05	0.016451	61.98	138.03	1134.2	0.2474	1.8294
8	182.80	0.016527	47.35	150.87	1139.3	0.2676	1.8060
10	193.21	0.016592	38.42	161.26	1143.3	0.2836	1.7879
20	227.96	0.016834	20.087	196.27	1156.3	0.3358	1.7320
30	250.34	0.017009	13.744	218.9	1164.1	0.3682	1.6995
40	267.25	0.017151	10.4965	236.1	1169.8	0.3921	1.6765
50	281.02	0.017274	8.5140	250.2	1174.1	0.4112	1.6586
60	292.71	0.017383	7.1736	262.2	1177.6	0.4273	1.6440
70	302.93	0.017482	6.2050	272.7	1180.6	0.4411	1.6316
80	312.04	0.017573	5.4711	282.1	1183.1	0.4534	1.6208
90	320.28	0.017659	4.8953	290.7	1185.3	0.4643	1.6113
100	327.82	0.01774	4.4310	298.5	1187.2	0.4743	1.6027
150	358.43	0.01809	3.0139	330.6	1194.1	0.5141	1.5695
200	381.80	0.01839	2.2873	355.5	1198.3	0.5438	1.5454
250	400.97	0.01865	1.84317	376.1	1201.1	0.5679	1.5264
300	417.35	0.01889	1.54274	394.0	1202.9	0.5882	1.5105
350	431.73	0.01912	1.32554	409.8	1204.0	0.6059	1.4968
400	444.60	0.01934	1.16095	424.2	1204.6	0.6217	1.4847
450	456.28	0.01954	1.03179	437.3	1204.8	0.6360	1.4738
500	467.01	0.01975	0.92762	449.5	1204.7	0.6490	1.4639
600	486.20	0.02013	0.76975	471.7	1203.7	0.6723	1.4461
700	503.08	0.02050	0.65556	491.6	1201.8	0.6928	1.4304
800	518.21	0.02087	0.56896	509.8	1199.4	0.7111	1.4163
900	531.95	0.02123	0.50091	526.7	1196.4	0.7279	1.4032
1000	544.58	0.02159	0.44596	542.6	1192.9	0.7434	1.3910
1200	567.19	0.02232	0.36245	571.9	1184.8	0.7714	1.3683
1400	587.07	0.02307	0.30178	598.8	1175.8	0.7966	1.3474
1600	604.87	0.02387	0.25545	624.2	1164.5	0.8199	1.3274
1800	621.02	0.02472	0.21861	648.5	1152.3	0.8417	1.3079

Saturated Steam (U.S. Units)—Pressure Table (cont'd)

Pressure	Temperature	Specific Volume, v $\frac{\text{ft}^3}{\text{lbm}}$		Specific Enthalpy, h $\frac{\text{Btu}}{\text{lbm}}$		Specific Entropy, s $\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}$	
		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor
2000	635.80	0.02565	0.18831	672.1	1138.3	0.8625	1.2881
2200	649.45	0.02669	0.16272	695.5	1122.2	0.8828	1.2676
2400	662.11	0.02790	0.14076	719.0	1103.7	0.9031	1.2460
2600	673.91	0.02938	0.1211	744.5	1082.0	0.9247	1.2225
2800	684.96	0.03134	0.10305	770.7	1055.5	0.9468	1.1958
3000	695.33	0.03428	0.08500	801.8	1020.3	0.9728	1.1619
3100	700.28	0.03681	0.07452	824.0	993.3	0.9914	1.1373
3200	705.08	0.04472	0.05663	875.5	931.6	1.0351	1.0832
3208.2	705.47	0.05078	0.05078	906.0	906.0	1.0612	1.0612

9.8.2 Properties of Saturated Steam (SI Units)

Saturated Steam (SI Units)—Temperature Table

Temperature	Pressure	Specific Volume, v $\frac{\text{m}^3}{\text{kg}}$		Specific Enthalpy, h $\frac{\text{kJ}}{\text{kg}}$		Specific Entropy, s $\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$	
		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor
0.01	0.6113	0.001000	206.14	0.01	2501.4	0	9.1562
5	0.8721	0.001000	147.12	20.98	2510.6	0.0761	9.0257
10	1.2276	0.001000	106.38	42.01	2519.8	0.151	8.9008
15	1.7051	0.001001	77.93	62.99	2528.9	0.2245	8.7814
20	2.339	0.001002	57.79	83.96	2538.1	0.2966	8.6672
25	3.169	0.001003	43.36	104.89	2547.2	0.3674	8.5580
30	4.246	0.001004	32.89	125.79	2556.3	0.4369	8.4533
35	5.628	0.001006	25.22	146.68	2565.3	0.5053	8.3531
40	7.384	0.001008	19.52	167.37	2574.3	0.5725	8.2570
45	9.593	0.001010	15.26	188.45	2583.2	0.6387	8.1648
50	12.349	0.001012	12.03	209.33	2592.1	0.7038	8.0763
55	15.758	0.001015	9.568	230.23	2600.9	0.7679	7.9913
60	19.94	0.001017	7.671	251.13	2609.6	0.8312	7.9096
65	25.03	0.001020	6.197	272.06	2618.3	0.8935	7.8310
70	31.19	0.001023	5.042	292.98	2626.8	0.9549	7.7553
75	38.58	0.001026	4.131	313.93	2635.3	1.0155	7.6824
80	47.39	0.001029	3.407	334.91	2643.7	1.0753	7.6122
85	57.83	0.001033	2.828	355.90	2651.9	1.1343	7.5445
90	70.14	0.001036	2.361	376.92	2660.1	1.1925	7.4791
95	84.55	0.001040	1.982	397.96	2668.1	1.2500	7.4159
100	101.35	0.001044	1.6729	419.04	2676.1	1.3069	7.3549
105	120.82	0.001048	1.4194	440.15	2683.8	1.3630	7.2958

Saturated Steam (SI Units)—Temperature Table (cont'd)

Temperature	Pressure	Specific Volume, v		Specific Enthalpy, h		Specific Entropy, s	
		$\frac{\text{m}^3}{\text{kg}}$		$\frac{\text{kJ}}{\text{kg}}$		$\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$	
°C	kPa	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor
110	143.27	0.001052	1.2102	461.30	2691.5	1.4185	7.2387
115	169.06	0.001056	1.0366	482.48	2699.0	1.4734	7.1833
120	198.53	0.001060	0.8919	503.71	2706.3	1.5276	7.1296
125	232.1	0.001065	0.7706	524.99	2713.5	1.5813	7.0775
130	270.1	0.001070	0.6685	546.31	2720.5	1.6344	7.0269
135	313.0	0.001075	0.5822	567.69	2727.3	1.6870	6.9777
140	361.3	0.001080	0.5089	589.13	2733.9	1.7391	6.9299
145	415.4	0.001085	0.4463	610.63	2740.3	1.7907	6.8833
150	475.8	0.001091	0.3928	632.20	2746.5	1.8418	6.8379
155	543.1	0.001096	0.3468	653.84	2752.4	1.8925	6.7935
160	617.8	0.001102	0.3071	675.55	2758.1	1.9427	6.7502
165	700.5	0.001108	0.2727	697.34	2763.5	1.9925	6.7078
170	791.7	0.001114	0.2428	719.21	2768.7	2.0419	6.6663
175	892.0	0.001121	0.2168	741.17	2773.6	2.0909	6.6256
180	1002.1	0.001127	0.194005	763.22	2778.2	2.1396	6.5857
185	1122.7	0.001134	0.174009	785.37	2782.4	2.1879	6.5465
190	1254.4	0.001141	0.156054	807.62	2786.4	2.2359	6.5079
195	1397.8	0.001149	0.141005	829.98	2790.0	2.2835	6.4698
200	1553.8	0.001157	0.127036	852.45	2793.2	2.3309	6.4323
205	1723.0	0.001164	0.115021	875.04	2796.0	2.3780	6.3952
210	1906.2	0.001173	0.104041	897.76	2798.5	2.4248	6.3585
215	2104	0.001181	0.094079	920.62	2800.5	2.4714	6.3221
220	2318	0.001190	0.086019	943.62	2802.1	2.5178	6.2861
225	2518	0.001199	0.078049	966.78	2803.3	2.5639	6.2503
230	2795	0.001209	0.071058	990.12	2804.0	2.6099	6.2146
235	3060	0.001219	0.065037	1013.62	2804.2	2.6558	6.1791
240	3344	0.001229	0.059076	1037.32	2803.8	2.7015	6.1437
245	3618	0.001240	0.054071	1061.23	2803.0	2.7472	6.1083
250	3973	0.001251	0.050130	1085.36	2801.5	2.7927	6.0730
255	4319	0.001263	0.045098	1109.73	2799.5	2.8383	6.0375
260	4688	0.001276	0.042021	1134.37	2796.9	2.8838	6.0019
265	5081	0.001289	0.038077	1159.28	2793.6	2.9294	5.9662
270	5499	0.001302	0.035064	1184.51	2789.7	2.9751	5.9301
275	5942	0.001317	0.032079	1210.07	2785.0	3.0208	5.8938
280	6412	0.001332	0.030017	1235.99	2779.6	3.0668	5.8571
285	6909	0.001348	0.027077	1262.31	2773.3	3.1130	5.8199
290	7436	0.001366	0.025057	1289.07	2766.2	3.1594	5.7821
295	7993	0.001384	0.023054	1316.3	2758.1	3.2062	5.7437
300	8581	0.001404	0.021067	1344.0	2749.0	3.2534	5.7045

Saturated Steam (SI Units)—Temperature Table (cont'd)

Temperature	Pressure	Specific Volume, v		Specific Enthalpy, h		Specific Entropy, s	
		$\frac{\text{m}^3}{\text{kg}}$		$\frac{\text{kJ}}{\text{kg}}$		$\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$	
°C	kPa	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor
305	9202	0.001425	0.019948	1372.4	2738.7	3.3010	5.6643
310	9856	0.001447	0.018350	1401.3	2727.3	3.3493	5.6230
315	10,547	0.001472	0.016867	1431.0	2714.5	3.3982	5.5804
320	11,274	0.001499	0.015488	1461.5	2700.1	3.4480	5.5362
330	12,845	0.001561	0.012996	1525.3	2665.9	3.5507	5.4417
340	14,586	0.001638	0.010797	1594.2	2622.0	3.6594	5.3357
350	16,513	0.001740	0.008813	1670.6	2563.9	3.7777	5.2112
360	18,651	0.001893	0.006945	1760.5	2481.0	3.9147	5.0526
370	21,030	0.002213	0.004925	1890.5	2332.1	4.1106	4.7971
374.14	22,090	0.003155	0.003155	2099.3	2099.3	4.4298	4.4298

Saturated Steam (SI Units)—Pressure Table

Pressure	Temperature	Specific Volume, v		Specific Enthalpy, h		Specific Entropy, s	
		$\frac{\text{m}^3}{\text{kg}}$		$\frac{\text{kJ}}{\text{kg}}$		$\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$	
kPa	°C	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor
0.6113	0.01	0.001000	206.14	1	2501.4	0	9.1562
1	6.98	0.001000	129.21	29.3	2514.2	0.1059	8.9756
1.5	13.03	0.001001	87.98	54.71	2525.3	0.1957	8.8279
2	17.50	0.001001	67	73.48	2533.5	0.2607	8.7237
2.5	21.08	0.001002	54.25	88.49	2540.0	0.3120	8.6432
3	24.08	0.001003	45.67	101.05	2545.5	0.3545	8.5776
4	28.96	0.001004	34.8	121.46	2554.4	0.4226	8.4746
5	32.88	0.001005	28.19	137.82	2561.5	0.4764	8.3951
7.5	40.29	0.001008	19.24	168.79	2574.8	0.5764	8.2515
10	45.81	0.001010	14.67	191.83	2584.7	0.6493	8.1502
15	53.97	0.001014	10.02	225.94	2599.1	0.7549	8.0085
20	60.05	0.001017	7.649	251.4	2609.7	0.8320	7.9085
25	64.97	0.001020	6.204	271.93	2618.2	0.8931	7.8314
30	69.10	0.001022	5.229	289.23	2625.3	0.9439	7.7686
40	75.87	0.001027	3.993	317.58	2636.8	1.0259	7.6700
50	81.33	0.001030	3.24	340.49	2645.9	1.0910	7.5939
75	91.78	0.001037	2.217	384.39	2663.0	1.2130	7.4564
100	99.63	0.001043	1.694	417.46	2675.5	1.3026	7.3594
125	105.99	0.001048	1.3749	444.32	2685.4	1.3740	7.2844
150	111.37	0.001053	1.1593	467.11	2693.6	1.4336	7.2233
175	116.06	0.001057	1.0036	486.99	2700.6	1.4849	7.1717
200	120.23	0.001061	0.8857	504.7	2706.7	1.5301	7.1271

Saturated Steam (SI Units)—Pressure Table (cont'd)

Pressure	Temperature	Specific Volume, v $\frac{\text{m}^3}{\text{kg}}$		Specific Enthalpy, h $\frac{\text{kJ}}{\text{kg}}$		Specific Entropy, s $\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$	
		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor
225	124.00	0.001064	0.7933	520.72	2712.1	1.5706	7.0878
250	127.44	0.001067	0.7187	535.37	2716.9	1.6072	7.0527
275	130.60	0.001070	0.6573	548.89	2721.3	1.6408	7.0209
300	133.55	0.001073	0.6058	561.47	2725.3	1.6718	6.9919
325	136.30	0.001076	0.5620	573.25	2729.0	1.7006	6.9652
350	138.88	0.001079	0.5243	584.33	2732.4	1.7275	6.9405
375	141.32	0.001081	0.4914	594.81	2735.6	1.7528	6.9175
400	143.63	0.001084	0.4625	604.74	2738.6	1.7766	6.8959
450	147.93	0.001088	0.4140	623.25	2743.9	1.8207	6.8565
500	151.86	0.001093	0.3749	640.23	2748.7	1.8607	6.8213
550	155.48	0.001097	0.3427	655.93	2753.0	1.8973	6.7893
700	164.97	0.001108	0.2729	697.22	2763.5	1.9922	6.7080
750	167.78	0.001112	0.2556	709.47	2766.4	2.0200	6.6847
800	170.43	0.001115	0.2404	721.11	2769.1	2.0462	6.6628
850	172.96	0.001118	0.2270	732.22	2771.6	2.0710	6.6421
900	175.38	0.001121	0.2150	742.83	2773.9	2.0946	6.6226
950	177.69	0.001124	0.2042	753.02	2776.1	2.1172	6.6041
1000	179.91	0.001127	0.194044	762.81	2778.1	2.1387	6.5865
1100	184.09	0.001133	0.177053	781.34	2781.7	2.1792	6.5536
1200	187.99	0.001139	0.163033	798.65	2784.8	2.2166	6.5233
1300	191.64	0.001144	0.151025	814.93	2787.6	2.2515	6.4953
1400	195.07	0.001149	0.140084	830.30	2790.0	2.2842	6.4693
1500	198.32	0.001154	0.131077	844.89	2792.2	2.3150	6.4448
1750	205.76	0.001166	0.113049	878.50	2796.4	2.3851	6.3896
2000	212.42	0.001177	0.099063	908.79	2799.5	2.4474	6.3409
2250	218.45	0.001187	0.088075	936.49	2801.7	2.5035	6.2972
2500	223.99	0.001197	0.079098	962.11	2803.1	2.5547	6.2575
3000	233.90	0.001217	0.066068	1008.42	2804.2	2.6457	6.1869
3500	242.60	0.001235	0.057007	1049.75	2804.2	2.7253	6.1253
4000	250.40	0.001252	0.049078	1087.31	2801.4	2.7964	6.0701
5000	263.99	0.001286	0.039044	1154.23	2794.3	3.9202	5.9734
6000	275.64	0.001319	0.032044	1213.35	2784.3	3.0267	5.8892
7000	285.88	0.001351	0.027037	1267.00	2772.1	3.1211	5.8133
8000	295.06	0.001384	0.023052	1316.64	2758.0	3.2068	5.7432
9000	303.40	0.001418	0.020048	1363.26	2742.1	3.2858	5.6772
10,000	311.06	0.001452	0.018026	1407.56	2724.7	3.3596	5.6141
11,000	318.15	0.001489	0.015987	1450.1	2705.6	3.4295	5.5527
12,000	324.75	0.001527	0.014263	1491.3	2684.9	3.4962	5.4924
13,000	330.93	0.001567	0.01278	1531.5	2662.2	3.5606	5.4323

Saturated Steam (SI Units)—Pressure Table (cont'd)

Pressure	Temperature	Specific Volume, v $\frac{\text{m}^3}{\text{kg}}$		Specific Enthalpy, h $\frac{\text{kJ}}{\text{kg}}$		Specific Entropy, s $\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$	
		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor
14,000	336.75	0.001611	0.011485	1571.1	2637.6	3.6232	5.3717
15,000	342.24	0.001658	0.010337	1610.5	2610.5	3.6848	5.3098
16,000	347.44	0.001711	0.009306	1650.1	2580.6	3.7461	5.2455
17,000	352.37	0.001770	0.008364	1690.3	2547.2	3.8079	5.1777
18,000	357.06	0.001840	0.007489	1732.0	2509.1	3.8715	5.1044
19,000	361.54	0.001924	0.006657	1776.5	2464.5	3.9388	5.0228
20,000	365.81	0.002036	0.005834	1826.3	2409.7	4.0139	4.9269
21,000	369.89	0.002207	0.004952	1888.4	2334.6	4.1075	4.8013
22,000	373.80	0.002742	0.003568	2022.2	2165.6	4.3110	5.5327
22,090	374.14	0.003155	0.003155	2099.3	2099.3	4.4298	4.4298

9.8.3 Superheated Steam (U.S. Customary Units)

Superheated Steam (U.S. Units) $v = \left(\frac{\text{ft}^3}{\text{lbm}} \right) \quad h = \left(\frac{\text{Btu}}{\text{lbm}} \right) \quad s = \left(\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}} \right)$

Pressure (psia)		Temperature (°F)											
Saturated Temp. (°F)		200	300	400	500	600	700	800	900	1000	1200	1400	1600
1 (101.74)	v	392.6	452.3	512.0	571.6	631.2	690.8	750.4	809.9	869.5	988.7	1107.8	1227.0
	h	1150.4	1195.8	1241.7	1288.3	1335.7	1383.8	1432.8	1482.7	1533.5	1637.7	1745.7	1857.5
	s	2.0512	2.1153	2.1720	2.2233	2.2702	2.3137	2.3542	2.3923	2.4283	2.4952	2.5566	2.6137
5 (162.24)	v	78.16	90.25	102.26	114.22	126.16	138.10	150.03	161.95	173.87	197.71	221.60	245.40
	h	1148.8	1195.0	1241.2	1288.0	1335.4	1383.6	1432.7	1482.6	1533.4	1637.7	1745.7	1857.4
	s	1.8718	1.9370	1.9942	2.0456	2.0927	2.1361	2.1767	2.2148	2.2509	2.3178	2.3792	2.4363
10 (193.21)	v	38.85	45.00	51.04	57.05	63.03	69.01	74.98	80.95	86.92	98.84	110.77	122.69
	h	1146.6	1193.9	1240.6	1287.5	1335.1	1383.4	1432.5	1482.4	1533.2	1637.6	1745.6	1857.3
	s	1.7927	1.8595	1.9172	1.9689	2.0160	2.0596	2.1002	2.1383	2.1744	2.2413	2.3028	2.3598
14.696 (212)	v		30.53	34.68	38.78	42.86	46.94	51.00	55.07	59.13	67.25	75.37	83.48
	h		1192.8	1239.9	1287.1	1334.8	1383.2	1432.3	1482.3	1533.1	1637.5	1745.5	1857.3
	s		1.8160	1.8743	1.9261	1.9734	2.0170	2.0576	2.0958	2.1319	2.1989	2.2603	2.3174
20 (227.96)	v		22.36	25.43	28.46	31.47	34.47	37.46	40.45	43.44	49.41	55.37	61.34
	h		1191.6	1239.2	1286.6	1334.4	1382.9	1432.1	1482.1	1533.0	1637.4	1745.4	1857.2
	s		1.7808	1.8396	1.8918	1.9392	1.9829	2.0235	2.0618	2.0978	2.1648	2.2263	2.2834
40 (267.25)	v		11.04	12.628	14.168	15.688	17.198	18.702	20.20	21.70	24.69	27.68	30.66
	h		1186.8	1236.5	1284.8	1333.1	1381.9	1431.3	1481.4	1532.4	1637.0	1745.1	1857.0
	s		1.6994	1.7608	1.8140	1.8619	1.9058	1.9467	1.9850	2.0212	2.0883	2.1498	2.2069
60 (292.71)	v		7.259	8.357	9.403	10.427	11.441	12.449	13.452	14.454	16.451	18.446	20.440
	h		1181.6	1233.6	1283.0	1331.8	1380.9	1430.5	1480.8	1531.9	1636.6	1744.8	1856.7
	s		1.6492	1.7135	1.7678	1.8162	1.8605	1.9015	1.9400	1.9762	2.0434	2.1049	2.1621
80 (312.03)	v			6.220	7.020	7.797	8.562	9.322	10.077	10.830	12.332	13.830	15.325
	h			1230.7	1281.1	1330.5	1379.9	1429.7	1480.1	1531.3	1636.2	1744.5	1856.5
	s			1.6791	1.7346	1.7836	1.8281	1.8694	1.9079	1.9442	2.0115	2.0731	2.1303
100 (327.81)	v			4.937	5.589	6.218	6.835	7.446	8.052	8.656	9.860	11.060	12.258
	h			1227.6	1279.1	1329.1	1378.9	1428.9	1479.5	1530.8	1635.7	1744.2	1856.2
	s			1.6518	1.7085	1.7581	1.8029	1.8443	1.8829	1.9193	1.9867	2.0484	2.1056

Superheated Steam (U.S. Units) (cont'd) $v = \left(\frac{\text{ft}^3}{\text{lbm}} \right)$ $h = \left(\frac{\text{Btu}}{\text{lbm}} \right)$ $s = \left(\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}} \right)$

Pressure (psia) Saturated Temp. (°F)		Temperature (°F)											
		200	300	400	500	600	700	800	900	1000	1200	1400	1600
120 (341.25)	<i>v</i>			4.081	4.636	5.165	5.683	6.195	6.702	7.207	8.212	9.214	10.213
	<i>h</i>			1224.4	1277.2	1327.7	1377.8	1428.1	1478.8	1530.2	1635.3	1743.9	1856.0
	<i>s</i>			1.6287	1.6869	1.7370	1.7822	1.8237	1.8625	1.8990	1.9664	2.0281	2.0854
140 (353.02)	<i>v</i>			3.468	3.954	4.413	4.861	5.301	5.738	6.172	7.035	7.895	8.752
	<i>h</i>			1221.1	1275.2	1326.4	1376.8	1427.3	1478.2	1529.7	1634.9	1743.5	1855.7
	<i>s</i>			1.6087	1.6683	1.7190	1.7645	1.8063	1.8451	1.8817	1.9493	2.0110	2.0683
160 (363.53)	<i>v</i>			3.008	3.443	3.849	4.244	4.631	5.015	5.396	6.152	6.906	7.656
	<i>h</i>			1217.6	1273.1	1325.0	1375.7	1426.4	1477.5	1529.1	1634.5	1743.2	1855.5
	<i>s</i>			1.5908	1.6519	1.7033	1.7491	1.7911	1.8301	1.8667	1.9344	1.9962	2.0535
180 (373.06)	<i>v</i>			2.649	3.044	3.411	3.764	4.110	4.452	4.792	5.466	6.136	6.804
	<i>h</i>			1214.0	1271.0	1323.5	1374.7	1425.6	1476.8	1528.6	1634.1	1742.9	1855.2
	<i>s</i>			1.5745	1.6373	1.6894	1.7355	1.7776	1.8167	1.8534	1.9212	1.9831	2.0404
200 (381.79)	<i>v</i>			2.361	2.726	3.060	3.380	3.693	4.002	4.309	4.917	5.521	6.123
	<i>h</i>			1210.3	1268.9	1322.1	1373.6	1424.8	1476.2	1528.0	1633.7	1742.6	1855.0
	<i>s</i>			1.5594	1.6240	1.6767	1.7232	1.7655	1.8048	1.8415	1.9094	1.9713	2.0287
220 (389.86)	<i>v</i>			2.125	2.465	2.772	3.066	3.352	3.634	3.913	4.467	5.017	5.565
	<i>h</i>			1206.5	1266.7	1320.7	1372.6	1424.0	1475.5	1527.5	1633.3	1742.3	1854.7
	<i>s</i>			1.5453	1.6117	1.6652	1.7120	1.7545	1.7939	1.8308	1.8987	1.9607	2.0181
240 (397.37)	<i>v</i>			1.9276	2.247	2.533	2.804	3.068	3.327	3.584	4.093	4.597	5.100
	<i>h</i>			1202.5	1264.5	1319.2	1371.5	1423.2	1474.8	1526.9	1632.9	1742.0	1854.5
	<i>s</i>			1.5319	1.6003	1.6546	1.7017	1.7444	1.7839	1.8209	1.8889	1.9510	2.0084
260 (404.42)	<i>v</i>				2.063	2.330	2.582	2.827	3.067	3.305	3.776	4.242	4.707
	<i>h</i>				1262.3	1317.7	1370.4	1422.3	1474.2	1526.3	1632.5	1741.7	1854.2
	<i>s</i>				1.5897	1.6447	1.6922	1.7352	1.7748	1.8118	1.8799	1.9420	1.9995
280 (411.05)	<i>v</i>				1.9047	2.156	2.392	2.621	2.845	3.066	3.504	3.938	4.37
	<i>h</i>				1260.0	1316.2	1369.4	1421.5	1473.5	1525.8	1632.1	1741.4	1854.0
	<i>s</i>				1.5796	1.6354	1.6834	1.7265	1.7662	1.8033	1.8716	1.9337	1.9912

Superheated Steam (U.S. Units) (cont'd) $v = \left(\frac{\text{ft}^3}{\text{lbm}} \right)$ $h = \left(\frac{\text{Btu}}{\text{lbm}} \right)$ $s = \left(\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}} \right)$

Pressure (psia) Saturated Temp. (°F)		Temperature (°F)											
		200	300	400	500	600	700	800	900	1000	1200	1400	1600
300 (417.33)	<i>v</i>				1.7675	2.005	2.227	2.442	2.652	2.859	3.269	3.674	4.078
	<i>h</i>				1257.6	1314.7	1368.3	1420.6	1472.8	1525.2	1631.7	1741.0	1853.7
	<i>s</i>				1.5701	1.6268	1.6751	1.7184	1.7582	1.7954	1.8638	1.9260	1.9835
350 (431.72)	<i>v</i>				1.4923	1.7036	1.898	2.084	2.266	2.445	2.798	3.147	3.493
	<i>h</i>				1251.5	1310.9	1365.5	1418.5	1471.1	1523.8	1630.7	1740.3	1853.1
	<i>s</i>				1.5481	1.6070	1.6563	1.7002	1.7403	1.7777	1.8463	1.9086	1.9663
400 (444.59)	<i>v</i>				1.2851	1.477	1.6508	1.8161	1.9767	2.134	2.445	2.751	3.055
	<i>h</i>				1245.1	1306.9	1362.7	1416.4	1469.4	1522.4	1629.6	1739.5	1852.5
	<i>s</i>				1.5281	1.5894	1.6398	1.6842	1.7247	1.7623	1.8311	1.8936	1.9513
450 (456.28)	<i>v</i>				1.1231	1.3005	1.4584	1.6074	1.7516	1.8928	2.1700	2.4430	2.7140
	<i>h</i>				1238.4	1302.8	1359.9	1414.3	1467.7	1521.0	1628.6	1738.7	1851.9
	<i>s</i>				1.5095	1.5735	1.6250	1.6699	1.7108	1.7486	1.8177	1.8803	1.9381
500 (467.01)	<i>v</i>				0.9927	1.1591	1.3044	1.4405	1.5715	1.6996	1.9504	2.1970	2.4420
	<i>h</i>				1231.3	1298.6	1357.0	1412.1	1466.0	1519.6	1627.6	1737.9	1851.3
	<i>s</i>				1.4919	1.5588	1.6115	1.6571	1.6982	1.7363	1.8056	1.8683	1.9262
550 (476.94)	<i>v</i>				0.8852	1.0431	1.1783	1.3038	1.4241	1.5414	1.7706	1.9957	2.2190
	<i>h</i>				1223.7	1294.3	1354.0	1409.9	1464.3	1518.2	1626.6	1737.1	1850.6
	<i>s</i>				1.4751	1.5451	1.5991	1.6452	1.6868	1.7250	1.7946	1.8575	1.9155
600 (486.21)	<i>v</i>				0.7947	0.9463	1.0732	1.1899	1.3013	1.4096	1.6208	1.8279	2.0330
	<i>h</i>				1215.7	1289.9	1351.1	1407.7	1462.5	1516.7	1625.5	1736.3	1850.0
	<i>s</i>				1.4586	1.5323	1.5875	1.6343	1.6762	1.7147	1.7846	1.8476	1.9056
700 (503.1)	<i>v</i>					0.7934	0.9077	1.0108	1.1082	1.2024	1.3853	1.5641	1.7405
	<i>h</i>					1280.6	1345.0	1403.2	1459.0	1513.9	1623.5	1734.8	1848.8
	<i>s</i>					1.5084	1.5665	1.6147	1.6573	1.6963	1.7666	1.8299	1.8881
800 (518.23)	<i>v</i>					0.6779	0.7833	0.8763	0.9633	1.0470	1.2088	1.3662	1.5214
	<i>h</i>					1270.7	1338.6	1398.6	1455.4	1511.0	1621.4	1733.2	1847.5
	<i>s</i>					1.4863	1.5476	1.5972	1.6407	1.6801	1.7510	1.8146	1.8729
900 (531.98)	<i>v</i>					0.5873	0.6863	0.7716	0.8506	0.9262	1.0714	1.2124	1.3509
	<i>h</i>					1260.1	1332.1	1393.9	1451.8	1508.1	1619.3	1731.6	1846.3
	<i>s</i>					1.4653	1.5303	1.5814	1.6257	1.6656	1.7371	1.8009	1.8595

Superheated Steam (U.S. Units) (cont'd) $v = \left(\frac{\text{ft}^3}{\text{lbm}} \right)$ $h = \left(\frac{\text{Btu}}{\text{lbm}} \right)$ $s = \left(\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}} \right)$

Pressure (psia) Saturated Temp. (°F)		Temperature (°F)											
		200	300	400	500	600	700	800	900	1000	1200	1400	1600
1000 (544.61)	<i>v</i>					0.5140	0.6084	0.6878	0.7604	0.8294	0.9615	1.0893	1.2146
	<i>h</i>					1248.8	1325.3	1389.2	1448.2	1505.1	1617.3	1730.0	1845.0
	<i>s</i>					1.4450	1.5141	1.5670	1.6121	1.6525	1.7245	1.7886	1.8474
1100 (556.31)	<i>v</i>					0.4532	0.5445	0.6191	0.6866	0.7503	0.8716	0.9885	1.1031
	<i>h</i>					1236.7	1318.3	1384.3	1444.5	1502.2	1615.2	1728.4	1843.8
	<i>s</i>					1.4251	1.4989	1.5535	1.5995	1.6405	1.7130	1.7775	1.8363
1200 (567.22)	<i>v</i>					0.4016	0.4909	0.5617	0.6250	0.6843	0.7967	0.9046	1.0101
	<i>h</i>					1223.5	1311.0	1379.3	1440.7	1499.2	1613.1	1726.9	1842.5
	<i>s</i>					1.4052	1.4843	1.5409	1.5879	1.6293	1.7025	1.7672	1.8263
1400 (587.1)	<i>v</i>					0.3174	0.4062	0.4714	0.5281	0.5805	0.6789	0.7727	0.8640
	<i>h</i>					1193.0	1295.5	1369.1	1433.1	1493.2	1608.9	1723.7	1840.0
	<i>s</i>					1.3639	1.4567	1.5177	1.5666	1.6093	1.6836	1.7489	1.8083
1600 (604.9)	<i>v</i>						0.3417	0.4034	0.4553	0.5027	0.5906	0.6738	0.7545
	<i>h</i>						1278.7	1358.4	1425.3	1487.0	1604.6	1720.5	1837.5
	<i>s</i>						1.4303	1.4964	1.5476	1.5914	1.6669	1.7328	1.7926
1800 (621.03)	<i>v</i>						0.2907	0.3502	0.3986	0.4421	0.5218	0.5968	0.6693
	<i>h</i>						1260.3	1347.2	1417.4	1480.8	1600.4	1717.3	1835.0
	<i>s</i>						1.4044	1.4765	1.5301	1.5752	1.6520	1.7185	1.7786
2000 (635.82)	<i>v</i>						0.2489	0.3074	0.3532	0.3935	0.4668	0.5352	0.6011
	<i>h</i>						1240.0	1335.5	1409.2	1474.5	1596.1	1714.1	1832.5
	<i>s</i>						1.3783	1.4576	1.5139	1.5603	1.6384	1.7055	1.7660
2500 (668.13)	<i>v</i>						0.1686	0.2294	0.2710	0.3061	0.3678	0.4244	0.4784
	<i>h</i>						1176.8	1303.6	1387.8	1458.4	1585.3	1706.1	1826.2
	<i>s</i>						1.3073	1.4127	1.4772	1.5273	1.6088	1.6775	1.7389
3000 (695.36)	<i>v</i>						0.0984	0.1760	0.2159	0.2476	0.3018	0.3505	0.3966
	<i>h</i>						1060.7	1267.2	1365.0	1441.8	1574.3	1698.0	1819.9
	<i>s</i>						1.1966	1.3690	1.4439	1.4984	1.5837	1.6540	1.7163

Superheated Steam (U.S. Units) (cont'd) $v = \left(\frac{\text{ft}^3}{\text{lbm}} \right)$ $h = \left(\frac{\text{Btu}}{\text{lbm}} \right)$ $s = \left(\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}} \right)$

Pressure (psia) Saturated Temp. (°F)		Temperature (°F)											
		200	300	400	500	600	700	800	900	1000	1200	1400	1600
3206.2 (705.4)	v							0.1583	0.1981	0.2288	0.2806	0.3267	0.3703
	h							1250.5	1355.2	1434.7	1569.8	1694.6	1817.2
	s							1.3508	1.4309	1.4874	1.5742	1.6452	1.7080
3500	v						0.0306	0.1364	0.1762	0.2058	0.2546	0.2977	0.3381
	h						780.5	1224.9	1340.7	1424.5	1563.3	1689.8	1813.6
	s						0.9515	1.3241	1.4127	1.4723	1.5615	1.6336	1.6968
4000	v						0.0287	0.1052	0.1462	0.1743	0.2192	0.2581	0.2943
	h						763.8	1174.8	1314.4	1406.8	1552.1	1681.7	1807.2
	s						0.9347	1.2757	1.3827	1.4482	1.5417	1.6154	1.6795
4500	v						0.0276	0.0798	0.1226	0.1500	0.1917	0.2273	0.2602
	h						753.5	1113.9	1286.5	1388.4	1540.8	1673.5	1800.9
	s						0.9235	1.2204	1.3529	1.4253	1.5235	1.5990	1.6640
5000	v						0.0268	0.0593	0.1036	0.1303	0.1696	0.2027	0.2329
	h						746.4	1047.1	1256.5	1369.5	1529.5	1665.3	1794.5
	s						0.9152	1.1622	1.3231	1.4034	1.5066	1.5839	1.6499
5500	v						0.0262	0.0463	0.0880	0.1143	0.1516	0.1825	0.2106
	h						741.3	985.0	1224.1	1349.3	1518.2	1657.0	1788.1
	s						0.9090	1.1093	1.2930	1.3821	1.4908	1.5699	1.6369

9.8.4 Superheated Steam (SI Units)

$$\text{Superheated Steam (SI Units)} \quad v = \left(\frac{m^3}{kg} \right) \quad h = \left(\frac{kJ}{kg} \right) \quad s = \left(\frac{kJ}{kg \cdot K} \right)$$

Pressure (MPa)		Temperature (°C)											
Saturated Temp. (°C)		50	100	150	200	250	300	400	500	600	700	800	900
0.01 (45.81)	<i>v</i>	14.8690	17.1960	19.5120	21.8250	24.1360	26.4450	31.0630	35.6790	40.2950	44.9110	49.5260	54.1410
	<i>h</i>	2592.6	2687.5	2783.0	2879.5	2977.3	3076.5	3279.6	3489.1	3705.4	3928.7	4159.0	4396.4
	<i>s</i>	8.1749	8.4479	8.6882	8.9038	9.1002	9.2813	9.6077	9.8978	10.1608	10.4028	10.6281	10.8396
0.05 (81.33)	<i>v</i>		3.41800	3.88900	4.35600	4.82000	5.28400	6.20900	7.13400	8.05700	8.98100	9.90400	10.8280
	<i>h</i>		2682.5	2780.1	2877.7	2976.0	3075.5	3278.9	3488.7	3705.1	3928.5	4158.9	4396.3
	<i>s</i>		7.6958	7.9401	8.1580	8.3556	8.5373	8.8642	9.1546	9.4178	9.6599	9.8852	10.0967
0.1 (99.63)	<i>v</i>		1.69580	1.93640	2.17200	2.40600	2.63900	3.10300	3.56500	4.02800	4.49000	4.95200	5.41400
	<i>h</i>		2676.2	2776.4	2875.3	2974.3	3074.3	3278.2	3488.1	3704.7	3928.2	4158.6	4396.1
	<i>s</i>		7.3614	7.6134	7.8343	8.0333	8.2158	8.5435	8.8342	9.0976	9.3398	9.5652	9.7767
0.2 (120.23)	<i>v</i>			0.95960	1.08030	1.19880	1.31620	1.54930	1.78140	2.01300	2.24400	2.47500	2.70600
	<i>h</i>			2768.8	2870.5	2971.0	3071.8	3276.6	3487.1	3704.0	3927.6	4158.2	4395.8
	<i>s</i>			7.2795	7.5066	7.7086	7.8926	8.2218	8.5133	8.7770	9.0194	9.2449	9.4566
0.3 (133.55)	<i>v</i>			0.63390	0.71630	0.79640	0.87530	1.03150	1.18670	1.34140	1.49570	1.64990	1.80410
	<i>h</i>			2761.0	2865.6	2967.6	3069.3	3275.0	3486.0	3703.2	3927.1	4157.8	4395.4
	<i>s</i>			7.0778	7.3115	7.5166	7.7022	8.0330	8.3251	8.5892	8.8319	9.0576	9.2692
0.4 (143.63)	<i>v</i>			0.47080	0.53420	0.59510	0.65480	0.77260	0.88930	1.00550	1.12150	1.23720	1.35290
	<i>h</i>			2752.8	2860.5	2964.2	3066.8	3273.4	3484.9	3702.4	3926.5	4157.3	4395.1
	<i>s</i>			6.9299	7.1706	7.3789	7.5662	7.8985	8.1913	8.4558	8.6987	8.9244	9.1362
0.5 (151.86)	<i>v</i>				0.42490	0.47440	0.52260	0.61730	0.71090	0.80410	0.89690	0.98960	1.08220
	<i>h</i>				2855.4	2960.7	3064.2	3271.9	3483.9	3701.7	3925.9	4156.9	4394.7
	<i>s</i>				7.0592	7.2709	7.4599	7.7938	8.0873	8.3522	8.5952	8.8211	9.0329
0.6 (158.85)	<i>v</i>				0.35200	0.39380	0.43440	0.51370	0.59200	0.66970	0.74720	0.82450	0.90170
	<i>h</i>				2850.1	2957.2	3061.6	3270.3	3482.8	3700.9	3925.3	4156.5	4394.4
	<i>s</i>				6.9665	7.1816	7.3724	7.7079	8.0021	8.2674	8.5107	8.7367	8.9486
0.8 (170.43)	<i>v</i>				0.26080	0.29310	0.32410	0.38430	0.44330	0.50180	0.56010	0.61810	0.67610
	<i>h</i>				2839.3	2950.0	3056.5	3267.1	3480.6	3699.4	3924.2	4155.6	4393.7
	<i>s</i>				6.8158	7.0384	7.2328	7.5716	7.8673	8.1333	8.3770	8.6033	8.8153

Superheated Steam (SI Units) (cont'd) $v = \left(\frac{m^3}{kg} \right)$ $h = \left(\frac{kJ}{kg} \right)$ $s = \left(\frac{kJ}{kg \cdot K} \right)$

Pressure (MPa) Saturated Temp. (°C)		Temperature (°C)											
		50	100	150	200	250	300	400	500	600	700	800	900
1.0 (179.91)	<i>v</i>				0.20600	0.23270	0.25790	0.30660	0.35410	0.40110	0.44780	0.49430	0.54070
	<i>h</i>				2827.9	2942.6	3051.2	3263.9	3478.5	3697.9	3923.1	4154.7	4392.9
	<i>s</i>				6.6940	6.9247	7.1229	7.4651	7.7622	8.0290	8.2731	8.4996	8.7118
1.2 (187.99)	<i>v</i>				0.16930	0.19234	0.21380	0.25480	0.29460	0.33390	0.37290	0.41180	0.45050
	<i>h</i>				2815.9	2935.0	3045.8	3260.7	3476.3	3696.3	3922.0	4153.8	4392.2
	<i>s</i>				6.5898	6.8294	7.0317	7.3774	7.6959	7.9435	8.1881	8.4148	8.6272
1.4 (195.07)	<i>v</i>				0.14302	0.16350	0.18228	0.21780	0.25210	0.28600	0.31950	0.35280	0.38610
	<i>h</i>				2803.3	2927.2	3040.4	3257.5	3474.1	3694.8	3920.8	4153.0	4391.5
	<i>s</i>				6.4975	6.7467	6.9534	7.3026	7.6027	7.8710	8.1160	8.3431	8.5556
1.6 (201.41)	<i>v</i>					0.14184	0.15862	0.19005	0.22030	0.25000	0.27940	0.30860	0.33770
	<i>h</i>					2919.2	3034.8	3254.2	3472.0	3693.2	3919.7	4152.1	4390.8
	<i>s</i>					6.6732	6.8844	7.2374	7.5390	7.8080	8.0535	8.2808	8.4935
1.8 (207.15)	<i>v</i>					0.12497	0.14021	0.16847	0.19550	0.22200	0.24820	0.27420	0.30010
	<i>h</i>					2911.0	3029.2	3250.9	3469.8	3691.7	3918.5	4151.2	4390.1
	<i>s</i>					6.6066	6.8226	7.1794	7.4825	7.7523	7.9983	8.2258	8.4386
2.0 (212.42)	<i>v</i>					0.11144	0.12547	0.15120	0.17568	0.19960	0.22320	0.24670	0.27000
	<i>h</i>					2902.5	3023.5	3247.6	3467.6	3690.1	3917.4	4150.3	4389.4
	<i>s</i>					6.5453	6.7664	7.1271	7.4317	7.7024	7.9487	8.1765	8.3895
2.5 (223.99°C)	<i>v</i>					0.08700	0.09890	0.12010	0.13998	0.15930	0.17832	0.19716	0.21590
	<i>h</i>					2880.1	3008.8	3239.3	3462.1	3686.3	3914.5	4148.2	4387.6
	<i>s</i>					6.4085	6.6438	7.0148	7.3234	7.5960	7.8435	8.0720	8.2853
3.0 (233.90)	<i>v</i>					0.07058	0.08114	0.09936	0.11619	0.13243	0.14838	0.16414	0.17980
	<i>h</i>					2855.8	2993.5	3230.9	3456.5	3682.3	3911.7	4145.9	4385.9
	<i>s</i>					6.2872	6.5390	6.9212	7.2338	7.5085	7.7511	7.9862	8.1999
3.5 (242.60)	<i>v</i>					0.05872	0.06842	0.08453	0.09918	0.11324	0.12699	0.14056	0.15402
	<i>h</i>					2829.2	2977.5	3222.3	3450.9	3678.4	3908.8	4143.7	4384.1
	<i>s</i>					6.1749	6.4461	6.8405	7.1572	7.4339	7.6837	7.9134	8.1276
4.0 (250.40)	<i>v</i>						0.05884	0.07341	0.08643	0.09885	0.11095	0.12287	0.13469
	<i>h</i>						2960.7	3213.6	3445.3	3674.4	3905.9	4141.5	4382.3
	<i>s</i>						6.3615	6.7690	7.0901	7.3688	7.6198	7.8502	8.0647

Superheated Steam (SI Units) (cont'd) $v = \left(\frac{m^3}{kg} \right)$ $h = \left(\frac{kJ}{kg} \right)$ $s = \left(\frac{kJ}{kg \cdot K} \right)$

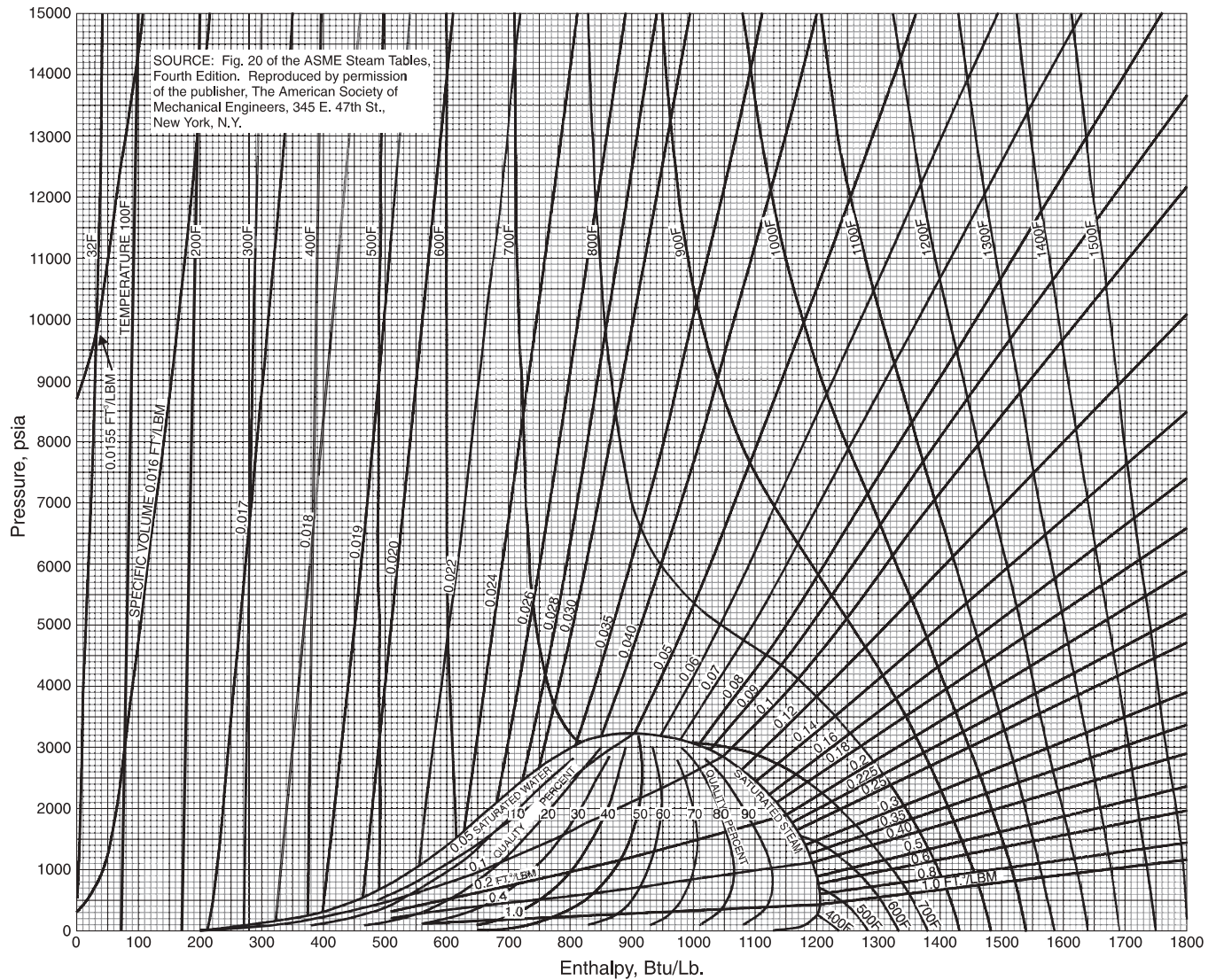
Pressure (MPa) Saturated Temp. (°C)		Temperature (°C)											
		50	100	150	200	250	300	400	500	600	700	800	900
4.5 (257.49)	<i>v</i>						0.05135	0.06475	0.07651	0.08765	0.09847	0.10911	0.11965
	<i>h</i>						2943.1	3204.7	3439.6	3670.5	3903.0	4139.3	4380.6
	<i>s</i>						6.2828	6.7047	7.0301	7.3110	7.5631	7.7942	8.0091
5.0 (263.99)	<i>v</i>						0.04532	0.05781	0.06857	0.07869	0.08849	0.09811	0.10762
	<i>h</i>						2924.5	3195.7	3433.8	3665.5	3900.1	4137.1	4378.8
	<i>s</i>						6.2084	6.6459	6.9759	7.2589	7.5122	7.7440	7.9593
6.0 (275.64)	<i>v</i>						0.03616	0.04739	0.05665	0.06525	0.07352	0.08160	0.08958
	<i>h</i>						2884.2	3177.2	3422.2	3658.4	3894.2	4132.7	4375.3
	<i>s</i>						6.0674	6.5408	6.8803	7.1677	7.4234	7.6566	7.8727
7.0 (285.88)	<i>v</i>						0.02947	0.03993	0.04814	0.05565	0.06283	0.06981	0.07669
	<i>h</i>						2838.4	3158.1	3410.3	3650.3	3888.3	4128.2	4371.8
	<i>s</i>						5.9305	6.4478	6.7975	7.0894	7.3476	7.5822	7.7991
8.0 (295.06)	<i>v</i>						0.02426	0.03432	0.04175	0.04845	0.05481	0.06097	0.06702
	<i>h</i>						2785.0	3138.3	3398.3	3642.0	3882.4	4123.8	4368.3
	<i>s</i>						5.7906	6.3634	6.7240	7.0206	7.2812	7.5173	7.7351
9.0 (303.40)	<i>v</i>							0.02993	0.03677	0.04285	0.04857	0.05409	0.05950
	<i>h</i>							3117.8	3386.1	3633.7	3876.5	4119.3	4364.8
	<i>s</i>							6.2854	6.6576	6.9589	7.2221	7.4596	7.6783
10.0 (311.06)	<i>v</i>							0.02641	0.03279	0.03837	0.04358	0.04859	0.05349
	<i>h</i>							3096.5	3373.7	3625.3	3870.5	4114.8	4361.2
	<i>s</i>							6.2120	6.5966	6.9029	7.1687	7.4077	7.6272
12.5 (327.89)	<i>v</i>							0.02000	0.02560	0.03029	0.03460	0.03869	0.04267
	<i>h</i>							3039.3	3341.8	3604.0	3855.3	4103.6	4352.5
	<i>s</i>							6.0417	6.4618	6.7810	7.0536	7.2965	7.5182
15.0 (342.24)	<i>v</i>							0.01565	0.02080	0.02491	0.02861	0.03210	0.03546
	<i>h</i>							2975.5	3308.6	3582.3	3840.1	4092.4	4343.8
	<i>s</i>							5.8811	6.3443	6.6776	6.9572	7.2040	7.4279

Superheated Steam (SI Units) (cont'd) $v = \left(\frac{m^3}{kg} \right)$ $h = \left(\frac{kJ}{kg} \right)$ $s = \left(\frac{kJ}{kg \cdot K} \right)$

Pressure (MPa) Saturated Temp. (°C)		Temperature (°C)											
		50	100	150	200	250	300	400	500	600	700	800	900
17.5 (354.75)	<i>v</i>							0.01245	0.01736	0.02106	0.02434	0.02738	0.03031
	<i>h</i>							2902.9	3274.1	3560.1	3824.6	4081.1	4335.1
	<i>s</i>							5.7213	6.2383	6.5866	6.8736	7.1244	7.3507
20.0 (365.81)	<i>v</i>							0.00994	0.01477	0.01818	0.02113	0.02385	0.02645
	<i>h</i>							2818.1	3238.2	3537.6	3809.0	4069.7	4326.4
	<i>s</i>							5.5540	6.1401	6.5048	6.7993	7.0544	7.2830
25.0	<i>v</i>							0.00604	0.01112	0.01414	0.01665	0.01891	0.02145
	<i>h</i>							2580.2	3162.4	3491.4	3777.5	4047.1	4309.1
	<i>s</i>							5.1418	5.9592	6.3602	6.6707	6.9345	7.1680
30.0	<i>v</i>							0.00279	0.00868	0.01145	0.01366	0.01562	0.01745
	<i>h</i>							2151.1	3081.1	3443.9	3745.6	4024.2	4291.9
	<i>s</i>							4.4728	5.7905	6.2331	6.5605	6.8332	7.0718
35.0	<i>v</i>							0.00210	0.00693	0.00953	0.01153	0.01328	0.01488
	<i>h</i>							1987.6	2994.4	3395.5	3713.5	4001.5	4274.9
	<i>s</i>							4.2126	5.6282	6.1179	6.4631	6.7450	6.9886
40.0	<i>v</i>							0.00191	0.00562	0.00894	0.00994	0.01152	0.01296
	<i>h</i>							1930.9	2903.3	3346.4	3681.2	3978.7	4257.9
	<i>s</i>							4.1135	5.4700	6.0114	6.3750	6.6662	6.9150

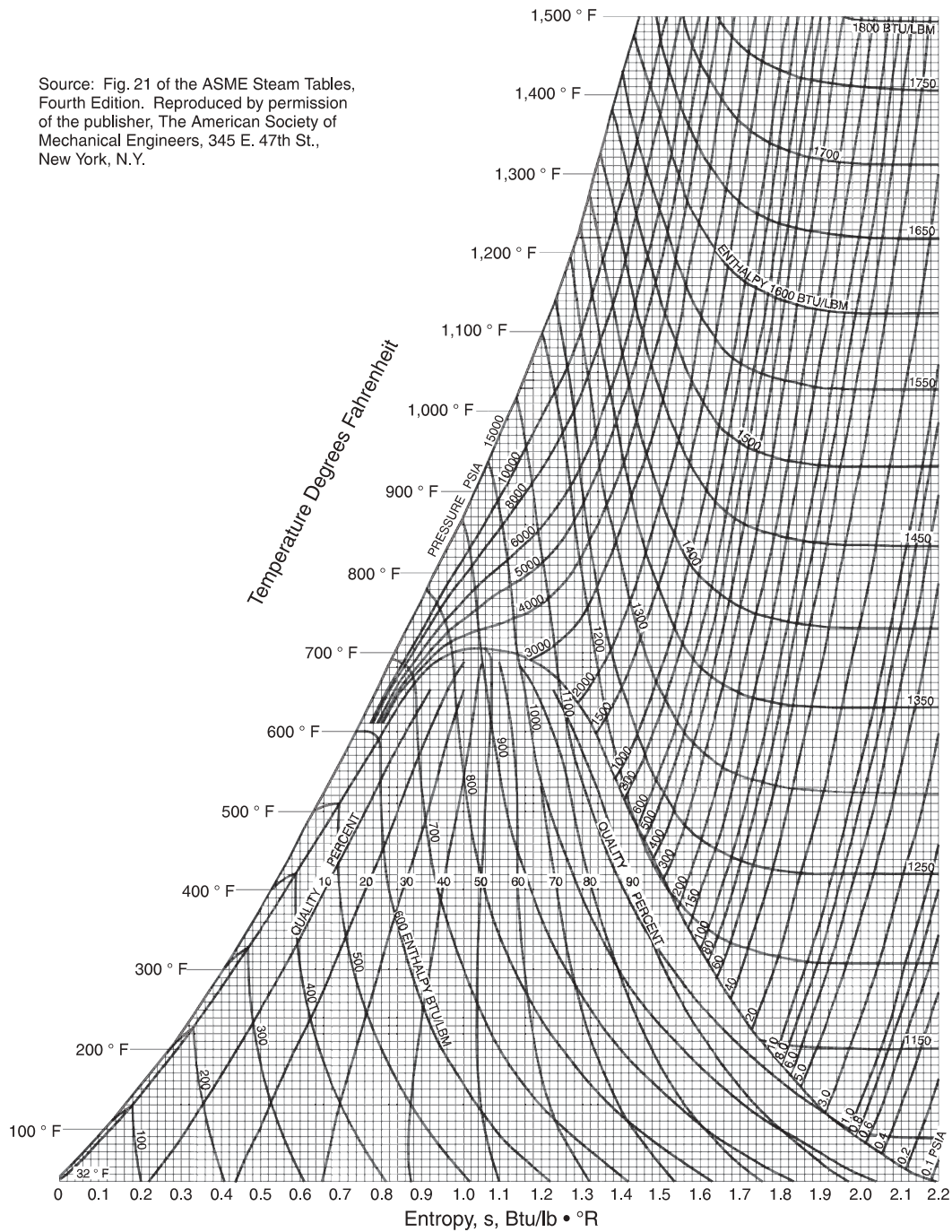
9.9 Diagrams for Water and Steam

Pressure-Enthalpy (p-H) Diagram (U.S. Customary Units)



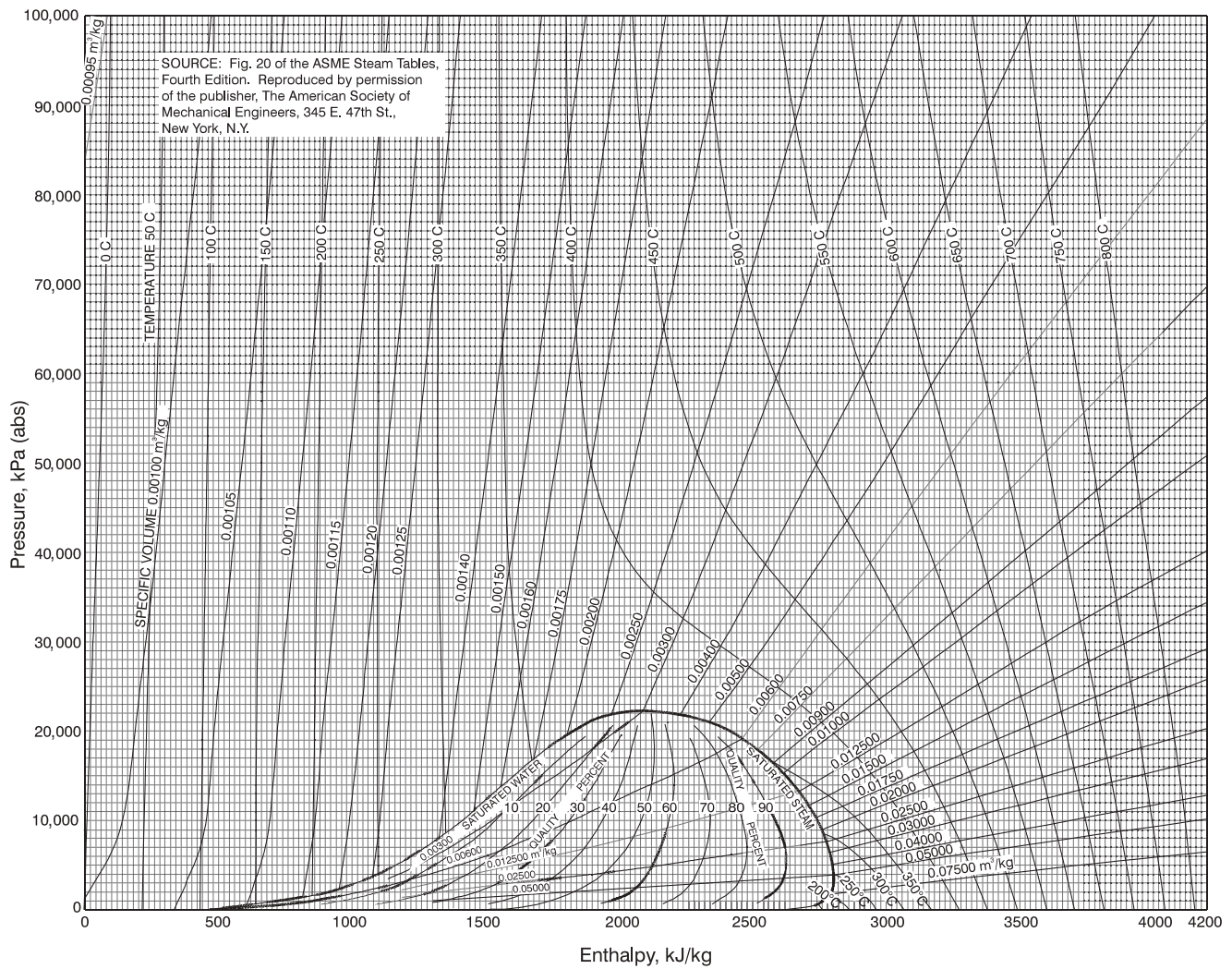
Temperature-Entropy (T-S) Diagram (U.S. Customary Units)

Source: Fig. 21 of the ASME Steam Tables, Fourth Edition. Reproduced by permission of the publisher, The American Society of Mechanical Engineers, 345 E. 47th St., New York, N.Y.



Source: ASME Steam Tables, 4th ed., New York: American Society of Mechanical Engineers, 1979.

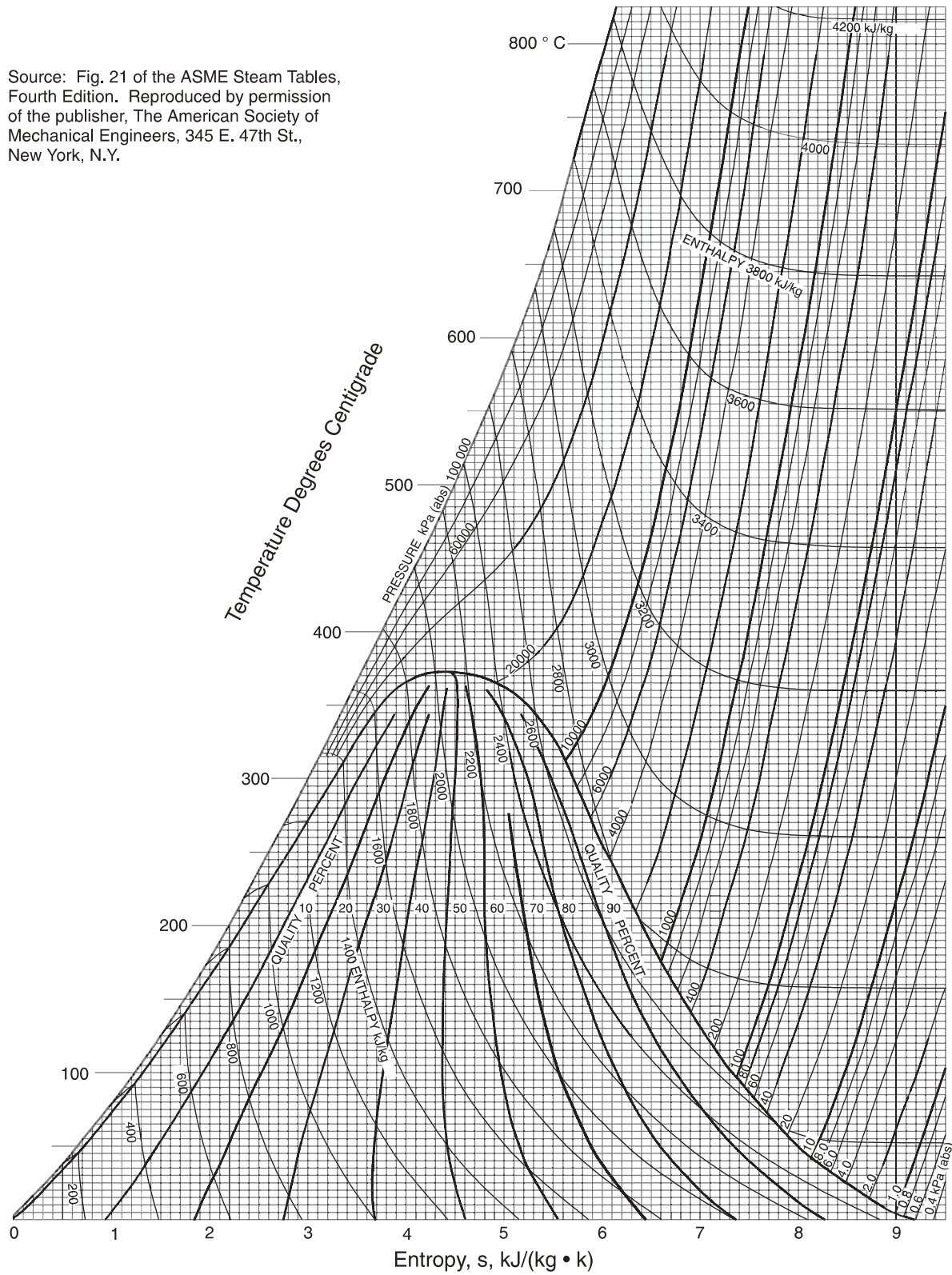
Pressure-Enthalpy (p-H) Diagram (SI Units)



Source: ASME Steam Tables, 4th ed., New York: American Society of Mechanical Engineers, 1979.

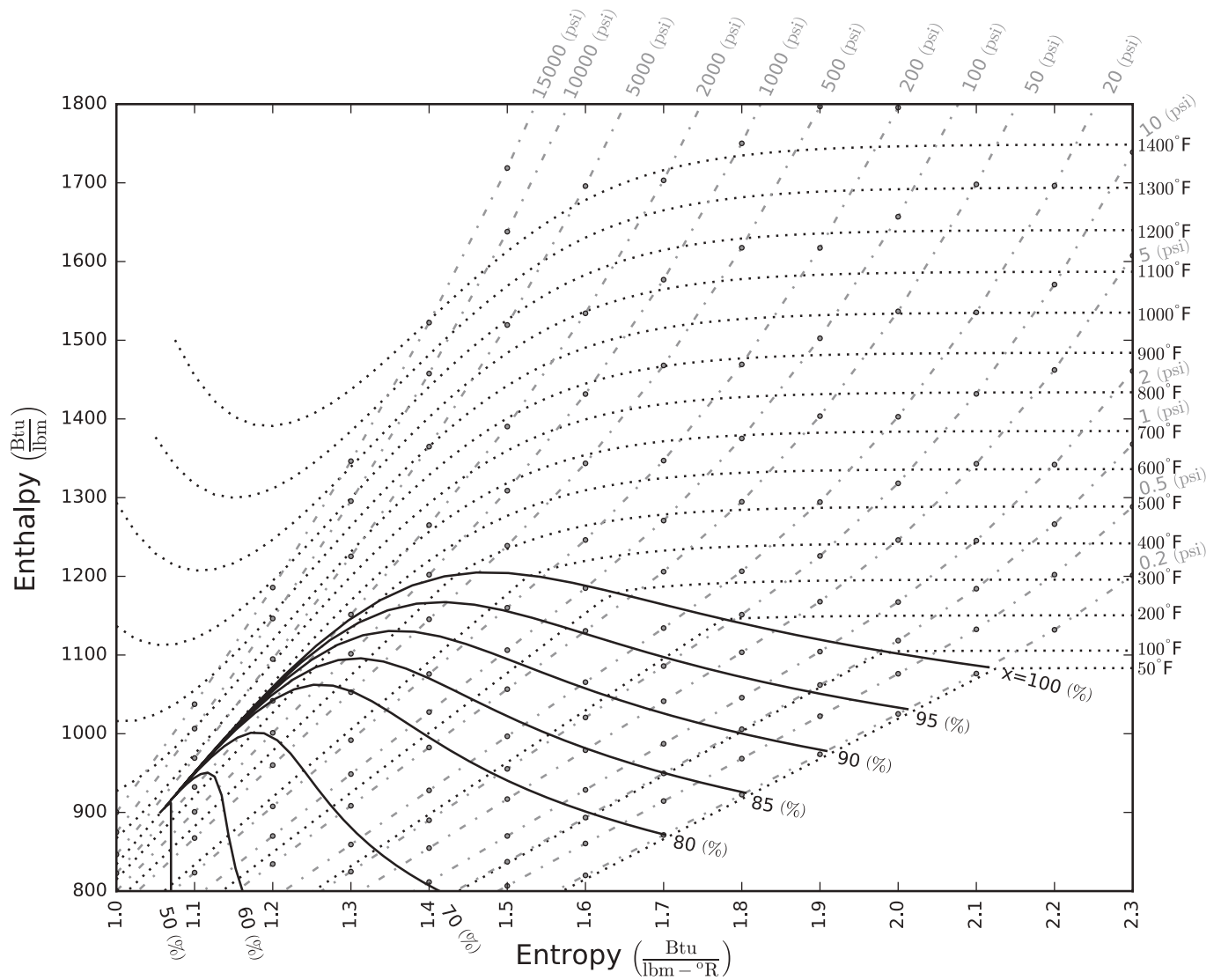
Temperature-Entropy (T-S) Diagram (SI Units)

Source: Fig. 21 of the ASME Steam Tables, Fourth Edition. Reproduced by permission of the publisher, The American Society of Mechanical Engineers, 345 E. 47th St., New York, N.Y.



Source: ASME Steam Tables, 4th ed., New York: American Society of Mechanical Engineers, 1979.

Mollier (h, s) Diagram for Steam (English Units)



Mollier (h, s) Diagram for Steam (Metric Units)

